Midterm #2 of EE101B

Winter, 2014

(Closed Book, 1 Hour 50 Min, Total 50 points)

Name : ________________

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Problem #1. [7 points] The electric field of an electromagnetic wave propagating in the free space is given by the following phasor form.

$$\vec{E}(z) = (\hat{x} + j\cdot\hat{y})\cdot 377\cdot e^{-j\frac{2\pi}{100}z} \quad [\frac{V}{m}].$$

(1) [3 points] Describe the polarization state of this wave.

Left - hand circular polarization.

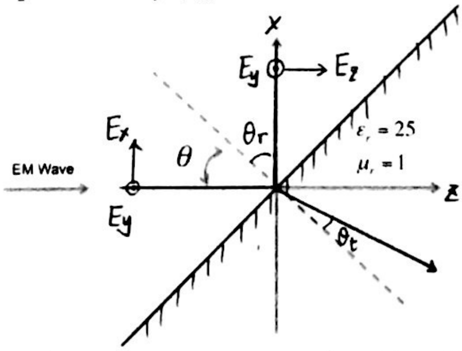
(2) [4 points] Find the associated magnetic field $\vec{H}(z)$. Note that the free space intrinsic impedance η_o is 377 [Ω].

$$\vec{H} = \frac{1}{\eta_o}\hat{z}\times\vec{E}$$

$$\vec{H} = \frac{1}{377}\hat{z}\times(\hat{x}+j\hat{y})\cdot 377\, e^{-j\frac{2\pi}{100}z} \quad (A/m)$$

$$\vec{H} = (\hat{y} - j\hat{x})\, e^{-j\frac{2\pi}{100}z} \quad (A/m)$$

Problem #2. [13 points] Suppose that the wave described in Problem #2 is obliquely incident on a dielectric medium with the angle θ as shown in the figure below. The medium has the relative permittivity (ε_r) of 25 and the relative permeability (μ_r) of 1.



$$\left\{\begin{array}{l} \frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \frac{1}{\sqrt{\varepsilon_r}} = \frac{1}{5} \\ \sin\theta_r = \sin\theta_i \end{array}\right.$$

$$\Rightarrow \theta_t = \sin^{-1}\left(\frac{1}{5}\sin 45°\right) = 8.13°$$

$$\theta_r = \theta_i = 45°$$

(1) [2 points] Draw two arrows representing the directions of the reflected and transmitted waves if θ = 45 degree. Please indicate the angles on the figure.

Problems are continued on next pag

(2) [6 points] Find the E-field of the **reflected** wave if θ = 45 degree. Equations below may be useful.

Transverse Electric (TE) wave	**Transverse magnetic (TM) wave**
E- field ⊥ to the incidence plane	E- field || to the incidence plane

$$\begin{cases} \Gamma_\perp = \dfrac{E^r_{\perp 0}}{E^i_{\perp 0}} = \dfrac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \\ \tau_\perp = \dfrac{E^t_{\perp 0}}{E^i_{\perp 0}} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \end{cases} \qquad \begin{cases} \Gamma_\parallel = \dfrac{E^r_{\parallel 0}}{E^i_{\parallel 0}} = \dfrac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \\ \tau_\parallel = \dfrac{E^t_{\parallel 0}}{E^i_{\parallel 0}} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \end{cases}$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{\cos\theta_i - \sqrt{\epsilon_r}\cos\theta_t}{\cos\theta_i + \sqrt{\epsilon_r}\cos\theta_t} = \frac{\cos 45^\circ - 5\cos 8.13^\circ}{\cos 45^\circ + 5\cos 8.13^\circ} = -0.75$$

$$\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{\cos\theta_t - \sqrt{\epsilon_r}\cos\theta_i}{\cos\theta_t + \sqrt{\epsilon_r}\cos\theta_i} = \frac{\cos 8.13^\circ - 5\cos 45^\circ}{\cos 8.13^\circ + 5\cos 45^\circ} = -0.56$$

$$\vec{E}_r = (\Gamma_\parallel \hat{z} + j\Gamma_\perp \hat{y}) \cdot 377 e^{-j\frac{2\pi}{100}x} \quad \left(\frac{V}{m}\right)$$

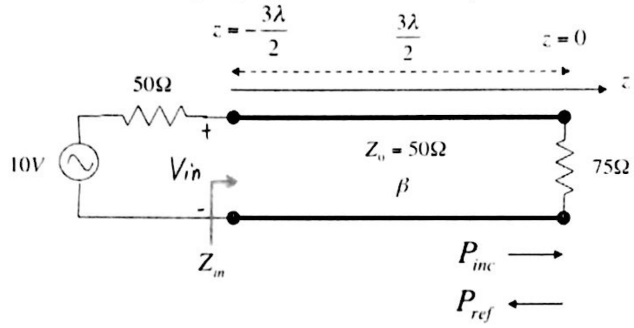
(3) [5 points] Is there any angle θ, which makes the reflected wave becomes linearly polarized? If yes, what is the angle θ?

Yes.

When the incident angle θ is equal to the Brewster angle θ_B, only $\hat{y}$ component exists in the reflected E field.

$$\theta_B = \tan^{-1}\sqrt{\frac{n_2}{n_1}} = \tan^{-1}\sqrt{\epsilon_r} = 78.69^\circ$$

Problem #3. [18 points] Consider the transmission line circuit shown below. Assume that transmission line is lossless with the propagation constant β=2π/λ.



(1) [8 points] Find the voltage V(z) as a function of z, where z is from −3λ/2 to 0.

Since $l = \frac{3}{2}\lambda$, $\tan(\beta l) = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{3\lambda}{2}\right) = 0$

$$Z_{in} = Z_o \frac{z_L + j\tan\beta l}{1 + jz_L \tan\beta l} = Z_L = 75\,\Omega$$

in the above equation, $z_L = \frac{Z_L}{Z_o}$

$$V_{in} = \frac{V_g}{Z_{in} + Z_g} Z_{in} = \frac{10}{50 + 75} \times 75 = 6\ V$$

Since $V_{in} = V(z = -\frac{3\lambda}{2}) = V_o^+ e^{j\beta \frac{3\lambda}{2}} + \Gamma V_o^+ e^{-j\beta\frac{3\lambda}{2}}$

$$= -(1 + \Gamma) V_o^+$$

$$\Rightarrow \quad V_o^+ = -\frac{V_{in}}{1 + \Gamma}$$

Also, $\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{75 - 50}{75 + 50} = 0.2$

$$V_o^+ = -\frac{6}{1 + 0.2} = -5\ V$$

Therefore, the total voltage on the transmission line as a function of the position z is :

$$V(z) = V_o^+ e^{-j\beta z} + \Gamma V_o^+ e^{+j\beta z}$$

$$= -5e^{-j\beta z} - e^{+j\beta z} \quad (V) \qquad -\frac{3\lambda}{2} \le z \le 0$$

Problems are continued on next page

(2) [4 points] What is the input impedance Z_{in} at $z=-3\lambda/2$ as indicated in the figure?

According to (1), $Z_{in} = Z_L = 75\,\Omega$.

(3) [6 points] Find the incident power P_{inc}, the reflected power P_{ref}, and the consumed power P_L at the load?

$$I_{in} = \frac{V_g}{Z_{in} + Z_g} = \frac{10}{50+75} = 0.08\ A$$

$$P_{inc} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0} = \frac{1}{2}\frac{5^2}{50} = 0.25\ W$$

$$P_{ref} = |\Gamma|^2 P_{inc} = 0.04\ P_{inc} = 0.01\ W$$

$$\text{or } P_{ref} = \frac{1}{2}\frac{|\Gamma V_0^+|^2}{Z_0} = 0.01\ W$$

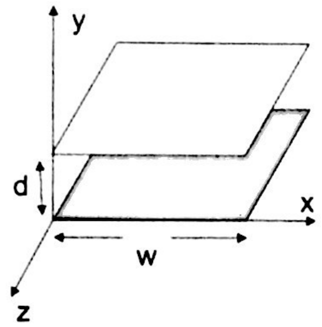
$$P_L = P_{inc} - P_{ref} = 0.25 - 0.01 = 0.24\ W$$

$$\text{or } P_L = \frac{1}{2}\frac{|V_L|^2}{Z_L} = \frac{1}{2}\frac{|(1+\Gamma)V_0^+|^2}{Z_L} = \frac{1}{2}\frac{6^2}{75} = 0.24\ W$$

$$\text{or } P_L = \frac{1}{2}|I_{in}|^2 Z_L = \frac{1}{2} \times 0.08^2 \times 75 = 0.24\ W$$

the transmission line is lossless.

Problem #4. [12 points] An EM wave is propagating in an air-filled parallel plate waveguide as shown below. The top and bottom plates of the waveguide are perfect conductors and the width w is assumed sufficiently larger than the thickness d, so that we can ignore the fringing effect near the waveguide edge. The wave has an electric field given by below.



$$\vec{E}(x,y,z)=\hat{y}\cdot E_o e^{-jk_o\cdot z}$$

where E_o is the constant, k_o is the wave number in the free space.

(1) [4 points] Show that the electric field specified satisfies the wave equation $\nabla^2\vec{E}+k_o^2\cdot\vec{E}=0$.

$$\nabla^2\vec{E} = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\vec{E}$$

$$= \frac{\partial^2}{\partial z^2}\hat{y}E_o e^{-jk_o z}$$

$$= -k_o^2 E_o e^{-jk_o z}\cdot\hat{y}$$

$$= -k_o^2\vec{E}$$

Therefore, $\nabla^2\vec{E}+k_o^2\vec{E}=0$ is valid

Problems are continued on next page

(2) [4 points] Using the Faraday's law, find the associated magnetic field $\vec{H}(x,y,z)$.

Faraday's law is given as

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

Since $\frac{\partial \vec{H}}{\partial t} = -j\omega \vec{H}$

$$\nabla \times \vec{E} = \hat{x}\left(-\frac{\partial E_y}{\partial z}\right) = \hat{x}\left(jk_0 E_0 e^{-jk_0 z}\right)$$

$$-j\omega\mu_0 \vec{H} = \hat{x}\left(\cancel{j}k_0 E_0 e^{-jk_0 z}\right)$$

$$\vec{H} = -\hat{x}\frac{k_0}{\mu_0\omega}E_0 e^{-jk_0 z} = -\hat{x}\frac{E_0}{\eta_0}e^{-jk_0 z}$$

(3) [2 points] Is this a TE, TM or TEM wave? Please explain.

TEM wave.

Explanation: Because $E_z = 0$, $H_z = 0$, this wave is a plane wave, i.e., TEM wave.

(4) [2 points] Find the wave impedance Z_w of this wave.

$$Z_W = -\frac{E_y}{H_x} = -\frac{E_0}{\frac{1}{\eta_0}E_0} = \eta_0 = 377\,\Omega$$