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Midterm #2 of EE101B

Winter, 2014

(Closed Book, 1 Hour 50 Min, Total 50 points)

Name :

UID :

$\omega t = 0: (1,0)$
$\omega t = \pi/2: (0,-1)$
$\omega t = \pi: (-1,0)$

Problem #1. [7 points] The electric field of an electromagnetic wave propagating in the free space is given by the following phasor form.

$$\vec{E}(z) = (\hat{x} + j\cdot\hat{y})\cdot 377\cdot e^{-j\frac{2\pi}{100}z} \quad [\frac{V}{m}]$$

(1) [3 points] Describe the polarization state of this wave.

Left-hand circularly polarized

(2) [4 points] Find the associated magnetic field $\vec{H}(z)$. Note that the free space intrinsic impedance η_o is 377 [Ω].

$\vec{H}(z) = \frac{\hat{k}\times\vec{E}(z)}{\eta_0} \quad \hat{k} = \hat{z}$

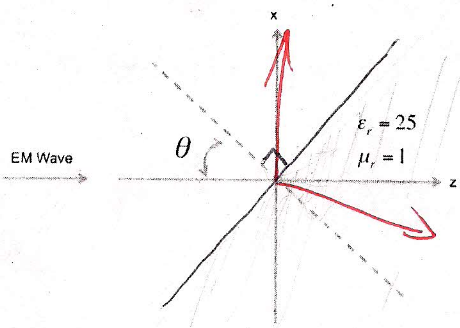
$$\boxed{\vec{H}(z) = (-j\hat{x}+\hat{y})\exp\left(-j\frac{2\pi}{100}z\right)\ (A/m)}$$

Problem #2. [13 points] Suppose that the wave described in Problem #1 is obliquely incident on a dielectric medium with the angle θ as shown in the figure below. The medium has the relative permittivity (ε_r) of 25 and the relative permeability (μ_r) of 1.

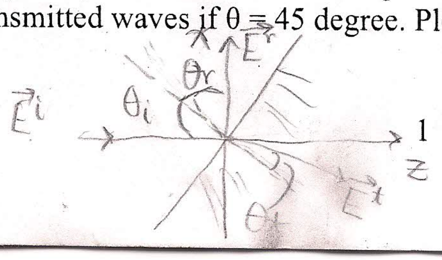
$n_1 \sin(\theta_i) = n_2 \sin(\theta_t)$

$\theta_t = \sin^{-1}\left[\frac{n_1}{n_2}\sin(\theta_i)\right]$

$\frac{n_1}{n_2} = \frac{c/u_{p1}}{c/u_{p2}} = \frac{\sqrt{\mu_{r1}\varepsilon_{r1}}}{\sqrt{\mu_{r2}\varepsilon_{r2}}}$



(1) [2 points] Draw two arrows representing the directions of the reflected and transmitted waves if θ = 45 degree. Please indicate the angles on the figure.



$\theta_i = \theta_r = 45°$

$\theta_t = 8.130°$

$\vec{E}^r$ $\vec{E}^t$ x z

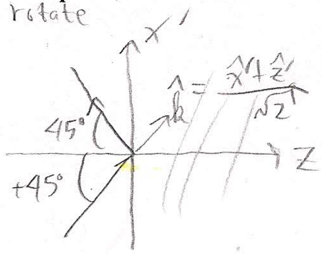
Problems are continued on next page

$$\frac{\eta_2}{\eta_1} = \frac{\sqrt{\mu_2/\varepsilon_2}}{\sqrt{\mu_1/\varepsilon_1}} = \sqrt{\varepsilon_1/\varepsilon_2} = \frac{1}{5}$$

Problems are continued from previous page

(2) [6 points] Find the E-field of the **reflected** wave if θ = 45 degree. Equations below may be useful. $-jk_1\cancel{z}$

rotate



Transverse Electric (TE) wave
E- field ⊥ to the incidence plane

$$\begin{cases} \Gamma_\perp = \dfrac{E^r_{\perp 0}}{E^i_{\perp 0}} = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \\ \tau_\perp = \dfrac{E^t_{\perp 0}}{E^i_{\perp 0}} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \end{cases}$$

Transverse magnetic (TM) wave
E- field || to the incidence plane

$$\begin{cases} \Gamma_\parallel = \dfrac{E^r_{\parallel 0}}{E^i_{\parallel 0}} = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \\ \tau_\parallel = \dfrac{E^t_{\parallel 0}}{E^i_{\parallel 0}} = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \end{cases}$$

$$\Gamma_\perp = \frac{\eta_2 \cos(\theta_i) - \eta_1 \cos(\theta_t)}{\eta_2 \cos(\theta_i) + \eta_1 \cos(\theta_t)} = -0.75 \qquad \Gamma_\parallel = -0.5625$$

$$\vec{\underset{\sim}{E}}{}^i_\perp = \hat{y} E^i_{\perp 0} \exp(-jk_1\cancel{z}) = \hat{y}\, j\, 377 \exp(-jk_1 z)$$

$$\vec{\underset{\sim}{E}}{}^r_\perp = -\hat{y}\, j\, 282.75 \exp(-jk_1 x)$$

$$\vec{\underset{\sim}{E}}{}^i_\parallel = \hat{x}\, 377 \exp(-jk_1 z) = \left[\hat{x}' \cos(45°) - \hat{z}' \sin(45°)\right] E^i_{\parallel 0} \exp(-jk_1 z)$$

$$\vec{\underset{\sim}{E}}{}^r_\parallel = \left[\hat{x}' \cos(\theta_r) + \hat{z}' \sin(\theta_r)\right] \Gamma_\parallel E^i_{\parallel 0} \exp(-jk_1 x) = \frac{\sqrt{2}}{2}\left(\hat{x}' + \hat{z}'\right)\cancel{\times}$$

$$= \hat{z}\, \Gamma_\parallel E^i_{\parallel 0} \exp(-jk_1 x) = -212.0625\, \hat{z} \exp(-jk_1 x)$$

$$\boxed{\vec{\underset{\sim}{E}}{}^r = -\left(j\hat{y}\, 282.75 + \hat{z}\, 212.0625\right) \exp\left(-j\frac{2\pi}{100} x\right) \ (\mathrm{V/m})}$$

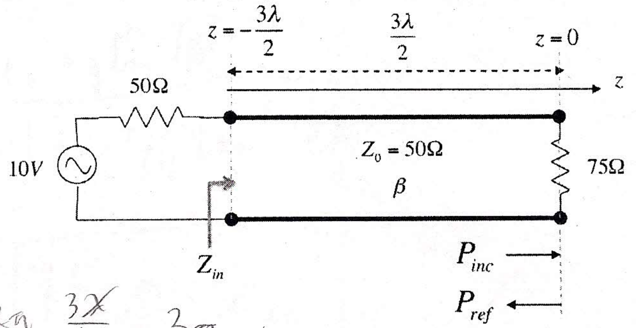
(3) [5 points] Is there any angle θ, which makes the reflected wave becomes linearly polarized? If yes, what is the angle θ?

$\boxed{\text{Yes.}}$ Brewster angle $\theta_{B\parallel} = \tan^{-1}\sqrt{\varepsilon_2/\varepsilon_1} = \boxed{78.69° = \theta_{B\parallel}}$

$$\Gamma_\parallel = 0 \rightarrow \eta_2 \cos(\theta_t) = \eta_1 \cos(\underset{\theta_{B\parallel}}{\theta_i})$$

Then $\vec{\underset{\sim}{E}}{}^r = -j\hat{y}\, 282.75 \exp\left(-j\frac{2\pi}{100} x\right) \left(\frac{V}{m}\right)$

Problem #3. [18 points] Consider the transmission line circuit shown below. Assume that transmission line is lossless with the propagation constant β=2π/λ.



$\Gamma = 0.2$

$\beta l = \frac{2\pi}{\lambda}\frac{3\lambda}{2} = 3\pi$

(1) [8 points] Find the voltage V(z) as a function of z, where z is from –3λ/2 to 0.

$$\underset{\sim}{V}(z) = V_0^+ \exp(-j\beta z) + \Gamma V_0^+ \exp(+j\beta z)$$

$$Z_{in} = Z_0 \frac{Z_L + j\tan(\beta l)}{1 + jZ_L\tan(\beta l)} = 75\ \Omega$$

$$V_0^+ = \frac{V_g Z_{in}}{Z_g + Z_{in}} \frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} = -5\ (V)$$

$$\underset{\sim}{V}(z) = V_0^+\left[\exp(-j\beta z) + \Gamma\exp(+j\beta z)\right]$$

$$= \boxed{-5\exp\left(-j\frac{2\pi}{\lambda}z\right) - \exp\left(+j\frac{2\pi}{\lambda}z\right) = \underset{\sim}{V}(z)\ \text{(Volts)}}$$

Problems are continued on next page

(2) [4 points] What is the input impedance Z_{in} at $z=-3\lambda/2$ as indicated in the figure?

$$Z_{in} = \frac{Z_L + j\tan(\beta l)}{1 + jZ_L\tan(\beta l)}$$

$$= \boxed{75\,\Omega = Z_{in}}$$

(3) [6 points] Find the incident power P_{inc}, the reflected power P_{ref}, and the consumed power P_L at the load?

$$P_{inc} = \frac{|V_0^+|^2}{2Z_0} \qquad \frac{25}{100} = \boxed{0.25\text{ W} = P_{inc}}$$

$$P_{ref} = -|\Gamma|^2 P_{inc} = \boxed{-0.01\text{ W} = P_{ref}}$$

$$V_L = V(z=0) = V_0^+(1+\Gamma) = (-5)(1.2) = -6\text{ V}$$

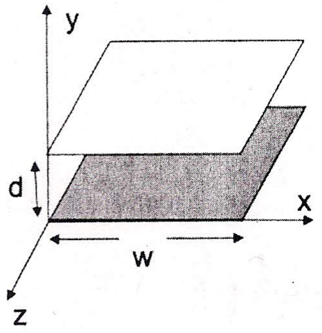
$$I_L = I(z=0) = \frac{V_0^+(1-\Gamma)}{Z_0} = \frac{-5(0.8)}{50} = -0.08$$

$$P_L = \frac{1}{2}\mathrm{Re}\{V_L I_L^*\} = \frac{1}{2}\mathrm{Re}\{6 \cdot 0.08\}$$

$$= \boxed{0.24\text{ W} = P_L}$$

Conservation of power

Problem #4. [12 points] An EM wave is propagating in an air-filled parallel plate waveguide as shown below. The top and bottom plates of the waveguide are perfect conductors and the width w is assumed sufficiently larger than the thickness d, so that we can ignore the fringing effect near the waveguide edge. The wave has an electric field given by below.



$$\vec{E}(x,y,z) = \hat{y} \cdot E_o e^{-jk_o \cdot z}$$

where E_o is the constant, k_o is the wave number in the free space.

(1) [4 points] Show that the electric field specified satisfies the wave equation $\nabla^2 \vec{E} + k_o^2 \cdot \vec{E} = 0$

$$\nabla^2 \vec{E} = \hat{y}\, E_0 e^{-jk_0 z} (-jk_0)^2$$

$$= \hat{y}\, -k_0^2 E_0{}^{-jk_0 z}$$

$$k_0^2 \vec{E} = \hat{y}\, k_0^2 E_0 e^{-jk_0 z}$$

$$\nabla^2 \vec{E} + k_0^2 \vec{E} = 0$$

Problems are continued on next page

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(2) [4 points] Using the Faraday's law, find the associated magnetic field $\vec{H}(x,y,z)$.

$\nabla \times \underset{\sim}{E} = -j\omega\mu \underset{\sim}{H}$ $\quad \nabla \times \underset{\sim}{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_0 e^{-jk_0 z} & 0 \end{vmatrix}$

$= \hat{x}\left[+(+jk_0)E_0 e^{-jk_0 z}\right]$

$= \hat{x}\, jk_0 E_0 e^{-jk_0 z}$

$\hat{x}\, \cancel{j} k_0 E_0 e^{-jk_0 z} = -\cancel{j}\omega\mu \underset{\sim}{H} \rightarrow \underset{\sim}{H} = -\hat{x}\frac{k_0}{\omega\mu}E_0 e^{-jk_0 z}$

$\rightarrow \boxed{\vec{H}(x,y,z) = -\hat{x}\frac{k_0}{\omega\mu_0}E_0 e^{-jk_0 z} \quad (A/m)}$ $\quad \frac{k_0}{\omega\mu_0} = \frac{1}{377}\ S$

(3) [2 points] Is this a TE, TM or TEM wave? Please explain.

TEM: $\vec{E}, \vec{H}$ fields are always perpendicular to direction of propagation in $+\hat{z}$ direction.

$E_z = H_z = 0$.

(4) [2 points] Find the wave impedance Z_w of this wave.

$Z_w = \frac{|\vec{E}|}{|\vec{H}|} = \frac{E_0}{\frac{k_0}{\omega\mu_0}E_0}$

$= \boxed{\frac{\omega\mu_0}{k_0} = Z_w}$

TEM $\longrightarrow$ $Z_w = \sqrt{\mu_0/\varepsilon_0} \approx 377\ \Omega$

End of Exam