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Midterm #1 of EE101B

Winter, 2014
(Closed Book, 1 Hr)

Name :

UID :

Problem #1. (10 points) Please answer following questions.

(1) [5 points] The left block below shows WRONG four Maxwell's equations. Please correct them and rewrite CORRECT four Maxwell's equations on the right block.

WRONG Maxwell's Eqs.	CORRECT Maxwell's Eqs.
$\nabla \cdot \vec{E} = \rho$	$\nabla \cdot \overleftrightarrow{D} = \rho$
$\nabla \cdot \vec{H} = \rho$	$\nabla \cdot \overleftrightarrow{B} = 0$
$\nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J}$	$\nabla \times \overleftrightarrow{H} = \overleftrightarrow{J} + \frac{\partial \overleftrightarrow{D}}{\partial t}$
$\nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$	$\nabla \times \overleftrightarrow{E} = -\frac{\partial \overleftrightarrow{B}}{\partial t}$

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(2) [5 points] Briefly describe the physical meaning of the intrinsic impedance η in a uniform plane electromagnetic wave. What is its unit?

η relates magnitude (and phase difference in lossy media) between the E and H fields in the wave. Specifically, $\overleftrightarrow{H}(z,t) = \hat{k} \times \frac{\overleftrightarrow{E}(z,t)}{\eta}$, where $\hat{k}$ is the propagation unit vector.

$[\eta] = \Omega$

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Problem #2. [20 points] The electric field of an electromagnetic wave propagating in a medium is given by

$$\vec{E}(x,y,z,t) = \hat{x}\cdot 4\cos\left(2\pi\cdot 3\times10^{6}t - \frac{2\pi}{50}z\right)\ [\frac{V}{m}],$$

where t is time in second and z is in meter. Please answer following questions. Make sure to write units if they are required.

(1) [2 points] Is this medium lossless or lossy?

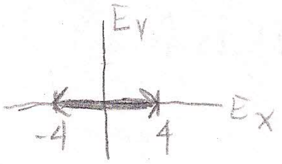
Lossless since there is no $e^{-\alpha z}$ decay term.

(2) [2 points] What is the propagating direction of this wave?

$+\hat{z}$ direction

(3) [2 points] Is this wave linearly polarized or circularly polarized or elliptically polarized?

Linearly polarized



(4) [2 points] What is the frequency, f?

$f = \frac{\omega}{2\pi} = 3\ MHz = f$

(5) [2 points] What is the wavelength, λ inside the medium?

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2\pi/50} = 50\ m = \lambda$

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(6) [2 points] What is the phase velocity, v_p?

$v_p = \lambda f = 150\times10^{6}\ m/s = 1.5\times10^{8}\ \frac{m}{s} = v_p\ (= c/2)$

(7) [2 points] Suppose this medium is nonmagnetic (relative permeability $\mu_r = 1$), what is the relative permittivity ε_r (also called dielectric constant)? We can use he following equation: $k = \beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v_p}$.

$$\frac{c}{2} = V_p = \frac{c}{\sqrt{\varepsilon_r}} \rightarrow \frac{1}{2} = \frac{1}{\sqrt{\varepsilon_r}}$$

$$\rightarrow \boxed{\varepsilon_r = 4}$$

(8) [2 points] Find the associated magnetic field $\vec{H}(x,y,z,t)$. Assume that the intrinsic impedance is η.

$\mu_r = 1 \rightarrow \eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{2}$ $\quad \eta_0 = 377\ \Omega$

$$\vec{H}(x,y,z,t) = \frac{\hat{z} \times \vec{E}(x,y,z,t)}{\eta} = \hat{y}\frac{4}{\eta}\cos\left(2\pi \times 3\times 10^6 t - \frac{2\pi}{50}z\right)\left(\frac{A}{m}\right) = \hat{y}\frac{8}{\eta_0}\cos\left(2\pi \times 3\times 10^6 t - \frac{2\pi}{50}z\right)(A/m)$$

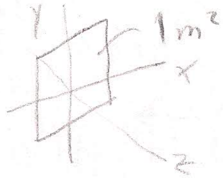
$$= \hat{y}\, 21.2\cos\left(2\pi \times 3\times 10^6 t - \frac{2\pi}{50}z\right)(mA/m)$$

(9) [2 points] What is the phasor form $\vec{E}(x,y,z)$ of this electric field?

$$\vec{E}(x,y,z) = \hat{x}\, 4 \exp\left(-j\frac{2\pi}{50}z\right)(V/m)$$

(10) [2 points] Suppose a 1 m^2 surface area on the x-y plane. What is the time-averaged power flow over this area? Assume that the intrinsic impedance is η.

$$\vec{S}_{av} = \hat{z}\frac{|E|^2}{2\eta} = \hat{z}\frac{8}{\eta}\ (W/m^2) \longrightarrow$$



$$\boxed{P_{av} = \frac{8}{\eta}\ (W)}$$

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