EE161 Midterm #2 Solutions (Fall 2006)

1. In this problem, we use the microstrip as a quarter wavelength transformer to match the impedance of a coax line to a resistive load.

   (a) The characteristic impedance of the coax line is

$$Z_{coax} = \eta \frac{\ln \frac{b}{a}}{2\pi} = \frac{\eta}{\sqrt{\varepsilon_r}} \frac{\ln \frac{b}{a}}{2\pi} = \frac{120\pi}{\sqrt{2.25}} \frac{\ln \frac{1.75}{0.5}}{2\pi} = 50.11\Omega$$

   (b) Parallel plate waveguide model is used to analyze the microstrip impedance transformer, therefore, the dominant mode is TEM mode.

$$\beta = k = \omega\sqrt{\mu\varepsilon} = 2\pi f \frac{\sqrt{\varepsilon_r}}{C} = 2\pi \times 3 \times 10^9 \frac{\sqrt{10.3}}{3 \times 10^8} = 201.65 \quad \text{rad/m}$$

$$u_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{C}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{10.3}} = 9.35 \times 10^8 \quad \text{m/s}$$

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{201.65} = 3.12 \quad \text{cm}$$

   (c) The microstrip works as a quarter wavelength transformer, therefore,

$$L = \frac{\lambda_g}{4} = \frac{3.12\text{cm}}{4} = 7.8 \quad \text{mm}$$

$$Z_{ms} = \sqrt{Z_L Z_{coax}} = \sqrt{18 \times 50.11} = 30.03 \quad \Omega$$

$$Z_{ms} = \frac{\eta_0 t}{\sqrt{\varepsilon_r} W} = \frac{120\pi \times 10^{-3}}{\sqrt{10.3} \times W} = 30.03$$

$$\Rightarrow \quad W = 3.91 \quad \text{mm}$$

2. 5GHz is the cutoff frequency of the dominant mode.

   (a) Since $b < 1.5b = a < 2b$, the dominant mode is TE_{10} mode.

$$f_{c,\text{TE10}} = \frac{C}{2a} \quad \Rightarrow \quad a = \frac{C}{2f_{c,\text{TE10}}} = \frac{3 \times 10^8}{2 \times 5 \times 10^9} = 3\,\text{cm}$$

$$b = a/1.5 = 2\,\text{cm}$$

(b) Because $a > b$, the dominant mode is TE_{10}, and the cutoff frequency is $f_{c,\text{TE10}} = 5\,\text{GHz}$.

Because $a = 1.5b < 2b$, the 2nd lowest mode is TE_{01}, and the cutoff frequency is

$$f_{c,\text{TE01}} = \frac{C}{2b} = \frac{3 \times 10^8}{2 \times 2 \times 10^{-2}} = 7.5\,\text{GHz}$$

For $\text{TE}_{11}/\text{TM}_{11}$ modes,

$$f_{c,\text{TE11/TM11}} = \frac{C}{2}\sqrt{(\frac{1}{a})^2 + (\frac{1}{b})^2} = \frac{3 \times 10^8}{2}\sqrt{(\frac{1}{0.03})^2 + (\frac{1}{0.02})^2} = 9.01\,\text{GHz}$$

For TE_{20} modes,

$$f_{c,\text{TE20}} = \frac{C}{a} = \frac{3 \times 10^8}{0.03} = 10\,\text{GHz}$$

Therefore, the propagating modes are TE_{10}, TE_{01}, and $\text{TE}_{11}/\text{TM}_{11}$.

(c) The dominant mode is TE_{10}.

$$k = 2\pi f/C = 2\pi \times 9.7 \times 10^9/3 \times 10^8 = 203.16, \quad k_c = 2\pi f_c/C = 2\pi \times 5 \times 10^9/3 \times 10^8 = 104.72$$

$$\Rightarrow \quad \beta = \sqrt{k^2 - k_c^2} = \sqrt{203.16^2 - 104.72^2} = 174.1\,\text{rad/m}$$

$$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 9.7 \times 10^9}{174.1} = 3.5 \times 10^8\,\text{m/s}$$

$$u_g = \frac{(C/\sqrt{\varepsilon_r})^2}{u_p} = \frac{(3 \times 10^8)^2}{3.5 \times 10^8} = 2.57 \times 10^8\,\text{m/s}$$

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{174.1} = 3.6\,\text{cm}$$

$$Z_{\text{TE10}} = \frac{k\eta}{\beta} = \frac{203.16 \times 120\pi}{174.1} = 439.9\,\Omega$$

(d) Two lowest modes are TE_{10} and TE_{01}.

$$u_{g\text{TE10}} = 2.57 \times 10^8\,\text{m/s}$$

$$u_{g\text{TE01}} = \frac{C}{\sqrt{\varepsilon_r}}\sqrt{1 - (\frac{f_{c,\text{TE01}}}{f})^2} = \frac{3 \times 10^8}{\sqrt{1}}\sqrt{1 - (\frac{7.5 \times 10^9}{9.7 \times 10^9})^2} = 1.9 \times 10^8\,\text{m/s}$$

$$\frac{L}{u_{g\text{TE01}}} - \frac{L}{u_{g\text{TE10}}} = 0.2\,\text{ns} \quad \Rightarrow \quad L = 0.2 \times 10^{-9}/(\frac{1}{1.9 \times 10^8} - \frac{1}{2.57 \times 10^8}) = 0.145\,\text{m}$$

3. Since the waveguide is filled with air, $\varepsilon_r = 1$.

(a) $a > 2b \quad \Rightarrow \quad$ dominant mode is TE_{10}.

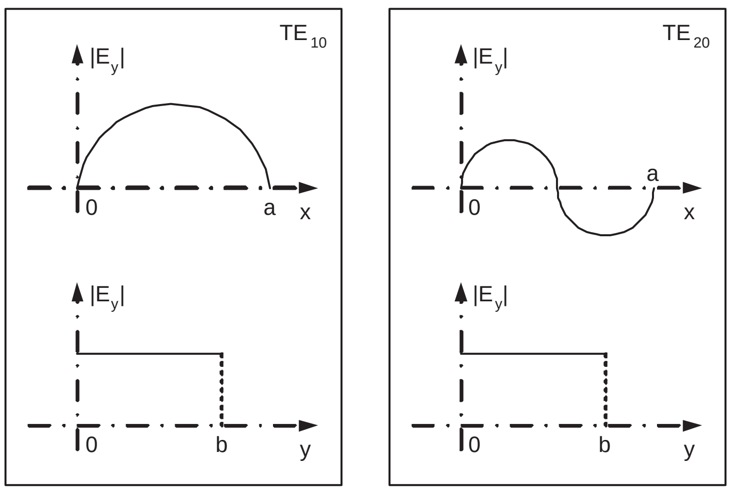
$$f_{c,\mathrm{TE}_{10}} = \frac{C}{2a} = \frac{3 \times 10^8}{2 \times 0.05} = 3\,\mathrm{GHz}$$

The second lowest mode is TE_{20}.

$$f_{c,\mathrm{TE}_{20}} = \frac{C}{a} = \frac{3 \times 10^8}{0.05} = 6\,\mathrm{GHz}$$

(b) Both TE_{10} and TE_{20} modes have only E_y component:

$$E_{y,\mathrm{TE}_{10}} \propto \sin\frac{\pi x}{a} \qquad E_{y,\mathrm{TE}_{20}} \propto \sin\frac{2\pi x}{a}$$



(c) To excite TE_{20} mode, place current probes at the maximum E-field. Therefore, $x = a/4 = 1.25\,\mathrm{cm}$ and $x = 3a/4 = 3.75\,\mathrm{cm}$.

4. It is assumed that $a, d > b$, therefore, the dominant mode is TE_{101}.

(a) For the dominant mode,

$$Q \approx \frac{f_r}{\Delta f} \quad \Rightarrow \quad f_{r,TE101} = Q\Delta f = 10000 \times 0.9 \times 10^6 = 9\,\mathrm{GHz}$$

(b) The waveguide is filled with air, therefore, $\varepsilon_r = 1$.

$$f_{r,mnp} = \frac{C}{\sqrt{\varepsilon_r}} \frac{\sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 + (\frac{p\pi}{d})^2}}{2\pi} \quad \Rightarrow$$

$$f_{r,TE101} = \frac{3 \times 10^8}{\sqrt{1}} \frac{\sqrt{(\frac{\pi}{0.04})^2 + (\frac{\pi}{d})^2}}{2\pi} = 9 \times 10^9 \quad \Rightarrow \quad d = 1.83\,\mathrm{cm}$$

(c) Since $a > 2b, d > 2b, a > d$, the second lowest mode is TE_{201}

$$f_{r,TE201} = \frac{3 \times 10^8}{\sqrt{1}} \frac{\sqrt{(\frac{2\pi}{0.04})^2 + (\frac{\pi}{0.0183})^2}}{2\pi} = 11.11\,\text{GHz}$$

and the third lowest mode is TE_{102}

$$f_{r,TE102} = \frac{3 \times 10^8}{\sqrt{1}} \frac{\sqrt{(\frac{\pi}{0.04})^2 + (\frac{2\pi}{0.0183})^2}}{2\pi} = 16.8\,\text{GHz}$$

(d) Now the cavity resonator is filled with dielectric $\varepsilon_r = 4$.

$$f_{r,TE101} = \frac{3 \times 10^8}{\sqrt{4}} \frac{\sqrt{(\frac{\pi}{0.04})^2 + (\frac{\pi}{d})^2}}{2\pi} = 9 \times 10^9 \quad \Rightarrow \quad d = 0.85\,\text{cm}$$

Since $a > b, d > b, a > 2b, d < 2b$, the second lowest mode is TM_{110}

$$f_{r,TM110} = \frac{3 \times 10^8}{\sqrt{4}} \frac{\sqrt{(\frac{\pi}{0.04})^2 + (\frac{\pi}{0.008})^2}}{2\pi} = 9.56\,\text{GHz}$$

and the third lowest mode is TE_{201}

$$f_{r,TE201} = \frac{3 \times 10^8}{\sqrt{4}} \frac{\sqrt{(\frac{2\pi}{0.04})^2 + (\frac{\pi}{0.0183})^2}}{2\pi} = 9.59\,\text{GHz}$$