EE161 Fall Quarter 2003, Prof. Brown
Midterm Exam, 3 November 2003
One Hour, Closed Book, Closed Notes, Calculators OK

1. Maxwell's dynamic equations
Consider a long, straight wire with round cross section made of a semimetal having conductivity σ, permittivity ε and permeability $\mu = \mu_0$. Assume that the electric field in the wire is sinusoidal in time with frequency ω, and is uniform everywhere (amplitude $\tilde{E}_0$) and pointed along the axis of the wire ?

a) Write down phasor expressions for the current densities flowing in the wire in terms of $\tilde{E}_0$ and consistent with Maxwell's equations.
b) Use a theorem of vector calculus to integrate Ampere's generalized equation over the cross section of the wire and equate this to a line integral over the periphery of the wire.
c) Use the results of b) to find a relationship between the azimuthal magnetic field on the surface of the wire (amplitude $\tilde{H}_0$) and the phasor electric field inside the wire $\tilde{E}_0$.

2. Plane Wave Propagation
A uniform plane wave having an electric field given below is propagating in seawater ($\varepsilon_r = 72, \mu_r = 1, \sigma = 4$): $\vec{E} = \hat{x} \cdot 10e^{-\alpha z}\cos(10^7 t - \beta z) + \hat{y} \cdot 10e^{-\alpha z}\cos(10^7 t - \beta z)$ [V/m]

a) What is the linear frequency of the wave?
b) What is the direction of propagation?
c) What is the polarization? Justify your answer.
d) Find the phase constant, the skin depth and the intrinsic impedance.
e) By what factor does the *power* decrease as the wave moves a distance of 1 m?
f) Find the magnetic field H in phasor form first, and then in instantaneous time domain. (Hint: Do not forget that the intrinsic impedance is complex)

3. Snell's Law and Fresnel Equations
Suppose plane wave radiation is propagating in glass ($\varepsilon r = 4.0$) and impinges upon a plane interface with silicon ($\varepsilon r = 12.0$). Both materials are assumed to be lossless, nonmagnetic dielectrics.

a) As a function of incident angle, does the magnitude of the reflection coefficient ever go to 1.0? If so, at what angle and what polarization (parallel, perpendicular to plane of incidence). If not, what are the maximum magnitude, and the angle and polarization where this occurs?
b) Does the magnitude of reflection coefficient ever go to 0 ? If not, what are the minimum magnitude, and the angle and polarization where this occurs? If so, at what incident angle and what polarization does the zero reflection occur?
c) Evaluate the field and power reflection coefficients at normal incidence.
d) Evaluate the field and power transmission coefficients at normal incidence.

EE161 Fall 2003 Midterm Solutions

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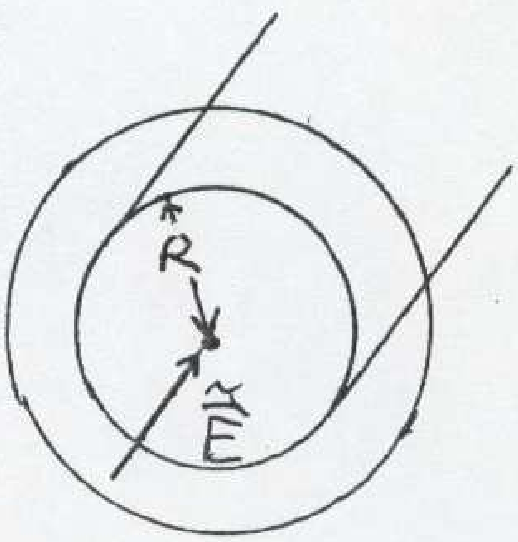
① Maxwell's dynamic equations $\boxed{25 \text{ points}}$



(a) From Ampere's generalized equation we know there are two current densities in the wire.

$\tilde{\vec{J}}_c = \sigma \tilde{\vec{E}}$, conduction current 4

$\tilde{\vec{J}}_D = j\omega\varepsilon \tilde{\vec{E}}$, displacement current 4

(b) If $\tilde{\vec{E}}$ is inside wire is uniform and directed along wire axis, then we can integrate Ampere's equation using $\boxed{\text{Stoke's theorem}}$ 4 over cross section of wire and along circular path at wire surface. By symmetry $\tilde{H}$ will not depend on position along this path so will be a constant, $\tilde{H}_o$. So Stokes theorem yields

$$\int \vec{\nabla}\times\tilde{\vec{H}}\cdot d\vec{S} = \oint \tilde{\vec{H}}\cdot d\vec{l} = \underbrace{\boxed{\tilde{H}_o \oint d\vec{l}}}_{4} \underset{\uparrow}{=} \boxed{\tilde{H}_o \cdot 2\pi R}$$

on circle at surface of wire.

Integration of other side of Ampere's equation yields

$$\underbrace{\boxed{\int (\tilde{\vec{J}}_c + \tilde{\vec{J}}_D)\cdot d\vec{S}}}_{4} = (\sigma + j\omega\varepsilon)\int \tilde{\vec{E}}\cdot d\vec{S} = (\sigma + j\omega\varepsilon)\tilde{E}_o \cdot A = \boxed{(\sigma + j\omega\varepsilon)\tilde{E}_o \pi R^2}$$

(c) Equating these two we get: $\boxed{\tilde{H}_o = (\sigma + j\omega\varepsilon)\tilde{E}_o \frac{R}{2}}$ 5

Since $\sigma\tilde{E}_o = \tilde{J}$, and $\tilde{J} = \frac{\tilde{I}}{\pi R^2}$, this can be written as:

$$\tilde{H}_o = \frac{\tilde{I}}{2\pi R} + j\omega\varepsilon\tilde{E}_o \frac{R}{2}$$

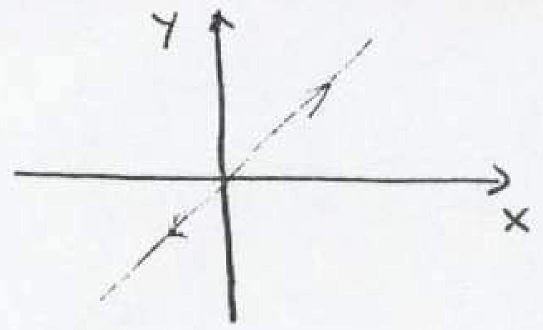
Problem 2 (Total: 35 pts)

a) linear frequency

$$f = \frac{\omega}{2\pi} = \frac{10^7}{2\pi} = 1.59\times10^6 \text{ hz} = \boxed{1.59 \text{ Mhz} = f}$$ (3)

b) Direction of propagation: $\boxed{+z}$ (2)

c) Polarization: $\boxed{\text{linear}}$ in x & y, as the amplitudes in the x & y directions are equal. (5)



d) We first need to find if the medium is a good conductor

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\epsilon} = \frac{4}{10^7 \times \frac{10^{-9}}{36\pi}\times 72} = \boxed{200\pi >>> 1 \Rightarrow \text{good conductor}}$$ (3)

$$\beta = \sqrt{\pi f \mu \sigma} = \boxed{5.01 \text{ rd/m}}$$ (3)

$\delta = \frac{1}{\alpha}$, but $\alpha = \beta$ (since good conductor)

$$= \frac{1}{5.01} = \boxed{1.99 \text{ m} = \delta}$$ (3)

$$\eta = (1+j)\frac{\alpha}{\cancel{\beta}\sigma} \simeq 1.25(1+j) = 1.767 e^{j\pi/4} \Rightarrow \boxed{\eta = 1.767 e^{j\pi/4}\ \Omega}$$ (3)

e) $H = \frac{1}{\eta}\vec{a_z}\times E$ (6)

$$\boxed{H(z) = -5.6 e^{-j(\beta z + \pi/4)} e^{-\alpha z}\vec{a_x} + 5.6 e^{-j(\beta z+\pi/4)} e^{-\alpha z}\vec{a_y}}$$

$$\boxed{H(z,t) = -5.6 e^{-\alpha z}\cos(\omega t - \beta z - \pi/4)\vec{a_x} + 5.6 e^{-\alpha z}\cos(\omega t - \beta z - \pi/4)}$$ (3)

e) power decreases by a factor $\boxed{e^{-2\alpha}} = 4.68\times10^{-5}$ times (4)

30 total ✓✓✓

3) Glass - Silicon Interface

a) From Snell's law, we expect total internal reflection where

$$\theta_i \geq \theta_{i,c} = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \sin^{-1}\sqrt{\frac{12}{4}}$$ which has no solution

But from Fresnel equations for $\theta_i = 90°$, $\cos\theta_i = 0$ and

4 $\boxed{\Gamma_\perp = \frac{-n_1\cos\theta_t}{n_1\cos\theta_t} = -1}$ and $\boxed{\Gamma_\parallel = \frac{n_2\cos\theta_t}{n_2\cos\theta_t} = +1}$ 4

So yes magnitude of Γ goes to 1.0 for <u>both</u> $\perp$ and $\parallel$ polarizations and for $\theta_i = 90°$ 2

(b) We know $\Gamma_\parallel$ goes to zero $\boxed{\text{(Brewster effect)}}$ for $\theta_i \equiv \theta_{i,B} = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$

So in this case $\boxed{\theta_{i,B} = \tan^{-1}\sqrt{\frac{12}{4}} = \tan^{-1}\sqrt{3} = 60°}$ 3 for $\boxed{\parallel \text{ polarization only}}$ 3

(c) At normal incidence $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1/\sqrt{12} - 1/\sqrt{4}}{1/\sqrt{12} + 1/\sqrt{4}} = \frac{1-\sqrt{3}}{1+\sqrt{3}} = \boxed{-0.27}$ 4

Power reflection $= R = |\Gamma|^2 = \boxed{0.072}$ 3

(d) $\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2/\sqrt{12}}{1/\sqrt{4} + 1/\sqrt{12}} = \frac{2}{\sqrt{3}+1} = \boxed{+0.73}$ 4

Power transmission $\equiv T = |\tau|^2 \frac{\eta_1}{\eta_2} = |0.73|^2\sqrt{3} = \boxed{0.928}$ 3

As check, note $R + T = 0.072 + 0.928 = 1.0$!