EE 141 – Midterm
Winter 2014

02/12/2014
Duration: 1 hour and 40 minutes

The midterm is closed book and closed lecture notes. No calculators.
You can use a single page of handwritten notes.
Please carefully justify all your answers.

Let us consider the problem of controlling a robot performing minimally invasive surgery. Each joint of the robot is actuated by a DC motor whose electrical circuit is depicted in Figure 1.

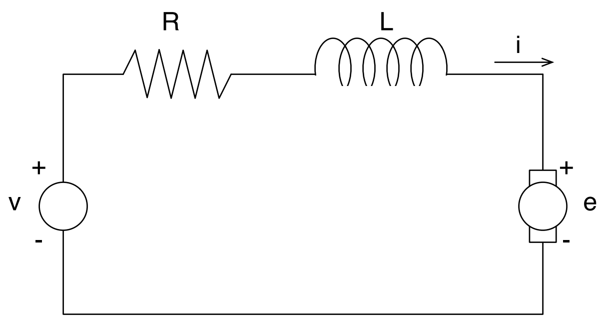

Figure 1: Electrical component of a DC motor.

1. Treating v and e as arbitrary voltages, write the linear differential equation modeling the evolution of the current.

2. Write the linear differential equation modeling the evolution of the motor shaft position θ knowing that the shaft's moment of inertial is J and that there are two torques applied on the shaft: a damping torque proportional to the shaft's angular velocity with proportionality constant a; and an electrical torque proportional to the current i with proportionality constant b.

3. Knowing that the voltage e is given by $b\dot{\theta}$, compute the transfer function from v to θ.

4. Using the following values for the parameters $J = 2$, $a = 1$, $L = 4$, $R = 2$, $b = 1$, determine if this system is stable.

5. Can you stabilize this system using a proportional controller in a unit feedback loop? If so, determine all the possible values for the controller gain.

6. Which value would you pick for the controller gain in order to place the closed-loop poles at $-0.5, -0.25 \pm 0.25j$?

7. Will the system be able to track step inputs using the gain computed in the previous question?

8. Verify the answer to the preceding question by computing what is the output of the closed-loop system in the time domain when the input is a step.

9. If you neglect the effect of the pole at -0.5, what is the rise-time and the settling-time?

10. For a general second order system with no zeros, sketch the region in the complex plane where the poles could be placed so as to ensure that the rise-time and the settling-time are no greater than the values you obtained in the previous question.