EE141 Winter 2014

FINAL

Problem 1

$$\tau_1 \dot{T} = -T + u \longrightarrow (1)$$
$$\tau_2 \ddot{z} + \dot{z} = aT + \omega \longrightarrow (2)$$

1. The Laplace transform of the two differential eqns gives:

$$\tau_1 s \mathcal{T}(s) = -\mathcal{T}(s) + U(s) \longrightarrow (3)$$
$$\tau_2 s^2 Z(s) + s Z(s) = a \mathcal{T}(s) \longrightarrow (4)$$

Here, we assume $z(0) = 0 = \dot{z}(0)$ and $T(t) = 0$. We set $T \xleftrightarrow{\mathcal{L}} \mathcal{T}$, and $\omega = 0$

Substitute $\mathcal{T}(s)$ from (3) into (4), to get:

$$\tau_2 s^2 Z(s) + s Z(s) = a\left(\frac{1}{\tau_1 s+1} U(s)\right)$$

$$\therefore Z(s) = \frac{a}{s(\tau_2 s+1)(\tau_1 s+1)} U(s)$$

Substituting $\tau_1 = 0.02$, $\tau_2 = 0.5$, and $a = 10$, we get

$$Z(s) = \frac{1000}{s(s+2)(s+50)} U(s)$$

$\therefore$ Poles at $0, -2$ and -50.

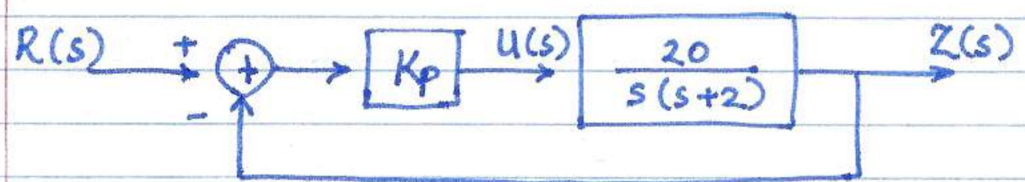
2. The pole at -50 is too far from the other two to affect the system significantly.

$\therefore$ Neglect the pole at -50.

$$\cancel{\therefore Z(s) \approx \frac{1000}{s(s+2)(s+50)}}$$

$$Z(s) \approx \frac{1000/50\; U(s)}{s(s+2)} = \frac{20}{s(s+2)} U(s)$$

Consider, first, a proportional controller K_p.



$$\therefore \frac{Z(s)}{R(s)} = \frac{20K_p}{s^2+2s+20K_p}$$

$$\therefore \zeta = \frac{1}{\sqrt{20K_p}} \quad \& \quad \omega_n = \sqrt{20K_p}$$

Overshoot $= \exp\left[-\pi \zeta / \sqrt{1-\zeta^2}\right]$

Zero overshoot $\Rightarrow \zeta = 1$

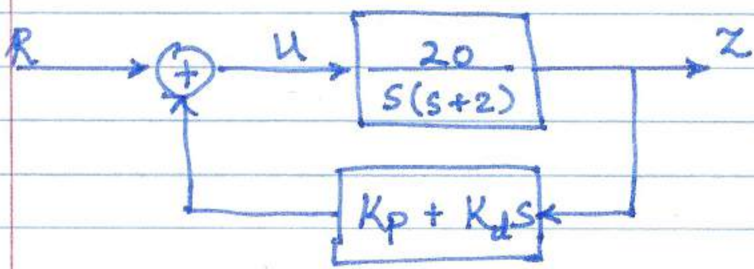
$\Rightarrow K_p = \frac{1}{20}$

$\Rightarrow \omega_n = 1$

Rise time $\approx \frac{1.8}{\omega_n} = 1.8$ s

$\therefore$ A proportional controller cannot satisfy the requirements.

We need a PD controller in order to reduce the rise time.
Place a PD controller $K_p + K_d s$ in the feedback loop as follows:



$$\therefore \frac{Z(s)}{R(s)} = \frac{\frac{20}{s(s+2)}}{1 + \frac{20}{s(s+2)}(K_p + K_d s)}$$

$$= \frac{20}{s^2 + (2 + 20K_d)s + 20K_p}$$

$$\omega_n = \sqrt{20K_p} \quad \text{and} \quad \zeta = \frac{1 + 10K_d}{\sqrt{20K_p}}$$

Zero overshoot $\Rightarrow \zeta = 1$

$$\Rightarrow K_d = \cancel{\tfrac{2}{10}}\ \frac{1}{10}\left(\sqrt{20K_p} - 1\right)$$

Rise time $\approx \frac{1.8}{\omega_n} \leq 0.9$

$$\Rightarrow \omega_n \geq 2$$
$$\therefore \sqrt{20K_p} \geq 2$$
$$\therefore K_p \geq \frac{1}{5}$$

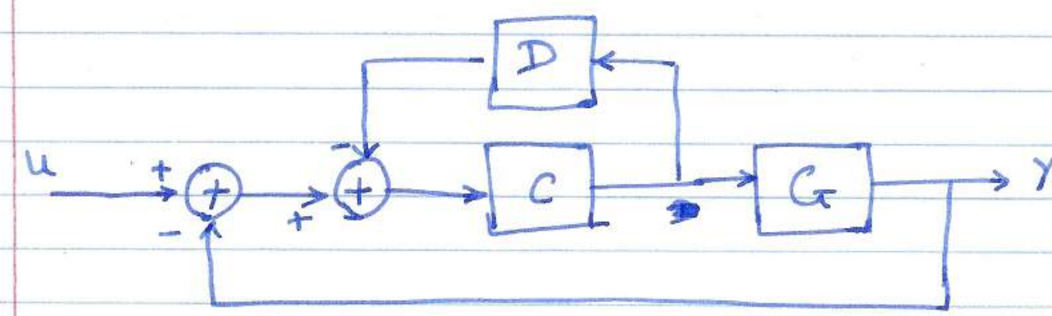
Substituting to get value of K_d

$$K_d = \frac{1}{10} \quad \text{for } K_p = \frac{1}{5}$$

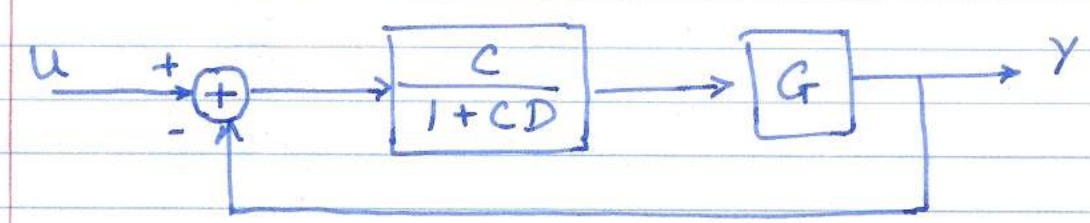
$\therefore$ The appropriate controller is $K_p + \frac{1}{10}\left(\sqrt{20K_p} - 1\right)s$

where $K_p \geq \frac{1}{5}$

Problem 2



1. Simplify the block diagram:



Hence, the transfer function is:

$$H(s) = \frac{\frac{GC}{1+CD}}{1+\frac{GC}{1+CD}}$$

$$\therefore H(s) = \frac{GC}{1+C(D+G)}$$

2. Assume $D(s) = 0$, $C(s) = 10^6$, $G(s) = \frac{s+1}{(s^2+10s+100)(s+100)^2}$

The open loop transfer function is

$Y(s) = CG\,U(s)$

$$\therefore \frac{Y(s)}{U(s)} = \frac{10^6 \ (s+1)}{(s^2+10s+100)(s+100)^2}$$

$$= \frac{100\ (s+1)}{(s^2+10s+100)\left(\frac{s}{100}+1\right)^2}$$

For the repeated real poles, the ~~to~~ break point is 100 rad/s

For the complex poles, $\omega_n^2 = 100 \Rightarrow \omega_n = 10$

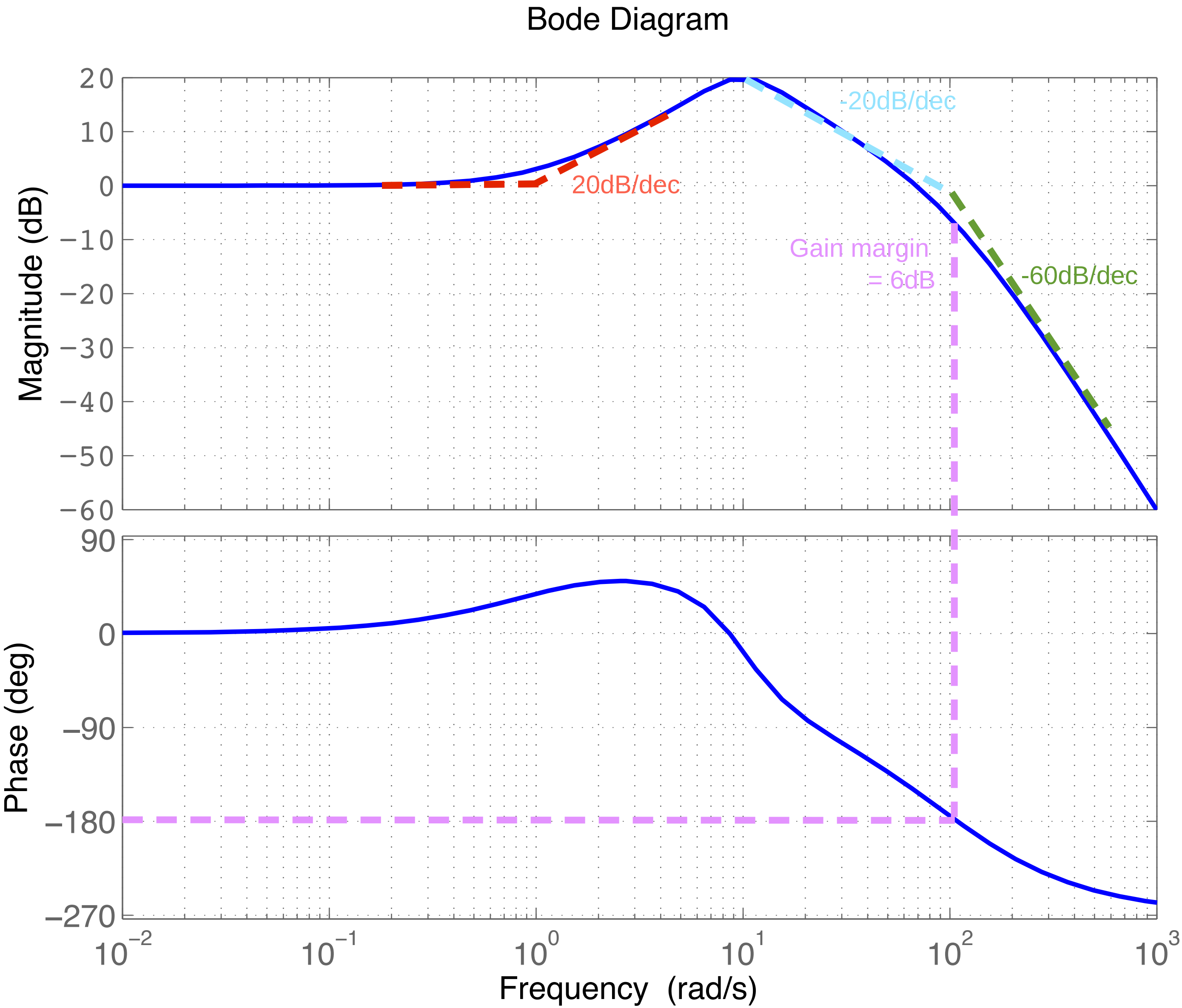
and $\zeta = \frac{1}{2} \Rightarrow \frac{1}{2\zeta} = 1$

Please see Bode plot on next page

3. The gain margin is 6 dB without D. Choose D such that 1+CD increases the gain margin to 30 dB.

This can be achieved by simply shifting the entire gain curve down by (30-6)=24dB. So set 1 + CD = 24dB, i.e., 1 + CD = 10^1.2
Therefore, set D = 10^(-4.8)

Q2: Open loop Bode plot



Problem 3

$$G(s) = \frac{s^3 + 6s^2 - 4s - 24}{s^3 + (k+6)s^2 + (4k-4)s + 5k - 24}$$

1. $$G(s) = \frac{(s+6)(s^2-4)}{(s+6)(s^2-4) + k(s^2+4s+5)}$$

$$\therefore G(s) = \frac{1}{1 + k\,\dfrac{s^2+4s+5}{(s+6)(s^2-4)}}$$

Define $b(s) = s^2+4s+5$ and $a(s) = (s+6)(s^2-4)$

Rule 1: No. of poles (n) = 3,
No. of zeros (m) = 2

Rule 2: No. of poles + zeros is odd
$\Rightarrow$ ~~Axis is~~ Root locus on axis

Rule 3: $n - m = 1 \Rightarrow$ 1 asymptote as part of root locus
~~Cent~~ Asymptote originates from $\frac{(-6) - (0)}{1} = -6$

at angle $\frac{180}{1} = 180^\circ$.

Rule 4: Angle of departure
For pole $= -6$
$= (\theta_1 - \theta_1) - (180 + 180) - 180$
$= -180^\circ$

For pole $= -2$
$= (90 - 90) - (0 + 180) - 180$
$= 0^\circ$

For pole $= 2$
$= (\theta_2 - \theta_2) - (0 + 0) - 180^\circ$
$= -180^\circ$

Angle of arrival
At zero $= -2 + j$
$= (\tan^{-1}(4) + 180 - \tan^{-1}(4) + 90) - 90 + 180$
$= 360^\circ$

* Symmetry forces zero at $-2-j$ to have same angle of arrival

Rule 5: Root locus intersection w/ imaginary axis found using Routh table

s^3	1	$4k-4$
s^2	$k+6$	$5k-24$
s^1	$4k-4 - \frac{5k-24}{k+6}$	
s^0	$5k-24$	

This implies that $5k-24=0$ is the point on the imaginary axis

Substituting $k = \frac{24}{5}$ gives the root locus point on the imaginary axis

$$s^3 + \left(\frac{24}{5} + 6\right)s^2 + \frac{76}{5}s + 0 = 0$$

$$\therefore s\left[s^2 + \frac{54}{5}s + \frac{76}{5}\right] = 0$$

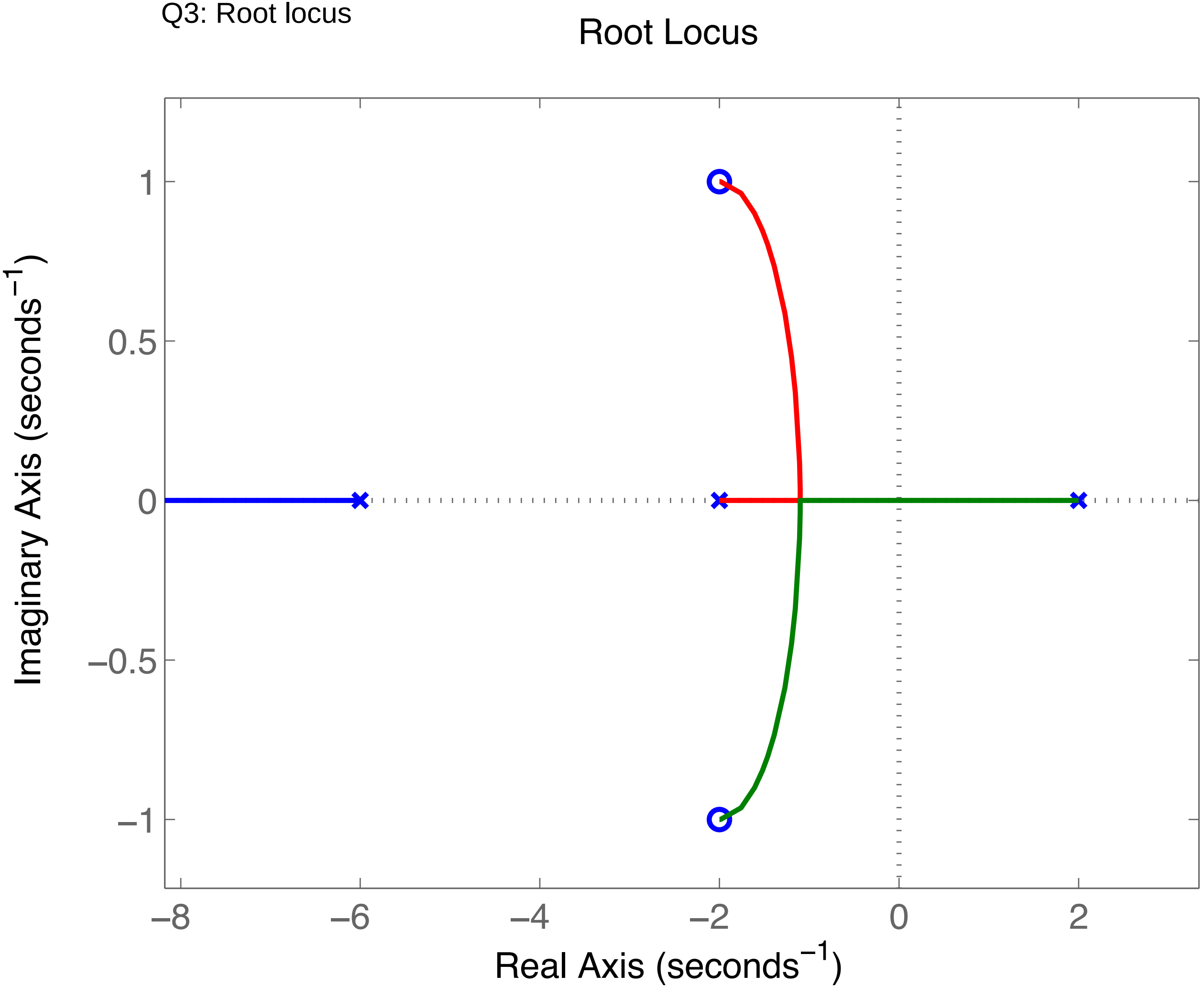
The only solutions on the imaginary axis is $s=0$

$\Rightarrow$ Break-out point is between -2 & 0.

Rule 6 should be tried, but need not be completed.

Please see root locus plot on next page

Q3: Root locus



2. No overshoot $\Rightarrow$ no complex poles
All poles are real $\Rightarrow$ All components of time domain response are of form $e^{-\sigma t}$

~~For rise time,~~
Settling time is dominated by the component with smallest $|\sigma|$
$\therefore$ Best operating point is the break-out point.

3. Consider the C.E in the form $a(s) + k b(s) = 0$ at $s = -1$:
i.e. $s^3 + (k+6) s^2 + (4k-4)s + 5k - 24 = 0$ at $s = -1$
$\therefore -1 + (k+6) - (4k-4) + 5k - 24 = 0$
$\therefore 2k - 15 = 0$
$\therefore k = \frac{15}{2}$