ECE141, Principles of Feedback Systems
UCLA Winter 2019
Midterm Exam
02/12/2019
Time Limit: 2 hours

(a) This exam booklet contains 14 pages (including this cover page) and 5 problems. Total of points is 100.

(b) All books and notes must remain closed. You are allowed to have a single double-sided letter-sized cheat sheet.

(c) Simple calculators are allowed. But no fancy calculators, smartphones, or any other smart and/or connected devices.

(d) Please fully justify your answers and clearly show all the intermediate steps in all your solutions. And, when appropriate, box your final answer.

(e) Please do NOT write your answers on the back of any pages. Answers written on the back will NOT be graded.

(f) **Good Luck...**

1. (10 points) **Quick Review**
Carefully read each statement below and identify it as *True* or *False* by clearly writing **T** or **F** in the box.

- T The BIBO stability of an LTI system is equivalent to the asymptotic stability in Lyapunov sense of the equilibrium point at the origin $X = 0$. T
- F A stable closed-loop system can track a ramp input with zero steady-state error as long as it has at least one pure integrator in the loop. F
- F Final Value Theorem can be used to find the steady-state response of a marginally stable system. F long as it's stable.
- F Any LTI system has a unique state-space representation. F
- T A lag compensator will reduce steady-state error. T
- T A Proportional-Integral (PI) compensator is a lag compensator whereas a Proportional-Derivative (PD) compensator is a lead compensator.
- F A unity feedback system with a non-minimum phase loop transfer function will always remain stable for large gains. F

6↓ t_s

- T As a pole of a 2nd-order system moves *closer* to the $j\omega$ axis, while its imaginary part remains constant, the settling time of the step response gets larger. T
- F An unstable pole in a plant can be canceled by a *zero* in the compensator as long as the transient performance remains acceptable. F
- T Making $|L(j\omega)| \ll 1$, where $L(s)$ is the loop transfer function, would improve the robust stability of a feedback system. T.

$\frac{L}{1+L} \sim L$ $\frac{1}{T} \to$ large $\frac{1+L}{L}$

10/10

2. (24 points) **Linearization, State-Space**

$30d - wd$ $\quad$ $0.5dw - 40w$

$30 - w$ $\quad$ $-d$ $\quad$ $0.5w$ $\quad$ $0.5d - 40$

Predator-prey models are commonly applied to ecological systems where there are two dominant species, one of which acts as herbivorous prey and the other as a carnivorous predator. Predator-prey systems have been studied for decades and are known to exhibit interesting dynamics. In this case, the system being modeled is the interaction of wolves and deer in an Alaskan wildlife preserve. The model is given as follows:

$$\dot{d} = (30 - w)d \tag{1}$$
$$\dot{w} = (0.5d - 40)w - f \tag{2}$$

$$\dot{x} = \begin{bmatrix} (30 - w)d \\ (0.5d - 40)w \end{bmatrix}$$

$\frac{16}{24}$

where d is the population of deer, w is the population of wolves, and f represents the control input used by conservation ecologists to control the population of wolves in order to ensure the health of the overall ecological system.

(a) (5 points) What will be the population of deer and wolves in the equilibrium with no input? i.e., with $f = 0$, find out the possible equilibrium points (d_e, w_e) for this nonlinear system.

$f = 0$ $\begin{cases} \dot{d} = (30 - w)d = 0 \\ \dot{w} = (0.5d - 40)w = 0 \end{cases}$

$\begin{cases} d_e = 0 \\ w_e = 0 \end{cases}$ trivial could be a possible equillbriumpoint.

$\begin{cases} d_e = 80 \\ w_e = 30 \end{cases}$ non-trivial equilibrium point.

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(b) (8 points) Linearize the model around the **non-zero** equilibrium point, and find matrices A and B such that $\delta\dot{\underline{x}} = A\delta\underline{x} + B\delta f$ where $\underline{x} \triangleq [d \quad w]^T$ and $\delta\underline{x} \triangleq \underline{x} - [d_e \quad w_e]^T$.

$$A = \frac{\partial S}{\partial x} = \left.\begin{bmatrix} 30-w & -d \\ 0.5w & 0.5d-40 \end{bmatrix}\right|_{d_e=80,\ w_e=30}$$

$$= \begin{bmatrix} 0 & -80 \\ 15 & 0 \end{bmatrix}$$

$$B = \frac{\partial S}{\partial f} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

(c) (5 points) Determine if the linearized system you found in Part (b) is stable, unstable, or marginally stable.

~~The system is not stable, since eigenvalues are -80, 15, 15>0.~~

x10!

$$\det(sI-A) = \det\begin{bmatrix} s & 80 \\ -15 & s \end{bmatrix}$$

$$= s^2 + 15 \cdot 80 = 0$$

$$s = \pm\sqrt{15 \cdot 80} = j\sqrt{(\ )}$$

marginally stable.

-4

$\delta f =$ $\delta x = \begin{bmatrix} 0 & -80 \\ 15 & 0 \end{bmatrix}\begin{bmatrix} \delta d \\ \delta w \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}\delta f$

(d) (6 points) Assuming the deviation of the deer population from its equilibrium δd as the output, find the Transfer Function from the input δf to the output δd in the linearized system.

$\delta d \sim \delta f$

$$\delta \dot{x} = \begin{bmatrix} 0 & -80 \\ 15 & 0 \end{bmatrix}\begin{bmatrix} \delta d \\ \delta w \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}\delta f$$

$15\delta d - \delta f = \delta \dot{w}$

$-80\delta w = \delta \dot{d} \Rightarrow \delta \ddot{d} = -80\delta \dot{w}$

$15\delta d - \delta f = \delta \ddot{d}$

$15\delta d - s^2 \delta d = \delta f$

$\cancel{\frac{\delta d}{\delta f} = \frac{1}{15-s^2}}$

$$\delta \dot{x} = \begin{bmatrix} \delta \dot{d} \\ \delta \dot{w} \end{bmatrix} = \begin{bmatrix} 0 & -80 \\ 15 & 0 \end{bmatrix}\begin{bmatrix} \delta d \\ \delta w \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix}\delta f$$

$\delta \ddot{d} = -80\delta \dot{w} - \cancel{\delta f}$

$\delta \dot{w} = 15\delta d - \delta f$

-4

$\delta \dot{w} = -\frac{1}{80}\delta \ddot{d}$

$= -\frac{s^2}{80}\delta d$

$-\frac{s^2}{80}\delta d = 15\delta d - \delta f$

$s^2\delta d + 15\times 80\delta d = 80\delta f$

$\frac{\delta d}{\delta f} = \frac{80}{s^2 + 15\times 80}$

(b) (4 points) Assuming zero initial conditions, write an Ordinary Differential Equation that relates the output $y(t)$ to the inputs $r(t)$, $w_1(t)$, and $w_2(t)$.

since $Y + CPY = CPR + Pw_1 + w_2 \Rightarrow Y + \frac{1}{(s+10)s} Y = \frac{1}{s(s+10)} R + w_1 \frac{1}{(s+10)} + w_2$

$\Rightarrow (s^2+10s+1) Y = R + s w_1 + (s^2+10s) w_2$

4

$\Leftrightarrow s\ddot{y} + 10\dot{y} + y = r + \dot{w}_1 + \ddot{w}_2 + 10\dot{w}_2$

(c) (7 points) Assume the following reference and disturbance inputs ($u(t)$ is the unit step function):

$$r(t) = (10 + \sin(t))u(t) \quad (3)$$
$$w_1(t) = 10u(t) \quad (4)$$
$$w_2(t) = \cos(1000t)u(t) \quad (5)$$

Find the the closed-loop system response in the steady-state $y_{ss}(t)$.

$H_1(s) = \frac{1}{s^2+10s+1} \Rightarrow H_1(j\omega) = \frac{1}{10j\omega + 1 - \omega^2}$

$\omega = 1 \quad H_1(j\omega) = \frac{1}{10j} = \frac{1}{10} e^{-j\frac{\pi}{2}}$

$H_2(s) = \frac{s}{s^2+10s+1} \Rightarrow H_2(j\omega) = \frac{j\omega}{10j\omega + 1 - \omega^2}$

$\omega = 0 \text{ (DC)} \quad H_2(j\omega) = 0.$

6

$H_3(s) = \frac{s^2+10s}{s^2+10s+1} \Rightarrow H_3(j\omega) = \frac{10j\omega - \omega^2}{10j\omega + 1 - \omega^2}$

$\omega = 1000 \quad H_3(j\omega) = \frac{10^4 j - 10^6}{10^4 j + 1 - 10^6} \sim 1$

$y_{ss}(t) = \frac{1}{10}(10 + \sin(t - \frac{\pi}{2}))u(t) + \cos(1000t)u(t)$

$\downarrow$

$10 + 0.1\sin(t - \frac{\pi}{2})u(t) + \cos(1000t)u(t).$

(d) (3 points) Can this closed-loop system track step reference input with zero steady-state error? i.e., Will we have $e(t) \to 0$ as $t \to \infty$ when $r(t) = u(t)$ and $w_1(t) = w_2(t) = 0$? If yes, why? If not, why not? Please explain. No need to calculate.

System is stable, $r(t) = u(t) \longleftrightarrow \frac{1}{s}$; $PC = \frac{1}{s(s+10)}$ includes unstable pole $s=0$,

Yes. it can track with 0 ss error.

3

(e) (4 points) Can this closed-loop system reject a ramp disturbance at the input to the plant in the steady-state? i.e., Will we have $y(t) \to 0$ as $t \to \infty$ for $w_1(t) = tu(t)$ and $r(t) = w_2(t) = 0$? If yes, why? If no, how would you propose to change the compensator $C(s)$ to make that happen?

4

NO, since C doesn't include all unstable poles of $w_1(t)$.

we can make $C = \frac{K_1}{s^2} + K_2$ for K_1, K_2 s.t. the system is stable.

4. (22 points) **Plotting Root Locus**
The primary mirror of a large telescope can have many smaller hexagonal segments with the orientation of each segment actively controlled. Suppose the unity feedback system for the mirror segments has the following Loop Transfer Function:

$s^2+2s+5=0 \quad \frac{-2 \pm \sqrt{4-20}}{2}$

$$L(s) = \frac{1}{s(s^2+2s+5)}$$

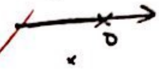
22/22

$s=0$

$-1 \pm \sqrt{1-5}$

$-1 \pm 2j$

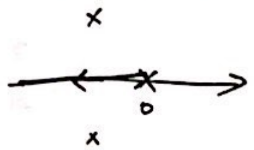
We want to plot the root locus of the characteristic equation $1 + KL(s) = 0$ for $K \geq 0$:



(a) (1 point) Determine the number of branches in the Root Locus: ...3..

(b) (2 points) Determine the portion of the real axis on the root locus: .1. portion

$(-\infty, 0)$



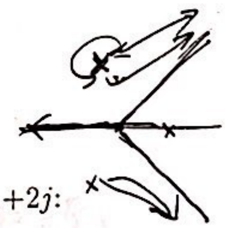
(c) (3 points) Determine the angle of each asymptote: $\theta_a = 60°, 180°, 300°$

$m=0$
$n=3$

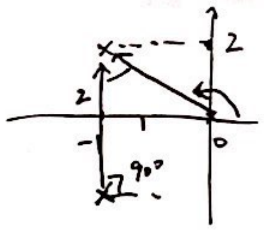
$\frac{180°}{3}(2l+1)$

(d) (3 points) Determine the asymptote centroid, i.e., the real axis intercept of the asymptotes: $\sigma_a = ... -\frac{2}{3}$

$\frac{0+(-1+2j)+(-1-2j)}{3} = \frac{-2}{3}$



(e) (4 points) Determine the angle of departure from the open-loop pole at $s = -1+2j$: (*note*: $\tan^{-1}(0.5) = 26.56°$)



$\tan^{-1}\frac{1}{2} = 26.56°$

$0 - 90° - 90° - 26.56° = 180°$ -30

$\angle D = 180° + 180° + 26.56° = 26.56°$

(f) (3 points) Using all of the above information, plot the root locus on the graph in the next page. Use arrows to show the direction of increasing gain.

(g) (6 points) Using Routh-Hurwitz method, estimate the minimum non zero gain that would make the closed-loop system unstable. Then find out the points at which the Root Locus crosses the jw axis.

$$1+KL = 1+\frac{K}{s(s^2+2s+5)} = 0 \Rightarrow s^3+2s^2+5s+K=0$$

s^3	1	5
s^2	2	K
s	$\frac{10-K}{2}$	0
s^0	K	0

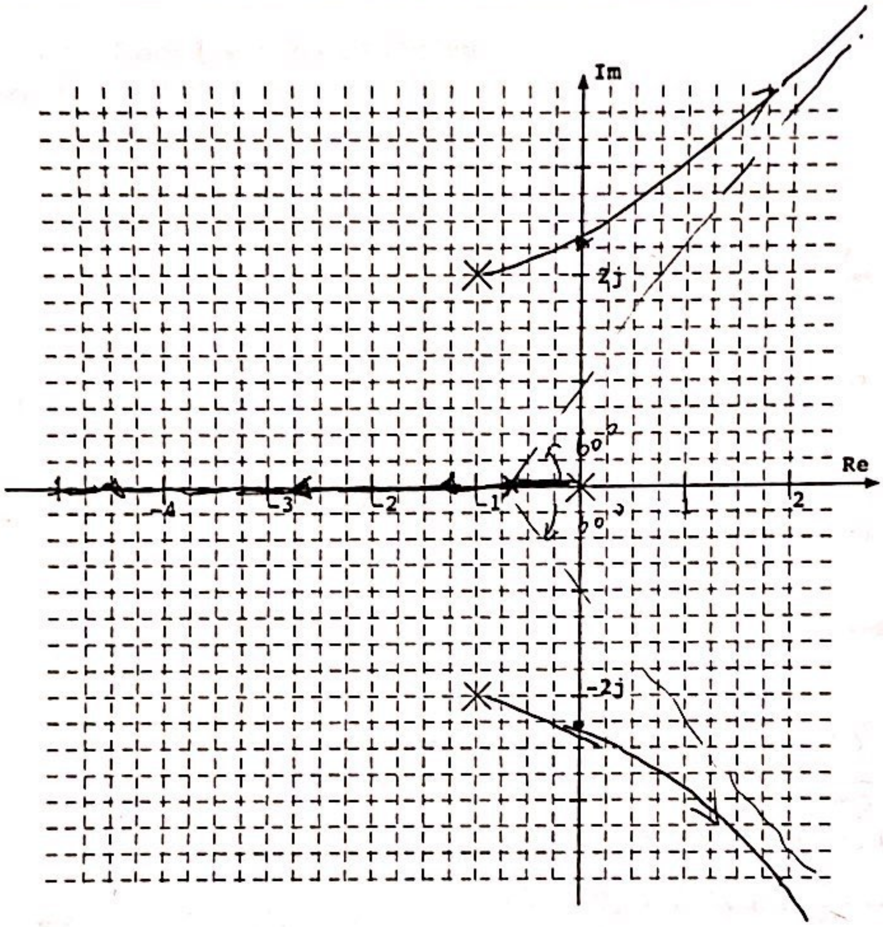
$$\begin{cases} \frac{10-K}{2} > 0 \\ K > 0 \end{cases} \Rightarrow \begin{cases} K < 10 \\ K > 0 \end{cases}$$

$0 < K < 10$. stable

ow unstable.

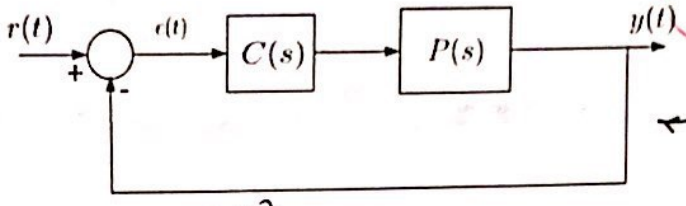
$\frac{10-K}{2} = 0$ $K = 10$ $2s^2 + K = 2s^2+10 = 0$ $s^2 = -5$ $s = \pm\sqrt{5}j$

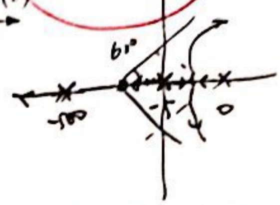
The cross point are $\pm\sqrt{5}j$


Im
Re
2j
-2j
-4
-3
-2
-1
1
2
60°

5. (20 points) **Root Locus-based Design**
Consider the following feedback system:

20/20





$PC = \frac{K(s+z)}{s(s+p)(s+5)(s+500)}$

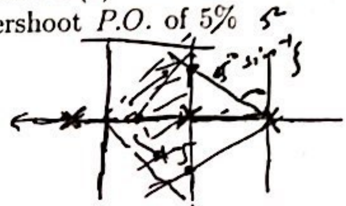
0 -5 -500

And assume:
$$P(s) = \frac{1}{s(s+5)(s+500)}$$

$\frac{PC}{1+PC}$ $C =$ $\frac{1}{s^2+5s}$

(a) (10 points) Using the Root Locus technique, design a compensator $C(s)$ such that the closed-loop system will have the maximum percentage overshoot $P.O.$ of 5% and the maximum 2% settling time t_s of 800 msec.

$s+5$
s^2



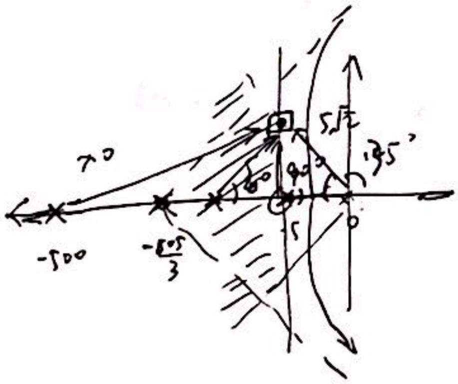
$C(s) = K\frac{s+z}{s+p}$ $PO \leq 5\% \Rightarrow \zeta \geq 0.7 \rightsquigarrow 45°$

$t_s(2\%) \leq 800ms \Rightarrow 6 \geq \frac{4}{800m} = 5$ $\zeta\omega_n \geq 5$

$P(s)C(s) = \frac{K(s+z)}{s(s+p)(s+5)(s+500)}$

we choose desired poles to be $-5 \pm 5j$

~~$\phi = -135° - 90° - \tan\frac{5}{500} - 180° = +45°$~~



let zero $z = 5$

$L(s) = \frac{K}{s(s+p)(s+500)}$

$\phi = 90° - 90° - 135° - \tan\frac{5}{500} - 180° = +45° \Rightarrow$

$P = 10$.

$|K| = \frac{5\sqrt{5^2+5^2}\times\sqrt{495^2+5^2}}{5} = \cancel{3500}\ 24750$

Therefore $L(s) = \frac{\cancel{K\,3500}\ 24750}{s(s+500)(s+10)}$

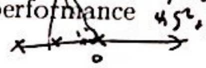
$C(s) = \frac{24750(s+5)}{(s+10)}$

(b) (3 points) With the compensator you designed in Part (a), find the velocity error constant (a.k.a., ramp error constant) $K_v = \lim_{s\to 0} sL(s)$

$$L(s) = \frac{24750(\cancel{s+5})}{s+10} \cdot \frac{1}{s(\cancel{s+5})(s+500)}$$

$$\lim_{s\to 0} sL(s) = \frac{24750}{(s+10)(s+500)} = \frac{24750}{5000} \approx 5$$

(c) (7 points) We have now been asked to reduce the closed-loop steady-state error to a ramp input by ten times, (i.e., to increase K_v ten times). Design a modified compensator which helps us achieve that while still meeting the transient performance specs in Part (a).



$K_{v0} = 5$ $K_v = 50$. $\lim_{s\to 0} sL(s)$ increase 10 times.

$$C(s) = \cancel{k} \frac{s+z}{s+p}$$

Assume $p = 0.1 \Rightarrow z = 1$ s.t. $C(s) = \frac{s+1}{s+0.1} \cdot \frac{24750}{(s+10)(s+500)}$