SOLUTIONS

# EE 141 Midterm Exam

## Thursday February 10, 2011

**You have 1 hr. and 45 min.**

**You are allowed only ONE sheet of notes**
**Lecture Notes, Homework solutions, Calculators and books are not allowed**

**Write your answer to each question in the space provided**

**Read the problem statements carefully**
**Show your work and reasoning clearly**

***For partial credit justify your results:***
**Simply writing down one line with the correct answer is not adequate**

**Your Name and Student Id#:** ______________________________

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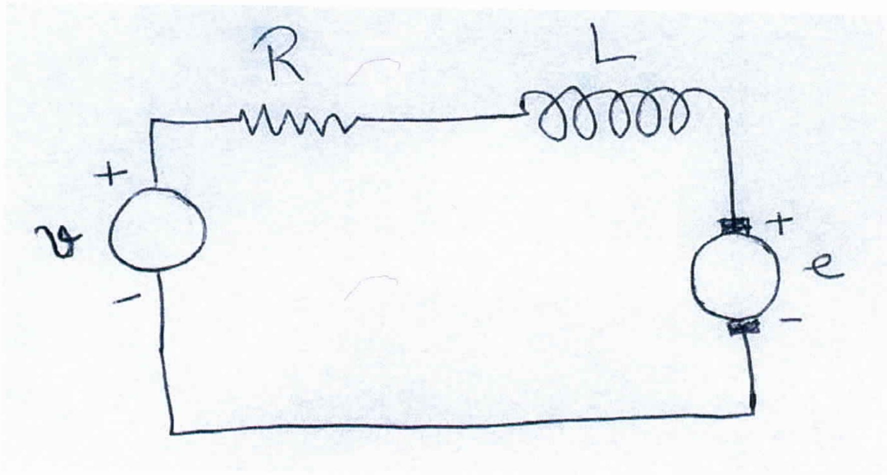
## Problem 1 (40 pts.)




Consider the above circuit representing a DC motor being driven by a voltage source.

*(i)* Write the differential equation governing the voltage on the circuit, i.e., $v(t)$, if the voltage drop across the DC motor is given by $\boldsymbol{e = K_e \frac{d\theta}{dt}}$, where $\theta$ is the angular position of the motor's axle.

*(ii)* Write the differential equation governing $\boldsymbol{\theta}$ if **(i)** the motor has moment of inertia ***J***, **(ii)** there is a frictional torque proportional to the angular velocity with constant of proportionality ***b***, and **(iii)** the circuit's current, ***i***, induces a torque given by $\boldsymbol{iK_t}$.

*(iii)* Compute the transfer function $\boldsymbol{\frac{\theta(s)}{v(s)}}$.

*(iv)* If $\boldsymbol{L = R = b = K_e = K_t = J = 1}$ what is the impulse response of the system?

(i) $v = Ri + L\,di/dt + K_e\,d\theta/dt$

(ii) $J\,d^2\theta/dt^2 = -b\frac{d\theta}{dt} + K_t i$

(iii) $V(s) = R\,I(s) + sL\,I(s) + sK_e\,\Theta(s)$

$s^2 J\Theta(s) = -sb\Theta(s) + K_t I(s)$

$\Rightarrow \quad \frac{\Theta(s)}{V(s)} = \frac{K_t}{s((Js+b)(Ls+R) + K_e K_t)}$

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(iv) $\frac{\theta(s)}{V(s)} = \frac{1}{s((s+1)^2+1)}$

$v(t) = \delta(t) \Rightarrow V(s) = 1$

$\Rightarrow \quad \theta(s) = \frac{1}{s((s+1)^2+1)}$

$\Rightarrow \quad \theta(t) = \int_0^t e^{-y} \sin y \, dy$

$= \left[ -e^{-y} \sin y + \int e^{-y} \cos y \, dy \right]$

$= \left[ -e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \sin y \, dy \right]_0^t$

$\Rightarrow \theta(t) = -\theta(t) - e^{-t}(\sin t + \cos t) + 1$

$\Rightarrow \quad \theta(t) = \frac{1}{2}\left[1 - e^{-t}(\cos t + \sin t)\right]$

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**Problem 2 (30 pts)**

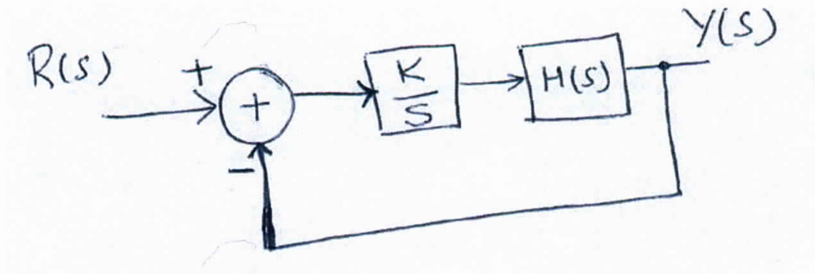
**(i)** **Consider a system described by the following transfer function:**

$$H(s) = \frac{2s+5}{s^3 + (p+20)s^2 + 10s + 20}$$

For what values of $p$ is the system stable?

**(ii)** What is the steady state error if the input is a step function?

**(iii)** Set p=0 and consider the following system with unit feedback:




What is the steady state error now if the input is a step function?

(i)

$$\begin{array}{c|cc} s^3 & 1 & 10 \\ s^2 & p+20 & 20 \\ s^1 & \frac{p+18}{p+20} & 0 \\ s^0 & 20 & \end{array} \Rightarrow \begin{array}{l} p+20>0 \Rightarrow p>-20 \\ p+18>0 \Rightarrow p>-18 \end{array}$$

(ii)

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} E(s) = \lim_{s\to 0} s\left(1 - \frac{Y(s)}{R(s)}\right)R(s) \qquad \boxed{R(s) = \frac{1}{s}}$$

$$= \lim_{s\to 0}\left(1 - \frac{Y(s)}{R(s)}\right) = 1 - \frac{5}{20} = \frac{3}{4}$$

(iii)

$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{s}H(s)}{1 + \frac{K}{s}H(s)} \qquad \frac{Y(s)}{R(s)} = \frac{K H(s)}{s + K H(s)}$$

$$\Rightarrow \left.\frac{Y(s)}{R(s)}\right|_{s=0} = 1 \Rightarrow \lim_{t\to\infty} e(t) = 0$$

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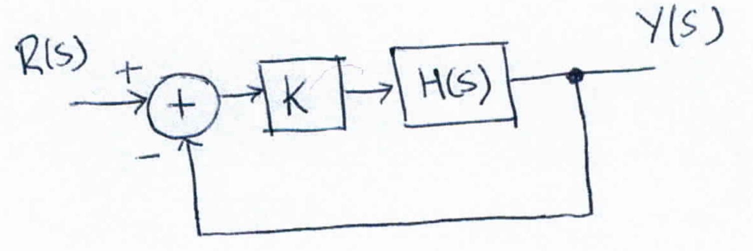
**Problem 3. (30 pts.) Consider the following system**




where

$$H(s) = \frac{s+1}{(s-1)(s+2)^2}$$

(i) Calculate the overall transfer function, and show that the denominator polynomial is given by: $s^3 + 3s^2 + Ks + (K-4)$. For what values of $\boldsymbol{K}$ is the feedback system stable?

(ii) Consider $\boldsymbol{K>4}$. Assume that the 3 poles of the system with feedback are $-\sigma \pm j\omega$, and $-a$, where, $\sigma, a, \omega > 0$ and are functions of $\boldsymbol{K}$. So the denominator of the overall transfer function of the feedback system can be expressed as $(s+\sigma+j\omega)(s+\sigma-j\omega)(s+a) = ((s+\sigma)^2+\omega^2)(s+a)$.

Show the following: $\lim_{K\to\infty} a = 1$, $\lim_{K\to\infty} \sigma = 1$, and $\lim_{K\to\infty} \omega = \infty$.

***Hint:*** Compare coefficients of the powers of $s$ in the denominator polynomial, as expressed in the two parts of the problem.

(i) $$\frac{Y(s)}{R(s)} = \frac{KH(s)}{1+KH(s)} = \frac{K(s+1)}{(s-1)(s+2)^2 + K(s+1)} = \frac{K(s+1)}{s^3+3s^2+Ks+(K-4)}$$

| | | |
|---|---|---|
| $s^3$ | $1$ | $K$ |
| $s^2$ | $3$ | $K-4$ |
| $s^1$ | $\frac{2K+4}{3}$ | $0$ |
| $s^0$ | $K-4$ | |

$\Rightarrow \quad K > 4$

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(ii) $\left((s+\sigma)^2+\omega^2\right)(s+a)$

$= \left(s^2+2\sigma s+(\sigma^2+\omega^2)\right)(s+a)$

$= s^3+(2\sigma+a)s^2+\left((\sigma^2+\omega^2)+2a\sigma\right)s + a(\sigma^2+\omega^2)$

$= s^3+3s^2+Ks+(K-4)$

$\Rightarrow$ $3=2\sigma+a$ $\Rightarrow$ a and $\sigma$ are bounded (*)

$K=(\sigma^2+\omega^2)+2a\sigma$ $\Rightarrow$ $aK=a(\sigma^2+\omega^2)+2a^2\sigma$ (**)

$K-4=a(\sigma^2+\omega^2)$ $\Rightarrow$ $a(\sigma^2+\omega^2)=K-4$ (***)

$\Rightarrow aK = K-4+2a^2\sigma$

$\Rightarrow K(1-a)=4-2a^2\sigma$

$\Rightarrow 1-a=\frac{4-2a^2\sigma}{K} \Rightarrow$ $\boxed{\lim_{K\to\infty} a=1}$

from (*) $\Rightarrow \lim_{K\to\infty}\sigma=1$ from (***) $\Rightarrow \lim_{K\to\infty}\omega=\infty$