EE141, Spring 2017 — Midterm Exam

May 9, Tuesday, 8:00am–9:50am

Name:**SOLUTIONS**________________ ID:________________

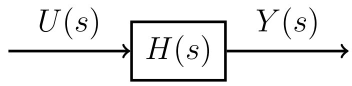
Instructions:

Closed book, no calculators allowed. One letter-sized sheet of notes are allowed. Show all your work and simplify your answers. Credit will be given for partial answers. Answers that are not simplified may not receive full credit. **Write only on the FRONT of each page.**

I am aware of and will abide by UCLA's policy on academic integrity during this exam. This means I will not use or give to others unauthorized materials, information, or study aids during this exam. I recognize that "Unauthorized materials" include other students' exam papers.

Signature:________________________________

Problem 1 (25 points). Consider the block diagram below:



Suppose the system H is characterized by the relationship:

$$y(t) = K \int_0^t e^{-c(t-\tau)} u(\tau) d\tau$$

for some $K > 0$ and $c \geq 0$. Notice that, when $c = 0$, the system is an integrator (when $c > 0$, the input is "forgotten" over time).

a) Show that this system is linear for any choice of parameters K and c.

b) Determine $h(t)$, the impulse response of the system. Plot $h(t)$. Indicate how the plot changes as c changes.

c) Find the transfer function $H(s)$ for the system.

Solution.

a) Consider two inputs $u_1(t)$ and $u_2(t)$ which generate $y_1(t)$ and $y_2(t)$ respectively, *i.e.*,

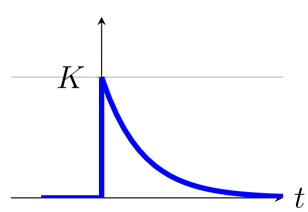
$$y_i(t) = K \int_0^t e^{-c(t-\tau)} u(\tau) d\tau, \qquad i = 1, 2.$$

Consider the input $u(t) = a_1 u_1(t) + a_2 u_2(t)$ for constants a_1, a_2. Then

$$\begin{aligned} y(t) &= K \int_0^t e^{-c(t-\tau)} \left(a_1 u_1(\tau) + a_2 u_2(\tau)\right) d\tau \\ &= K a_1 \int_0^t e^{-c(t-\tau)} u_1(\tau) d\tau + K a_2 \int_0^t e^{-c(t-\tau)} u_2(\tau) d\tau \\ &= a_1 y_1(t) + a_2 y_2(t). \end{aligned}$$

b) The impulse response satisfies $y(t) = h(t) * u(t) = \int_0^\infty h(t-\tau) u(\tau) d\tau$ (the lower bound is 0 because we assume $u(t) = 0$ for $t < 0$). From the equation for $y(t)$, this implies

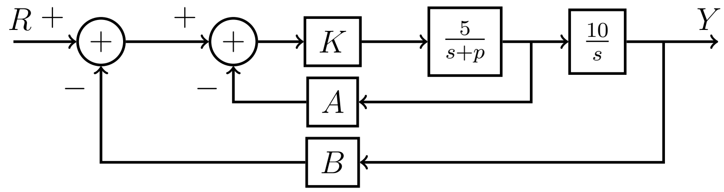
$$h(t) = K e^{-ct} \mathbf{1}(t).$$



As c increases, the plot gets steeper and decreases faster.

c) $H(s) = \frac{K}{c+s}$.

Problem 2 (25 points). Consider the nested feedback system below:



K, A, B, and p are constants.

a) Determine the closed-loop transfer function in terms of K, A, B, and p

b) Assume $p = 2$. Find K, A, and B such that:

- $\omega_n = 10$ rad/s, and
- $\zeta = 0.7$, and
- There is zero steady-state error to a step input.

c) Suppose $p > 2$, but the *same* parameters K, A, B as found above are used. Determine if each of the following will: increase, decrease, or stay the same:

i) Peak time? ______________________________

ii) Percent overshoot? ______________________________

iii) Settling time? ______________________________

iv) Steady state error, $\lim_{t\to\infty}(r(t)-y(t))$? ______________________________

Solution.

a)

$$\frac{50K}{s^2 + s(p + KA) + 50KB}$$

b) Zero steady-state error implies $\boxed{B = 1}$. $\omega_n = 10$ then implies $\boxed{K = 2}$. With $p = 2$, we must have $p + KA = 2\zeta\omega_n$ so $\boxed{A = 6/5}$.

c) From part a, p only affects damping ratio. Increasing p increases the damping ratio, which decreases ω_d and increases σ. Thus:

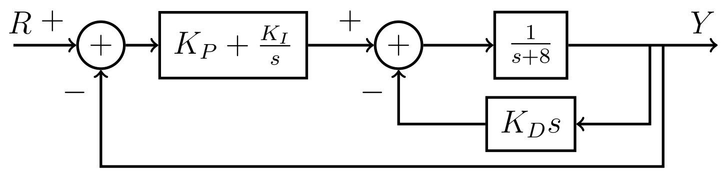
i) Peak time $\boxed{\text{increases}}$

ii) Percent overshoot $\boxed{\text{decreases}}$

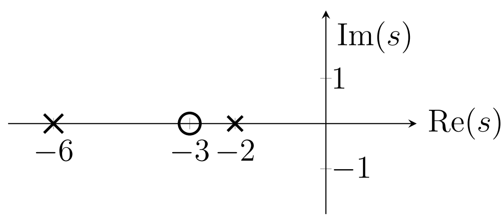
iii) Settling time $\boxed{\text{decreases}}$

iv) Steady state error, $\lim_{t\to\infty}(r(t) - y(t))$ $\boxed{\text{stays the same}}$

Problem 3 (25 points). Sometimes PID control is implemented as in the following feedback diagram:



a) Find K_D, K_I, and K_P so that the transfer function of this closed-loop system has the pole-zero plot shown below.



b) Compute explicitly the step response $y(t)$ of the closed-loop system with the parameters chosen above.

c) Suppose $r(t) = t\mathbf{1}(t)$, a ramp input, and assume the same parameters as chosen above. Compute $\lim_{t\to\infty}(r(t) - y(t))$.

Solution.

a) The closed-loop transfer function is:

$$\frac{sK_P + K_I}{s^2(1 + K_D) + s(8 + K_P) + K_I}.$$

Thus, we require

$$(s + K_I/K_P) = (s + 3) \implies \frac{K_I}{K_P} = 3$$

and

$$(1 + K_D)(s + 6)(s + 2) = s^2(1 + K_D) + s(8 + K_P) + K_I$$
$$\implies 8(1 + K_D) = 8 + K_P,$$
$$12(1 + K_D) = K_I.$$

Solving this system of equations gives

$$K_D = 1, \quad K_I = 24, \quad K_P = 8.$$

b) The closed-loop transfer function is

$$\frac{4(s+3)}{(s+6)(s+2)}$$

so that the Laplace transform of the step response is

$$\frac{4(s+3)}{s(s+6)(s+2)} = \frac{1}{s} + \frac{-\frac{1}{2}}{s+6} + \frac{-\frac{1}{2}}{s+2}$$

and the step response is

$$y(t) = \mathbf{1}(t) - \frac{1}{2}e^{-6t}\mathbf{1}(t) - \frac{1}{2}e^{-2t}\mathbf{1}(t).$$

c) Define $e(t) = r(t) - y(t)$. The transfer function for e is given by

$$\frac{E(s)}{R(s)} = 1 - \frac{Y(s)}{R(s)} = \frac{s(s+4)}{s^2+8s+12}.$$

For a ramp, $R(s) = 1/s^2$ so we have

$$E(s) = \frac{(s+4)}{s(s^2+8s+12)}.$$

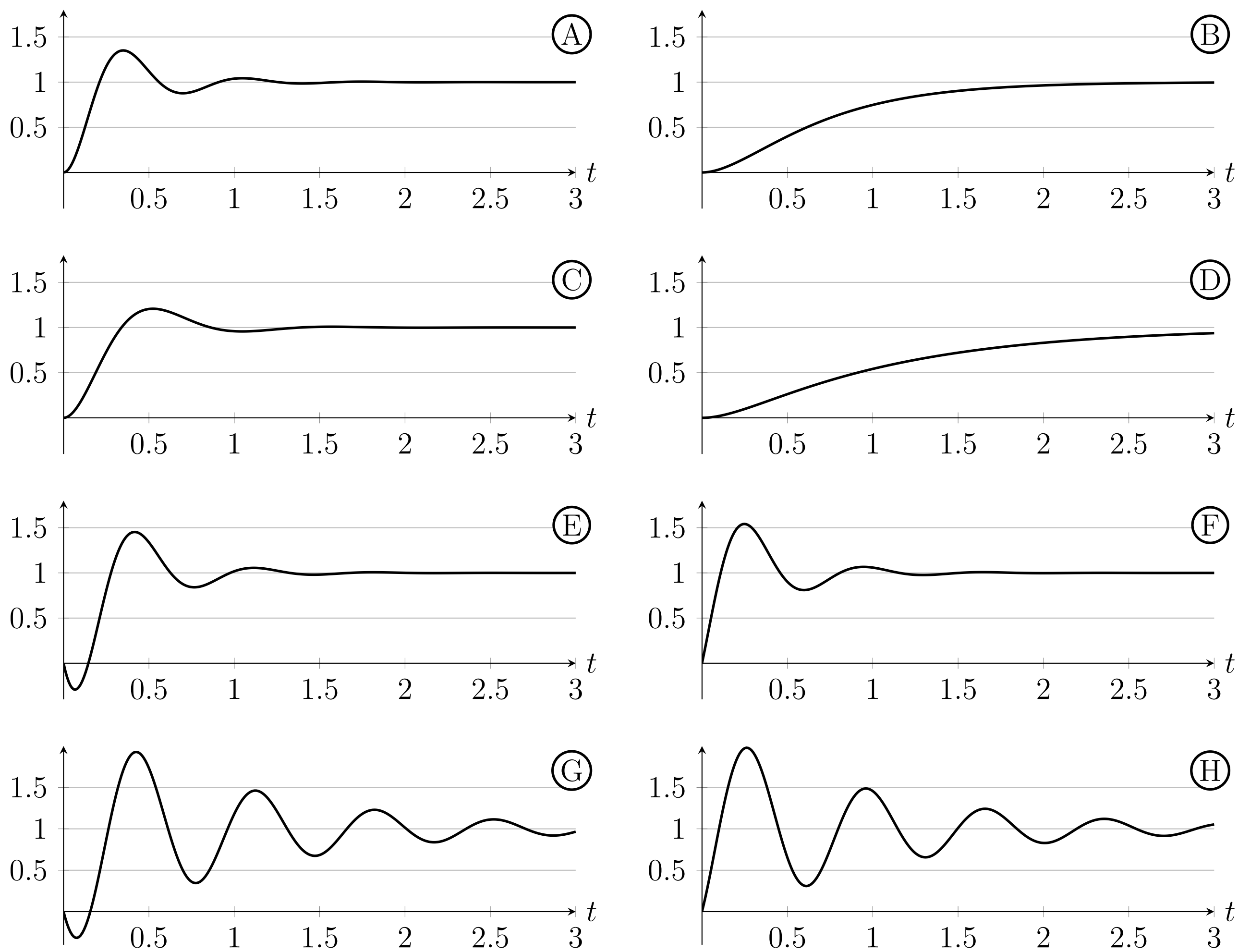
Since $E(s)$ has one pole at $s = 0$ and the remaining poles are in the left-half plane, the final value theorem applies and

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{3}.$$

Problem 4 (25 points). Match the following transfer functions with the step responses shown below.

$$\frac{45}{(s+3)^2+6^2} \qquad \frac{90}{(s+3)^2+9^2} \qquad \frac{9(s+10)}{(s+3)^2+9^2} \qquad \frac{-9(s-10)}{(s+3)^2+9^2}$$

$$\frac{8.2(s+10)}{(s+1)^2+9^2} \qquad \frac{-8.2(s-10)}{(s+1)^2+9^2} \qquad \frac{8}{(s+2)(s+4)} \qquad \frac{5}{(s+1)(s+5)}$$



Solution.

$\dfrac{45}{(s+3)^2+6^2}$	$\dfrac{90}{(s+3)^2+9^2}$	$\dfrac{9(s+10)}{(s+3)^2+9^2}$	$\dfrac{-9(s-10)}{(s+3)^2+9^2}$
C	A	F	E
$\dfrac{8.2(s+10)}{(s+1)^2+9^2}$	$\dfrac{-8.2(s-10)}{(s+1)^2+9^2}$	$\dfrac{8}{(s+2)(s+4)}$	$\dfrac{5}{(s+1)(s+5)}$
H	G	B	D