UCLA — Electrical Engineering Dept.
EE132A: Communication Systems
Midterm Exam

---

1. *The signal $m(t) = A_m \cos(2\pi f_m t)$, with $f_m = 5$ kHz, frequency-modulates a carrier with amplitude $A_c = 10$ V and frequency $f_c = 1$ MHz. The frequency deviation constant is $k_f = 1000$ Hz/V.*

   (a) *Estimate the bandwidth of the modulated signal, $u(t)$, when the amplitude of the message is $A_m = 1$ V.*
   $W = f_m = 5$ kHz. From Carson's rule, $B_c = 2W(\beta_f + 1) = 2(f_m + k_f A_m) =$ 10.2 kHz.

   (b) *Determine the value of A_m which causes the frequency component of the modulated signal at $f = f_c$ to vanish.*
   From Table 1 we have that $J_0(\beta_f) = 0$ when $\beta_f = 2.4$. Because $\beta_f = k_f A_m/f_m$, we have that $A_m = f_m\beta_f/k_f = 5000 \times 2.4/100 = 120$ V.

   (c) *Assume that the modulator is followed by an ideal band-pass filter of bandwidth 12 kHz. Write down the expression of the filter's output, $x(t)$, when $A_m = 12$ V. Try to write the output in the most compact form.*
   Only the frequency components at frequencies $f_c - f_m$, f_c, and $f_c + f_m$ go through the bandpass filter. With $A_m = 12$ V, $\beta_f = 12 \times 100/5000 = 0.24$, and $x(t) = A_c J_0(\beta_f) + A_c J_1(\beta_f)\cos(2\pi(f_c + f_m)t) - A_c J_1(\beta_f)\cos(2\pi(f_c - f_m)t) = A_c J_0(\beta_f) - A_c J_1(\beta_f)\sin(2\pi f_m t)\sin(2\pi f_c t)$.

   (d) *Is there a value of A_m which would cause the output of the band-pass filter in the previous part to be a pure sinusoid, and if so, what is that value?*
   From Table 1, $J_1(\beta_f) = 0$ when $\beta_f = 3.8$, which means $A_m = 5000 \times 3.8/100 =$ 190 V.

2. *Consider a conventional AM scheme , where the modulated carrier is given by $u(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t) = A_c[1 + a m_n(t)]\cos(2\pi f_c t)$ and the signal $m(t)$ is the triangular wave in Fig. 1.*

   (a) *If $k_a = 1/(4A)$, what is the value of the modulation index a?*
   Because $\min m(t) = -A$, $m_n(t) = m(t)/A$ and $u(t) = A_c[1+0.25 m_n(t)]\cos(2\pi f_c t)$, so $a = 0.25$.

   (b) *What is the ratio of sideband power to carrier power?*
   $P_c = A_c^2/2$ and $P_{SB} = A_c^2 a^2 P_{m_n}/2$, where $P_{m_n} = (1/2)\int_0^2 (1-t)^2\,\mathrm{d}t = 1/3$. Therefore $P_{SB}/P_c = a^2/3 = 1/48$.

   (c) *What would the sideband-to-carrier power ratio be if we increased the modulation index a to 100%?*
   From the previous part, if $a = 1$ we have that $P_{SB}/P_c = 1/3$.

3. *Consider a two-tone message $m(t) = A_1\cos(2\pi f_1 t) + A_2\cos(2\pi f_2 t)$, with $f_1 = 10$ kHz, $f_2 = 100$ kHz, $A_1 = 1$ V, and $A_2 = 2$ V.*

   (a) *What is the bandwidth W of $m(t)$?*
   $W = f_2 = 100$ kHz, the largest frequency in $m(t)$.

(b) *Let $m(t)$ be used to frequency-modulate a carrier of amplitude $A_c = 10$ V and frequency $f_c = 1$ MHz. Let the frequency deviation constant k_f be $k_f = 1000$ Hz/V. Let the demodulator output be $y(t) = k_f m(t) + n_o(t)$, where the output noise power spectral density is given by*

$$S_{n_o}(f) = \begin{cases} \frac{N_0 f^2}{A_c^2}, & |f| < W, \\ 0, & \text{elsewhere}, \end{cases}$$

where W is the bandwidth of $m(t)$ and $N_0 = 10^{-6}$ W/Hz. What is the SNR at the output of the demodulator?
$P_s = k_f^2(A_1^2 + A_2^2)/2 = 2.5 \times 10^6$ W. $P_{n_o} = \int_{-W}^{W} \frac{N_0}{A_c^2} f^2 \, \mathrm{d}f = (2/3) \times 10^7$ W. Therefore SNR $= 3.75 \times 10^{-1}$ or -4.25 dB.

(c) *Let the same message be used by a DSB-SC modulator. The demodulator is coherent with perfect phase estimation. The in-phase component of the noise, $n_c(t)$, has power spectral density equal to $N_0 = 10^{-6}$ W/Hz in the bandwidth of the message. What is the SNR at the output of the DSB-SC coherent receiver?*
$P_s = A_c^2(A_1^2 + A_2^2)/8 = 62.5$ W. $P_n = N_0 W/2 = 0.05$ W. SNR $= 1250$ or 30.9 dB.

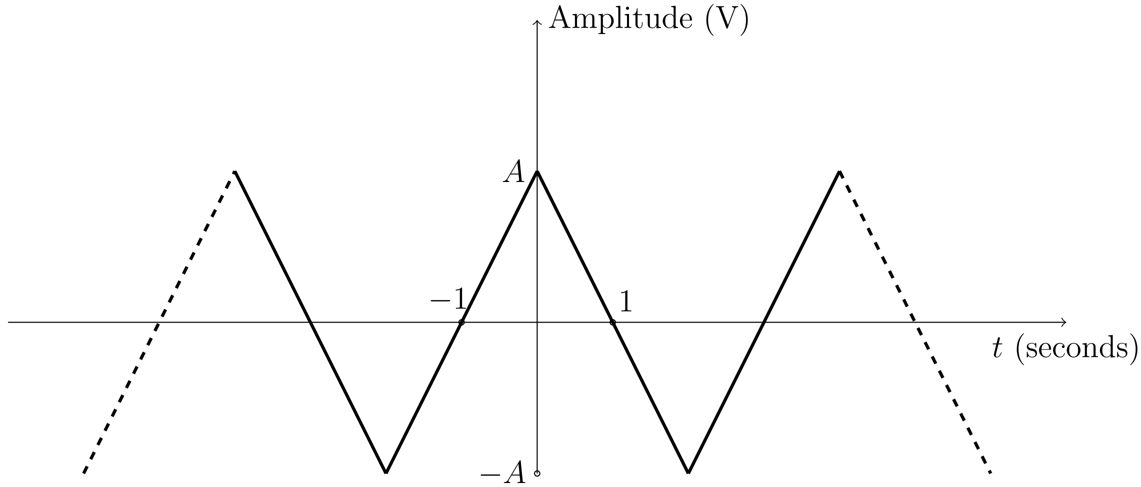

Figure 1: Triangular signal

Table 1: Bessel function $J_n(x)$

x	$J_0(x)$	$J_1(x)$	$J_2(x)$	$J_3(x)$
0.0	1	0	0	0
0.1	0.997502	0.049938	0.001249	0.0000208
0.2	0.990025	0.099501	0.004983	0.000166
0.3	0.977626	0.148319	0.011166	0.000559
0.4	0.960398	0.196027	0.019735	0.00132
0.5	0.93847	0.242268	0.030604	0.002564
0.6	0.912005	0.286701	0.043665	0.0044
0.7	0.881201	0.328996	0.058787	0.00693
0.8	0.846287	0.368842	0.075818	0.010247
0.9	0.807524	0.40595	0.094586	0.014434
1.0	0.765198	0.440051	0.114903	0.019563
1.1	0.719622	0.470902	0.136564	0.025695
1.2	0.671133	0.498289	0.159349	0.032874
1.3	0.620086	0.522023	0.183027	0.041136
1.4	0.566855	0.541948	0.207356	0.050498
1.5	0.511828	0.557937	0.232088	0.060964
1.6	0.455402	0.569896	0.256968	0.072523
1.7	0.397985	0.577765	0.281739	0.08515
1.8	0.339986	0.581517	0.306144	0.098802
1.9	0.281819	0.581157	0.329926	0.113423
2.0	0.223891	0.576725	0.352834	0.128943
2.1	0.166607	0.568292	0.374624	0.145277
2.2	0.110362	0.555963	0.395059	0.162325
2.3	0.05554	0.539873	0.413915	0.179979
2.4	0.002508	0.520185	0.43098	0.198115
2.5	-0.04838	0.497094	0.446059	0.2166
2.6	-0.0968	0.470818	0.458973	0.235294
2.7	-0.14245	0.441601	0.469562	0.254045
2.8	-0.18504	0.409709	0.477685	0.272699
2.9	-0.22431	0.375427	0.483227	0.291093
3.0	-0.26005	0.339059	0.486091	0.309063
3.1	-0.29206	0.300921	0.486207	0.326443
3.2	-0.32019	0.261343	0.483528	0.343066
3.3	-0.3443	0.220663	0.478032	0.358769
3.4	-0.3643	0.179226	0.469723	0.373389
3.5	-0.38013	0.137378	0.458629	0.38677
3.6	-0.39177	0.095466	0.444805	0.398763
3.7	-0.39923	0.053834	0.42833	0.409225
3.8	-0.40256	0.012821	0.409304	0.418026
3.9	-0.40183	-0.02724	0.387855	0.425044
4.0	-0.39715	-0.06604	0.364128	0.430171