UCLA — Electrical and Computer Engineering Dept.
ECE132A: Introduction to Communication Systems
Final Exam
Thursday, March 19, 2020

This exam has 4 questions, for a total of 40 points.

Open book. No calculator needed. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
Please, write your name and ID on the top of each loose sheet!

Question	Points	Score
1	14	
2	8	
3	8	
4	10	
Total:	40	

1. **True/false questions. Write T or F on the line preceding each question. One point for the correct answer and one for the correct explanation.**

   (a) (2 points) ___ There is a Hamming code with $n = 15$ and $k = 9$.

   (b) (2 points) ___ The Viterbi decoder is an optimal maximum likelihood decoder.

   (c) (2 points) ___ Hamming codes have a minimum distance of 4.

   (d) (2 points) ___ If the parity-check matrix of a linear block code is a matrix with 4 rows and 7 columns, then the rate of the code is $\frac{4}{7}$.

   (e) (2 points) ___ Given the following parity-check matrix

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix},$$

   The following generator matrix is consistent with H:

$$G = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

   (f) (2 points) ___ Given the same parity-check matrix H as in the previous part, the following generator matrix is consistent with H:

$$G = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

   (g) (2 points) ___ Given the same parity-check matrix H as in the previous part, the following generator matrix is consistent with H:

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

2. (8 points) *MAP detection.* Consider two equiprobable signals $s_0(t)$ and $s_1(t)$ sent through an additive noise channel. The corresponding matched-filter outputs can be written as $s_0 + n$ if $s_0(t)$ was sent and $s_1 + n$ if $s_1(t)$ was sent, where $s_0 = [0, 0]^\mathsf{T}$, $s_1 = [1, 1]^\mathsf{T}$, and $n = [n_0, n_1]^\mathsf{T}$. The noise joint pdf is given by

$$p(n_0, n_1) = \begin{cases} \exp(-n_0 - n_1), & \text{if } n_0 > 0 \text{ and } n_1 > 0, \\ 0, & \text{otherwise.} \end{cases}$$

What is the MAP decision rule and the corresponding probability of symbol error?

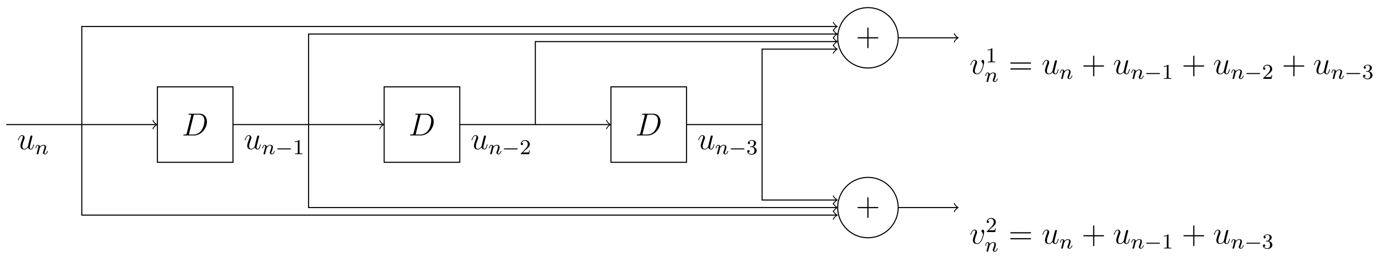
3. *Optimal receiver.* In an AWGN channel with noise power spectral density $\frac{N_0}{2}$ the following two equiprobable messages are transmitted:

$$s_0(t) = \begin{cases} \frac{At}{T}, & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases} \text{ and } s_1(t) = \begin{cases} A\left(1 - \frac{t}{T}\right), & 0 \leq t < T \\ 0, & \text{otherwise} \end{cases}.$$

(a) (4 points) Determine the structure of the optimal receiver.

(b) (4 points) Determine the probability of symbol error.

4. *Convolutional code.* Consider the convolutional code whose encoder is described by the following diagram:



(a) (2 points) What is the rate of this code?

(b) (4 points) Draw the state diagram.

(c) (4 points) Draw the trellis diagram of this code.