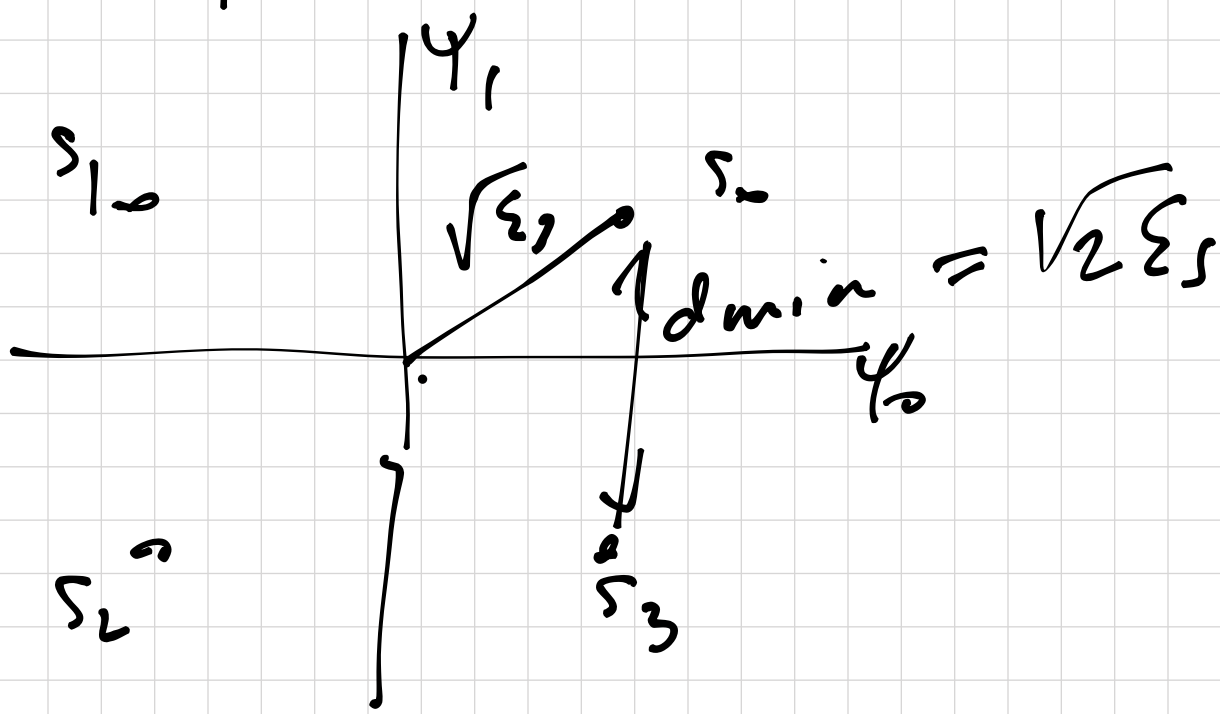
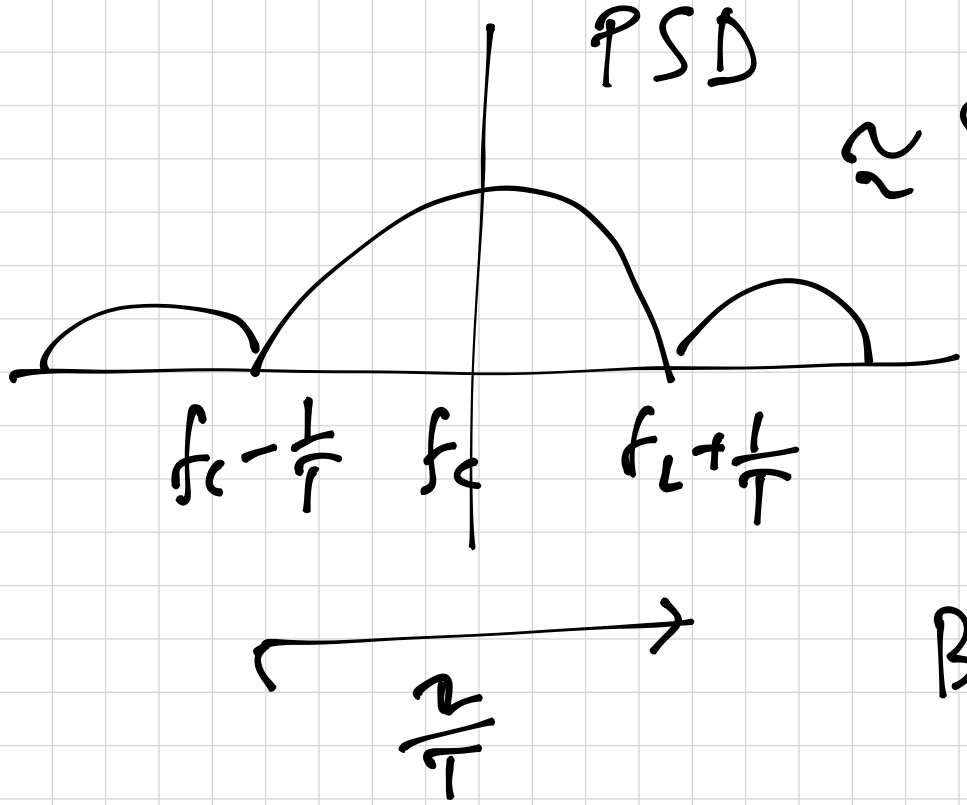
1 a) $\sqrt{\frac{2\mathcal{E}_s}{T}}\cos(2\pi f_c t) = \psi_0(t)$

$-\sqrt{\frac{2\mathcal{E}_s}{T}}\sin(2\pi f_c t) = \psi_1(t)$



b) $R_b = 1000\ b/s$ $\quad R_s = 500\ sym/s$

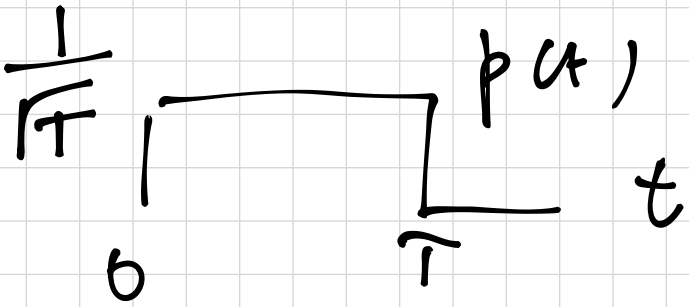


$\simeq S\,\mathrm{sinc}(fT)^2$

$T = \frac{1}{500}\ s$

$B = \frac{2}{T} = 1000\ Hz$

$$PSD = \frac{\mathcal{E}_s}{T} |P(f)|^2$$

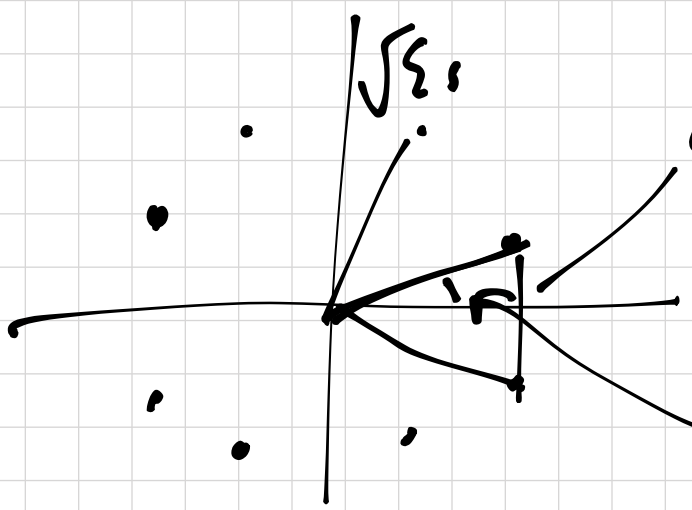


$$P(f) \propto sinc(\pi f T)$$

$$sinc(x) = \frac{\sin(\pi x)}{\pi x}$$

$$sinc(x) = \frac{\sin(x)}{x}$$

c)



$$d_{min} = 2\sqrt{\xi_s} \sin\left(\frac{\pi}{8}\right)$$

$$\alpha = \frac{\pi}{8}$$

$$2\sqrt{\xi_s^{(8)}} \sin\left(\frac{\pi}{8}\right) = \sqrt{2\,\xi_s^{(4)}}$$

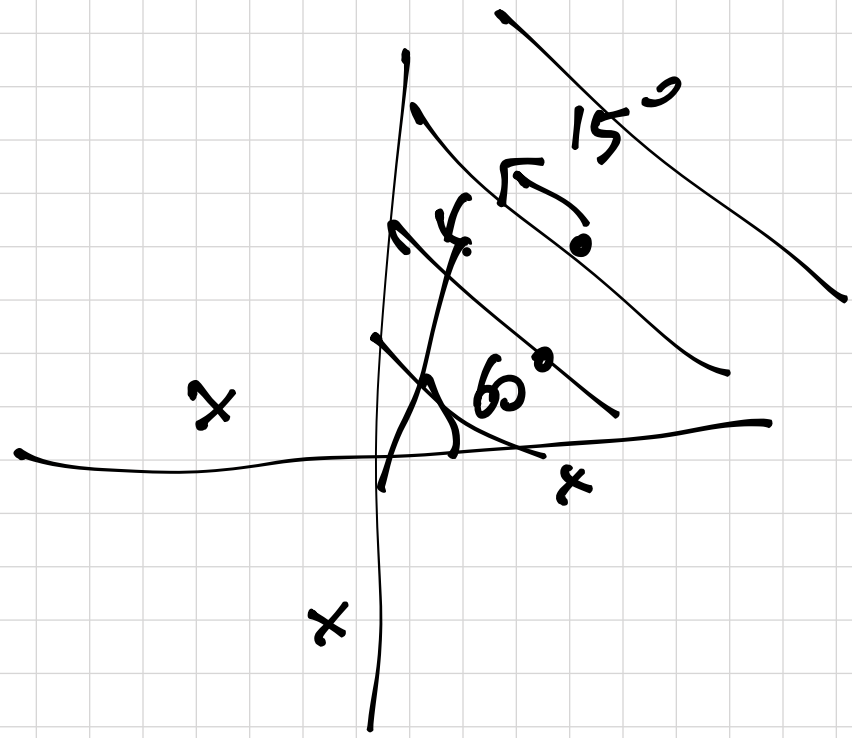
$\xi_s^{(8)}$ ↑ 8PSK

$\xi_s^{(4)}$ ↑ QPSK

⋮

⋮

d)



$n_1, n_2 \sim \mathcal{N}(0, \frac{N_0}{2})$

$$P(e|0) = 1 - P(c|0)$$

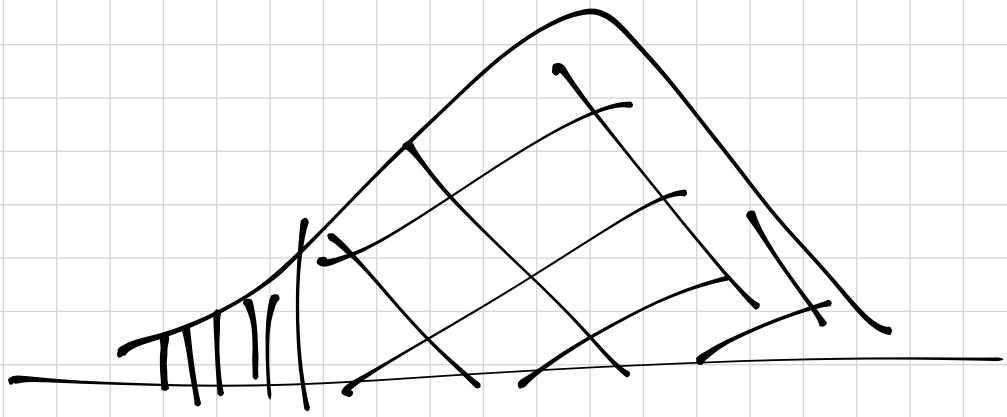
↑ symbol index

$$P(c|0) = P(n_1 > -\frac{\sqrt{3}}{2}\sqrt{\xi_1})$$

$$P(n_2 > -\frac{1}{2}\sqrt{\xi_1})$$

$$= \left(1 - Q\left(\frac{\frac{\sqrt{3}}{2}\sqrt{\xi_1}}{\sqrt{N_0/2}}\right)\right) \times$$

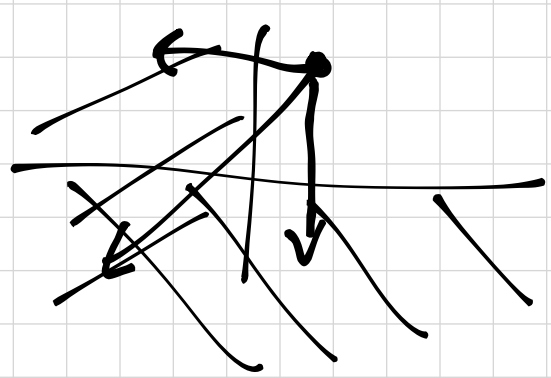
$$\left(1 - Q\left(\frac{\frac{1}{2}\sqrt{\xi_1}}{\sqrt{N_0/2}}\right)\right)$$



$$P(e) = \frac{1}{4} P(e|0) + \frac{1}{4} P(e|1)$$
$$+ \frac{1}{4} P(e|2) + \frac{1}{4} P(e|3)$$
$$= P(e|0)$$

$P(e) < P(\text{error in } n_1 \text{ direction})$
$+ P(\text{error in } n_2 \text{ direction})$
$= Q(\ ) + Q(\ )$

$P(n_1 < -\frac{\sqrt{3}}{2}\sqrt{\xi_r})$



2) OFDM 4 subcarriers

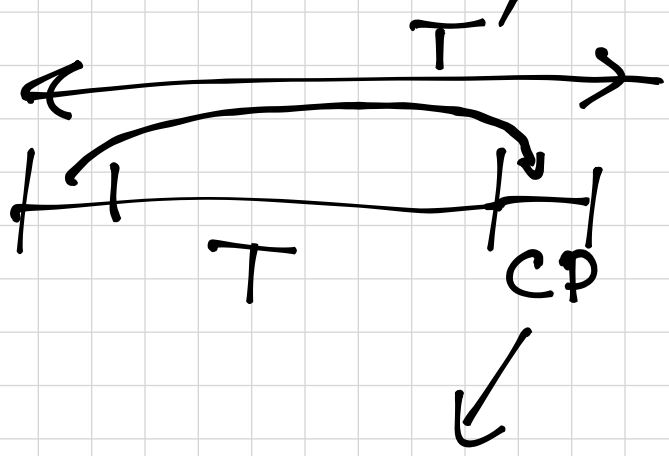
a) BPSK 5000 b/s

4 bits to simultaneously

$\Rightarrow \frac{5000}{4}$ sym/s

1250 sym/s

b) $T = \frac{1}{1250}\,s = 800\,ms$



1000 ms

$\frac{4 \text{ bit}}{1000\,\mu s} = 4000$ b/s

length of impulse response

$\Delta f = \frac{1}{T} = 1250$ Hz w/o CP

$\Delta f = \frac{1}{T'} = 1000$ Hz w/ CP

c) gains = 0, 1, 1/2, 1/3

tx $\mathcal{E}_b$ joules/bit

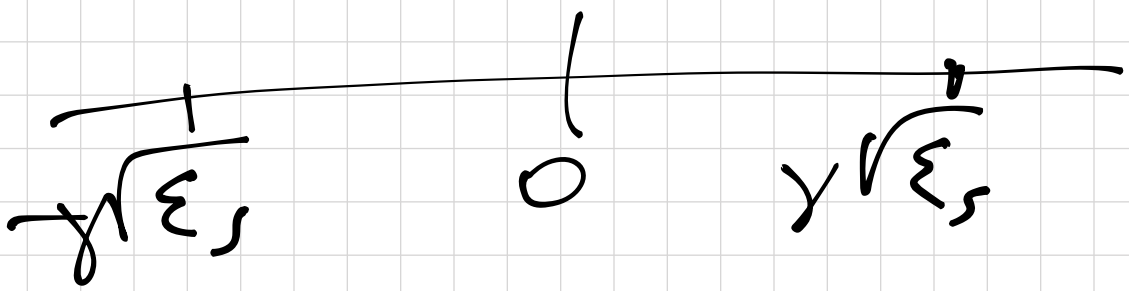
rx 0 joules/bit in channel 1

$\mathcal{E}_b$ 2

$\frac{1}{4}\mathcal{E}_b$ 3

$\frac{1}{9}\mathcal{E}_b$ 4

$$P(e) = Q\left(\sqrt{\frac{2\mathcal{E}_s}{N_0}}\right) \text{ for BPSK}$$



1. $P(e) = Q(0) = 0.5$

2: $P(e) = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$

3: $P(e) = Q\left(\sqrt{\frac{2\frac{1}{4}\mathcal{E}_b}{N_0}}\right)$

4: $P(e) = Q\left(\sqrt{\frac{2\frac{1}{9}\mathcal{E}_b}{N_0}}\right)$

a) $T' = 1000\ \mu s = 1 ms$

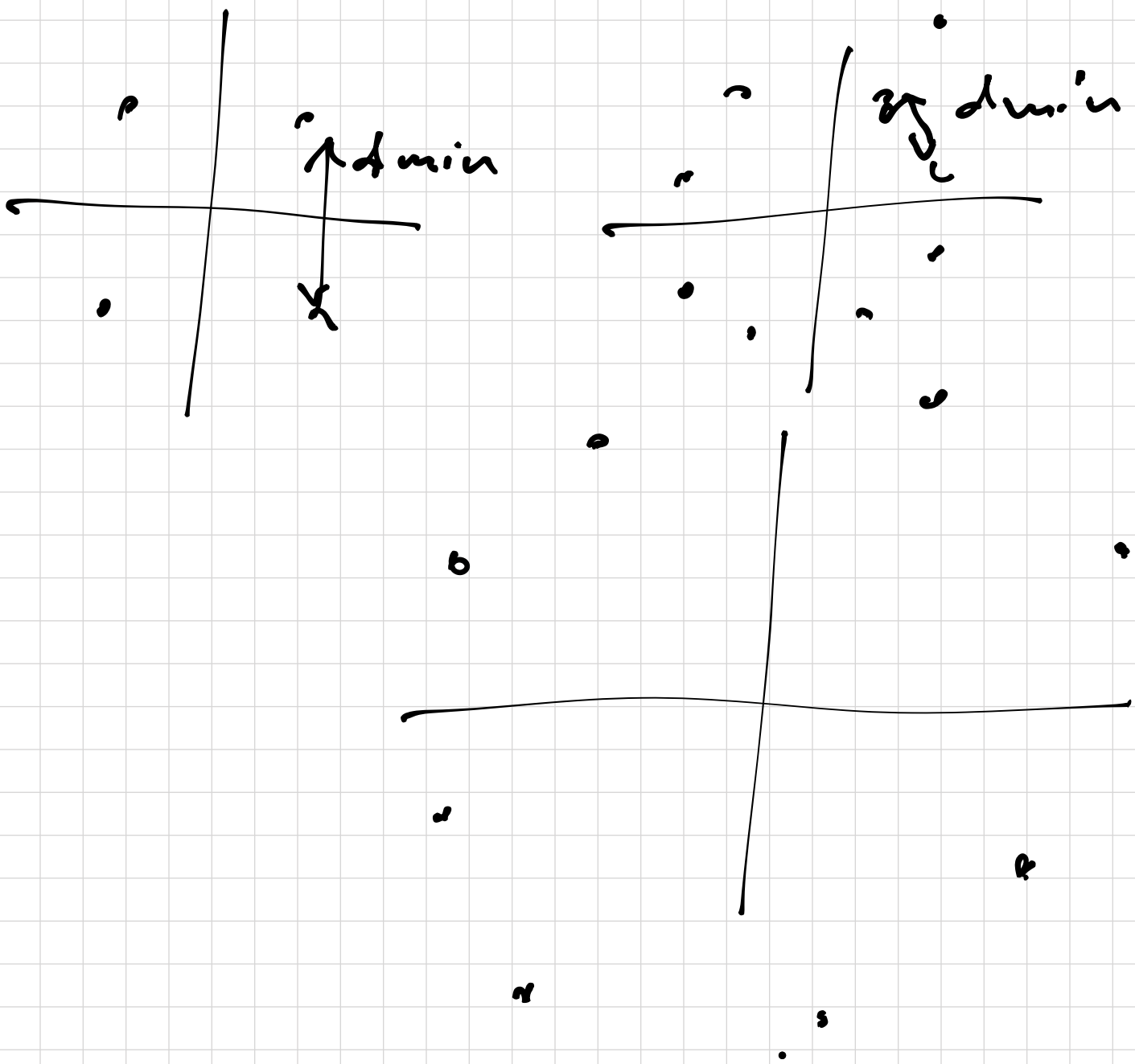
tx 6 bits / ms

allocate 0 bits on channel 1

3 bits to channel 2 (8 PSK)

2 bit to channel 3 (4 PSK)

1 bit to channel 4 (BPSK)



3) $G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$

$\underbrace{\quad\quad}_{I} \qquad \underbrace{\quad\quad}_{P}$

$k = 3 \qquad n = 6 \qquad r = \frac{k}{n} = \frac{1}{2}$

a) $H: \quad GH^T = 0 \leftarrow$

$H = \left[P^T \mid I\right]$

$\left[I \mid P\right]\left(\begin{matrix} P \\ I \end{matrix}\right) = P + P = 0$

H^T (k)

$\uparrow$

G

$H = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$ $(n-k) \times n$

3×6

b) $c_1 \in$ code

$c_2 \in$ code

then $c_1 + c_2 \in$ code

$\Rightarrow 0 \in$ code

d^H_{min} = minimum weight among all the codewords

$k = 3$ $\quad 2^k$ codewords $\quad u \downarrow G$ (info)

weight						
0	0	0	0	0	0	0
4	1	0	0	1	1	1
4	0	1	0	1	1	1
3	0	0	1	1	0	1
2	1	1	0	0	0	0
3	1	0	1	0	1	0
3	0	1	1	0	1	0
5	1	1	1	0	1	1

add 1 here

(row 1 + row 2) $u = [1\ 1\ 0]$

(row 1 + row 3) $u = [1\ 0\ 1]$

(row 2 + row 3) $u = [0\ 1\ 1]$

Sum all the rows $u = [1\ 1\ 1]$

$d^H_{min} = 2$

c) $u G_{ext} = \underline{c}_{ext} = [\underline{c} \; \underline{c'}]$

$\underline{c}$: $k \times n$; $\underline{c'}$: new bit

$$\underline{c'} = v \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 1$$

$u = [1 \; 1 \; 0]$ ← corresponds to

$\underline{c} = [1 \; 1 \; 0 \; 0 \; 0 \; 0]$

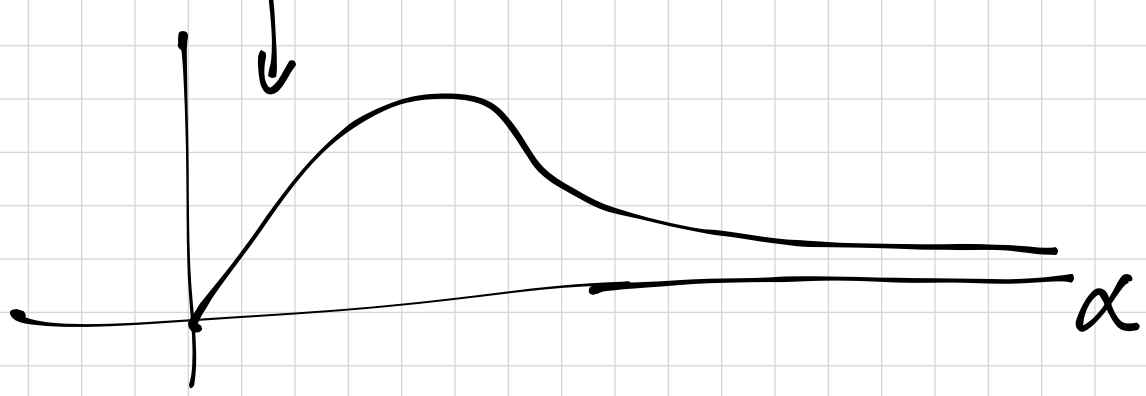
$a = 1, b = 0$

or $a = 0, b = 1$

c anything

4) $P(e|\alpha) = \frac{1}{2} e^{-\alpha^2 \frac{\mathcal{E}_b}{2N_0}}$ ← bit energy

$$f(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}, \quad \alpha > 0$$



$$P(e) = \int_0^\infty P(e|\alpha) f(\alpha) \, d\alpha$$

$$= \int_0^\infty \frac{1}{2} e^{-\alpha^2 \frac{\mathcal{E}_b}{2N_0}} \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} d\alpha$$

use formula with $n = 0$

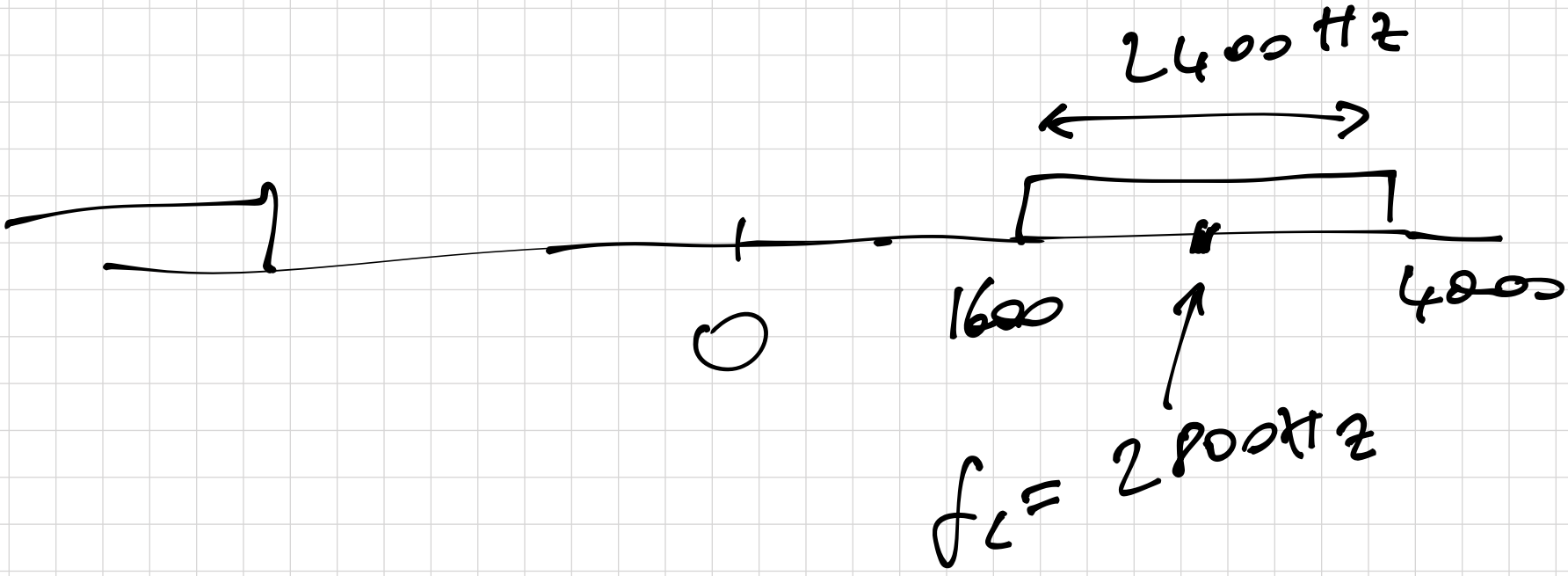
$$= \frac{1}{2\sigma^2} \int_0^\infty \alpha \, e^{-\alpha^2 \left(\frac{\mathcal{E}_b}{2N_0} + \frac{1}{2\sigma^2} \right)} d\alpha$$

← a

$$\int_0^\infty \alpha \, e^{-a\alpha^2} d\alpha = \frac{1}{2a}$$

5) 151

bandpass frequency 1600 Hz
to 4000 Hz

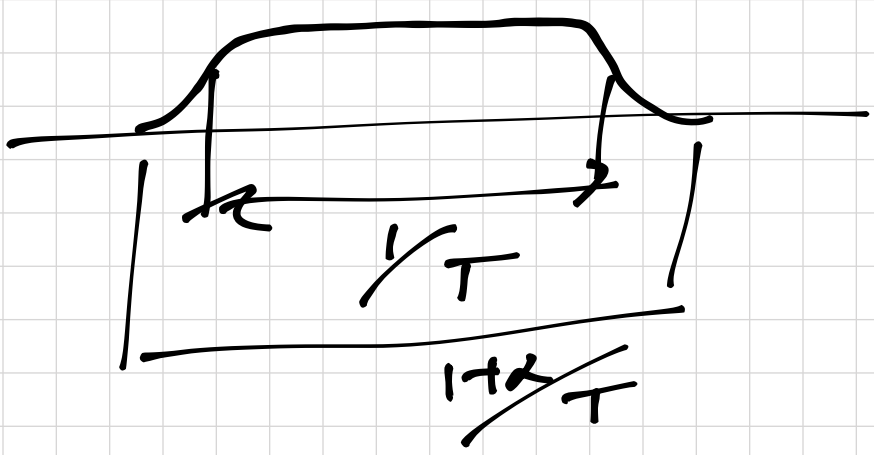


a) QPSK $f_c = 2800$ Hz

root raised cosine

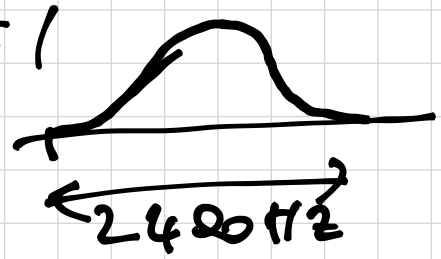
bit rate = 2400 b/s

$R_s = 1200$ sym/s $\quad \frac{1}{T} = 1200$ Hz



$\frac{1+\alpha}{T} = 2400$

$\alpha = 1$

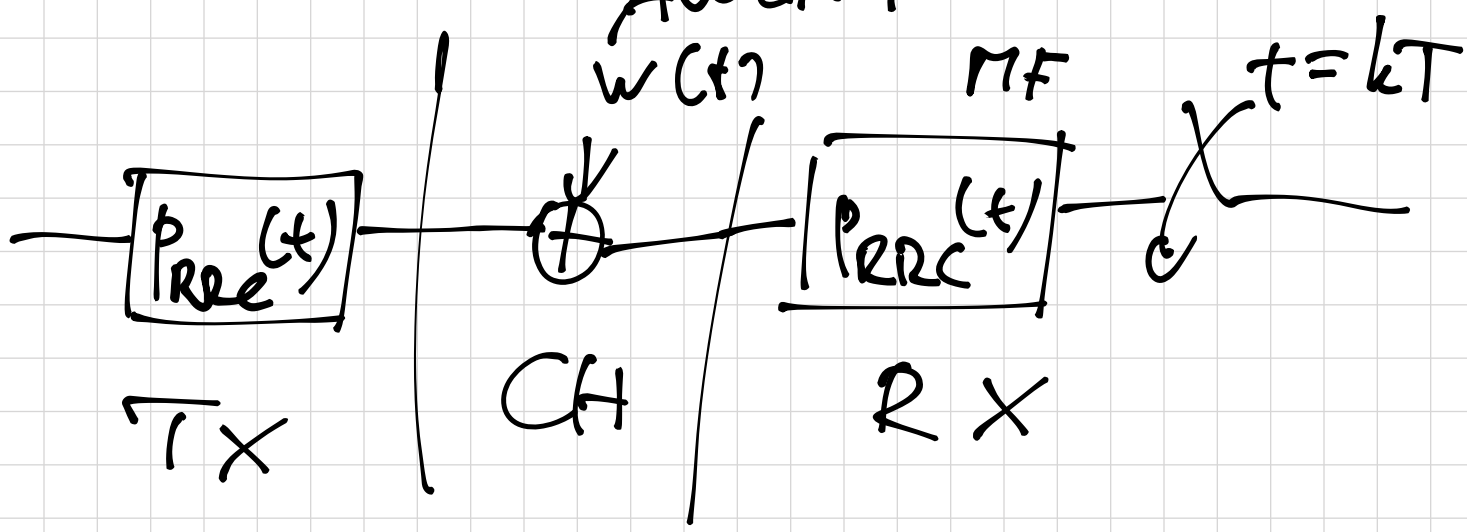


b) $R_b = 4800$ b/s

$R_s = 2400$ sym/s

$\frac{1}{T} = 2400$ Hz

$\frac{1+\alpha}{T} = 2400 \Rightarrow \alpha = 0$



c) 2400 sym/s $\rightarrow R_b = 9600$ b/s

$\Rightarrow$ 4 bits/sym $\Rightarrow$ 16 QAM

$\frac{1}{T} = 2400$ Hz $\qquad \frac{1+\alpha}{T} = 2400$ Hz

$\Rightarrow \alpha = 0$

$PSD = \frac{\mathcal{E}_s}{T} |P(f)|^2$