UCLA — Electrical and Computer Engineering Dept.
ECE132A: Introduction to Communication Systems
Final Exam
Monday, March 19, 2018

This exam has 5 questions, for a total of 100 points.

Closed book. No calculator. One two-sided cheat sheet allowed. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
Please, write your name and ID on the top of each loose sheet!

Name and ID: ____________________

Name of person on your left: ____________________

Name of person on your right: ____________________

Question	Points	Score
1	25	
2	25	
3	30	
4	5	
5	15	
Total:	100	

1. *Signal Constellation.* Consider a QPSK modulation with signals

$$s_k(t) = \sqrt{\frac{2\mathcal{E}_s}{T}} \cos\left(2\pi f_c t + k\frac{\pi}{2} + \frac{\pi}{4}\right), 0 \le t < T, k = 0, 1, 2, 3.$$

(a) (1 point) Draw the constellation and label the axes.

(b) (3 points) Suppose that the information bit rate is 1000 bit/s. What is the null-to-null bandwidth of the transmitted symbol? (Hint: consider the PSD of the modulated signal.)

(c) (6 points) Find the symbol energy of an 8PSK constellation with the same minimum Euclidean distance as the QPSK constellation above.

(d) (15 points) Consider the QPSK constellation above. Suppose the receiver's local frequency oscillator has an error of exactly 15° with respect to the transmitter. Find the new probability of symbol error. Your answer can be a bound, as opposed to a precise error probability. Assume AWGN with noise power spectral density of $\frac{N_0}{2}$. (Hint: consider a constellation that is rotated by 30° but where the decision boundaries are left unchanged.)

2. *OFDM.* Consider an OFDM transmission scheme with $N = 4$ subcarriers.

   (a) (5 points) If all subcarriers are BSPK-modulated, and the target bit rate is 5000 bit/s, what is the OFDM symbol rate? Or, equivalently, how many times per second will an OFDM symbol be transmitted?

   (b) (5 points) With the BPSK symbol durations of part (a), if the channel impulse response lasts 200 µs, what is the minimum subcarrier spacing, in hertz, that can be used? What is the new OFDM symbol rate? If each subcarrier is BPSK-modulated, what is the new bit rate?

   (c) (10 points) Suppose the channel gain is constant in each of the four subbands, and equal to 0, 1, 1/2, and 1/3, respectively. Let each subcarrier be BPSK-modulated, the energy per bit be $\mathcal{E}_\mathrm{b}$ (before entering the channel), and the noise power spectral density be $\frac{N_0}{2}$. Find the bit error probability in each subband. Simplify as much as you can.

   (d) (5 points) Using your knowledge of the channel gains, how would you reallocate the bits to each of the four OFDM subcarriers to transmit 6000 bit/s? In each subband, you can choose among BPSK, QPSK, and 8PSK. The channel impulse response and the modulated symbols have the same durations as in part (b).

3. *Block codes.* Consider a linear block code with the following generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

(a) (5 points) Find the parity-check matrix H for this generator matrix.

(b) (7 points) What is the minimum Hamming distance of this code? Provide an explanation for your answer.

(c) (18 points) Now extend the code by adding one more parity check bit. This is obtained by adding one column to the right of the generator matrix G to produce a new generator matrix

$$G_{\text{ext}} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & a \\ 0 & 1 & 0 & 1 & 1 & 1 & b \\ 0 & 0 & 1 & 1 & 0 & 1 & c \end{bmatrix},$$

where a, b, and c can be either 0 or 1. Among all the possible choices, provide the one(s) that correspond to the largest minimum Hamming distance for the extended code.

4. (5 points) *Rayleigh fading.*

The probability of error for a binary FSK modulation scheme with non-coherent detection on an AWGN channel is given by

$$\mathrm{P}(e) = \frac{1}{2}\mathrm{e}^{-\rho/2},$$

where $\rho = \alpha^2 \mathcal{E}_\mathrm{b}/N_0$, α is the channel attenuation factor, and $\mathcal{E}_\mathrm{b}$ is the average transmitted bit energy. In a flat-fading channel, the amplitude α is Rayleigh distributed with pdf:

$$f(\alpha) = \begin{cases} \frac{\alpha}{\sigma^2}\mathrm{e}^{-\alpha^2/(2\sigma^2)}, & \alpha > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Compute the expression for the bit error probability for binary FSK with non-coherent detection in a Rayleigh fading channel. (Hint: you may need $\int_0^\infty x^{2n+1}\mathrm{e}^{-ax^2}\,\mathrm{d}x = n!/(2a^{n+1})$, $a > 0$.)

5. *ISI channel.* Consider a channel with bandpass frequency spanning frequencies 1600 Hz to 4000 Hz.

   (a) (5 points) Let the data be QPSK-modulated with carrier frequency $f_{\mathrm{c}} = 2800\,\mathrm{Hz}$. Sketch and describe the block diagram of a system that uses root-raised cosine pulses to transmit at a bit rate of $R_{\mathrm{b}} = 2400\,\mathrm{bit/s}$. Make sure the entire frequency bandwidth is utilized.

   (b) (5 points) Repeat the previous part for a bit rate of $R_{\mathrm{b}} = 4800\,\mathrm{bit/s}$.

   (c) (5 points) Consider now a system that transmits at a symbol rate of 2400 symbols per second, with the target bit rate of $R_{\mathrm{b}} = 9600\,\mathrm{bit/s}$. Select an appropriate QAM signal constellation and the roll-off factor of the root-raised cosine pulse that utilizes the entire frequency band. Sketch the spectrum of the transmitted signal.

Q function table

***Q*(0.0)**	0.500000000	1/2.0000
***Q*(0.1)**	0.460172163	1/2.1731
***Q*(0.2)**	0.420740291	1/2.3768
***Q*(0.3)**	0.382088578	1/2.6172
***Q*(0.4)**	0.344578258	1/2.9021
***Q*(0.5)**	0.308537539	1/3.2411
***Q*(0.6)**	0.274253118	1/3.6463
***Q*(0.7)**	0.241963652	1/4.1329
***Q*(0.8)**	0.211855399	1/4.7202
***Q*(0.9)**	0.184060125	1/5.4330

***Q*(1.0)**	0.158655254	1/6.3030
***Q*(1.1)**	0.135666061	1/7.3710
***Q*(1.2)**	0.115069670	1/8.6904
***Q*(1.3)**	0.096800485	1/10.3305
***Q*(1.4)**	0.080756659	1/12.3829
***Q*(1.5)**	0.066807201	1/14.9684
***Q*(1.6)**	0.054799292	1/18.2484
***Q*(1.7)**	0.044565463	1/22.4389
***Q*(1.8)**	0.035930319	1/27.8316
***Q*(1.9)**	0.028716560	1/34.8231

***Q*(2.0)**	0.022750132	1/43.9558
***Q*(2.1)**	0.017864421	1/55.9772
***Q*(2.2)**	0.013903448	1/71.9246
***Q*(2.3)**	0.010724110	1/93.2478
***Q*(2.4)**	0.008197536	1/121.9879
***Q*(2.5)**	0.006209665	1/161.0393
***Q*(2.6)**	0.004661188	1/214.5376
***Q*(2.7)**	0.003466974	1/288.4360
***Q*(2.8)**	0.002555130	1/391.3695
***Q*(2.9)**	0.001865813	1/535.9593

***Q*(3.0)**	0.001349898	1/740.7967
***Q*(3.1)**	0.000967603	1/1033.4815
***Q*(3.2)**	0.000687138	1/1455.3119
***Q*(3.3)**	0.000483424	1/2068.5769
***Q*(3.4)**	0.000336929	1/2967.9820
***Q*(3.5)**	0.000232629	1/4298.6887
***Q*(3.6)**	0.000159109	1/6285.0158
***Q*(3.7)**	0.000107800	1/9276.4608
***Q*(3.8)**	0.000072348	1/13822.0738
***Q*(3.9)**	0.000048096	1/20791.6011
***Q*(4.0)**	0.000031671	1/31574.3855