EE 132A
Introduction to Communication Systems
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Diagnostic Quiz

1. For continuous signal

$$x(t) = \begin{cases} 1, & -\frac{T}{2} \le t \le \frac{T}{2} \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the convolution of $x(t)$ with itself, $y(t) = x(t) * x(t)$.

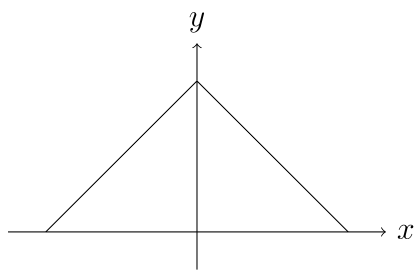
   (b) Find the Fourier transform of $y(t)$.

   Solution:

   (a)

$$y(t) = x(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau = \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} x(\tau)d\tau$$

$$= \begin{cases} 0, & t < -T \\ t+T, & -T \le t < 0 \\ -t+T, & 0 \le t \le T \\ 0, & t > T \end{cases} = \begin{cases} |t| + T, & |t| \le T \\ 0, & \text{otherwise} \end{cases}$$



   (b)

$$X(f) = \int_{\alpha}^{\alpha+T} x(t)e^{-j2\pi ft}dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-j2\pi ft}dt = \frac{1}{-j2\pi f}\left(e^{-j\pi fT} - e^{j\pi fT}\right)$$

$$= \frac{1}{\pi f}\sin(\pi fT) = T\text{sinc}(fT)$$

$$Y(f) = X(f)^2 = T^2\text{sinc}^2(fT)$$

2. Compute the Fourier series coefficients of $x(t) = 2\sin(2t)$.
   Solution:

The period of the signal is equal to $T_0 = \pi$.

$$x(t) = 2\sin(2t) = \frac{1}{j}e^{j2t} - \frac{1}{j}e^{-j2t} = \sum_{n=-\infty}^{\infty} x_n e^{j2\pi\frac{n}{\pi}t}$$

$$x_n = \begin{cases} \frac{1}{j}, & n = 1 \\ \frac{-1}{j}, & n = -1 \\ 0, & \text{otherwise.} \end{cases}$$

3. Suppose X is a continuous random variable uniformly distributed on the interval $(0, 1)$. Compute $E[X^n]$ for arbitrary positive integer n.
   Solution:

$$f_x(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

$$E[X^n] = \int_{-\infty}^{\infty} x^n f_x(x)dx = \int_0^1 x^n dx = \left.\frac{x^{n+1}}{n+1}\right|_0^1 = \frac{1}{n+1}$$

4. Suppose $X_1, \cdots, X_n$ are i.i.d. Bernouli (p) random variables. Do you remember the distribution of $Y = X_1 + X_2 + \cdots + X_n$? What is $P(Y = k)$ for arbitrary k?
   Solution:
   The distribution of Y is binomial.

$$X_i = \begin{cases} 1, & \text{with probability } p \\ 0, & \text{with probability } 1 - p. \end{cases}$$

$$P(Y = k) = \binom{n}{k} p^k (1 - p)^{n-k}.$$