EE 132A
Introduction to Communication Systems
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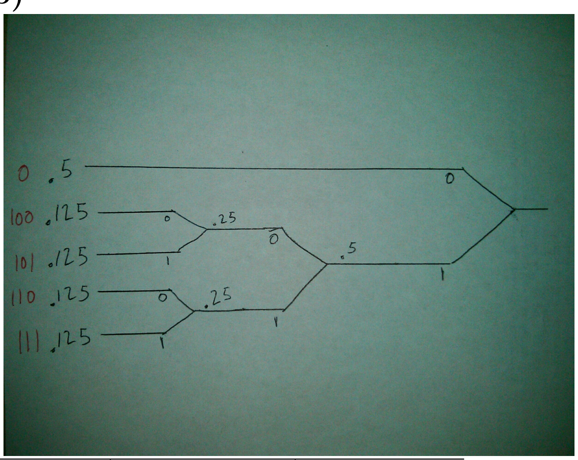
Spring 2015 Midterm Solutions
Wednesday, May 6, 2015

Problem 1
(a)

$$\begin{aligned} E[X] &= \frac{1}{2}(-4) + \frac{1}{8}(-2+0+2+4) \\ &= -2 + \frac{4}{8} \\ &= -1.5 \end{aligned}$$

$$\begin{aligned} H(X) &= \frac{1}{2}\log_2(2) + \frac{4}{8}\log_2(8) \\ &= \frac{1}{2}(1) + \frac{1}{2}(3) \\ &= 2 \text{ bits} \end{aligned}$$

(b)



Symbol	Probability	Codeword
A	0.5	0
B	0.125	100
C	0.125	101
D	0.125	010
E	0.125	111

This Huffman coding is not unique, but any correct answer will have a symbol with length 1 for the most common value, and the other 4 symbols with have lengths 3. In addition, the code must be a prefix code.

$$
\begin{aligned}
E(L) &= \frac{1}{2}(1) + \frac{1}{8}(4)(3) \\
&= \frac{1}{2} + \frac{3}{2} \\
&= 2 \text{ bits}
\end{aligned}
$$

(c)

$p(y)$	Y
$\frac{5}{8}$	16
$\frac{2}{8}$	4
$\frac{1}{8}$	0

$$
\begin{aligned}
E[Y] &= \frac{5}{8}(16) + \frac{1}{8}(4) + \frac{1}{8}(0) \\
&= 10 + 1 + 0 \\
&= 11
\end{aligned}
$$

$$
\begin{aligned}
H(Y) &= \frac{5}{8}\log_2\left(\frac{8}{5}\right) + \frac{1}{4}\log_2(4) + \frac{1}{8}\log(8) \\
&= \frac{5}{8}\log_2\left(\frac{8}{5}\right) + \frac{1}{2} + \frac{3}{8} \\
&= \frac{5}{8}\log_2\left(\frac{8}{5}\right) + \frac{7}{8} \qquad \text{bits}
\end{aligned}
$$

Problem 2

(a)

$$
\begin{aligned}
\mathrm{COV}(X(t_i), X(t_j)) &= R_X(t_i, t_j) - \mu_X(t_i)\mu_X(t_j) \\
\mathrm{COV}(X(t_i), X(t_j)) &= R_X(t_i, t_j) \\
\sigma^2 \max(t_i, t_j) &= R_X(t_i, t_j)
\end{aligned}
$$

(b)

X(t) is **not** WSS because the autocorrelation of X(t) is not a function of the time difference $t_i - t_j$, but rather it depends on the actual values of t_i and t_j.

Problem 3
We **cannot** conclude from the given information that $X(t)$ is strictly stationary.

We are not given any information about the joint distributions between different time instances. We are only given that the process $X(t)$ is zero-mean with variance σ^2 at any particular time instance. However, we could have different probability distributions for the random process at different time instances that satisfy the mean and variance requirements. Thus in this situation the statistics of the process would not be time invariant, and hence we cannot conclude that $X(t)$ is strictly stationary.

(Additionally, we cannot even conclude that $X(t)$ is WSS. We do not have enough to conclude that the aurocorrelation of $X(t)$, $R_X(t_1, t_2)$ only depends on the time difference between t_1 and t_2.)

Problem 4
Let $\tau = t_1 - t_2$.

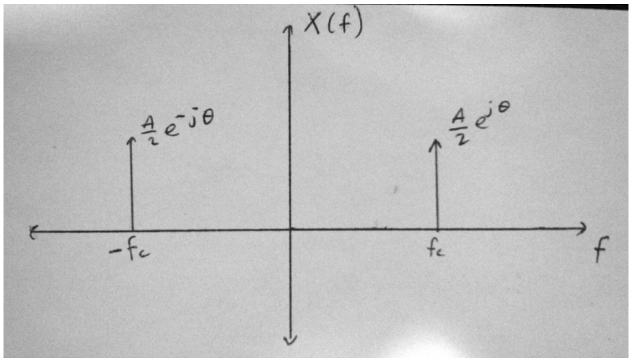
$$\begin{aligned} R_{YX}(\tau) &= E[Y(t_1)X(t_2)] \\ &= E[X(t_2)Y(t_1)] \\ &= R_{XY}(t_2, t_1) \\ &= R_{XY}(-\tau) \end{aligned}$$

We see that $R_{YX}(\tau) = R_{XY}(-\tau)$. Thus $R_{YX}(\tau) = R_{XY}(u)$ for $u = -\tau$.

Note that the cross-correlation function of jointly WSS processes is not necessarily an even function (this is only true for auto-correlation functions of WSS processes), so the answer $R_{YX}(\tau) = R_{XY}(\tau)$ (hence $u = \tau$) is incorrect.

Problem 5

$$\begin{aligned} x(t) &= A\cos(2\pi f_c t + \theta) \\ x(t) &= \frac{A}{2}e^{j\theta}e^{j2\pi f_c t} + \frac{A}{2}e^{-j\theta}e^{-j2\pi f_c t} \\ X(f) &= \frac{A}{2}[e^{j\theta}\delta(f - f_c) + e^{-j\theta}\delta(f + f_c)] \end{aligned}$$



To represent $x(t)$ in terms of its in-phase and quadrature components, we can decompose $x(t)$ into the following form: $x(t) = x_c(t)\cos(2\pi f_c t) - x_s(t)\sin(2\pi f_0 t)$.

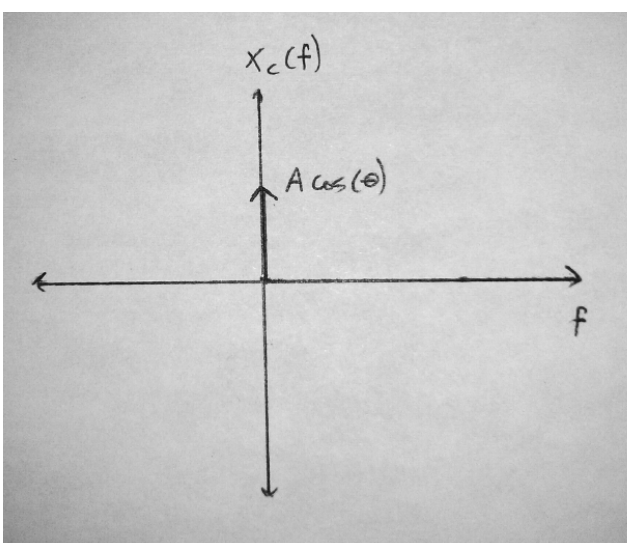
$$x(t) = A\cos(2\pi f_c t + \theta)$$
$$x(t) = A\cos(\theta)\cos(2\pi f_c t) - A\sin(\theta)\sin(2\pi f_c t)$$

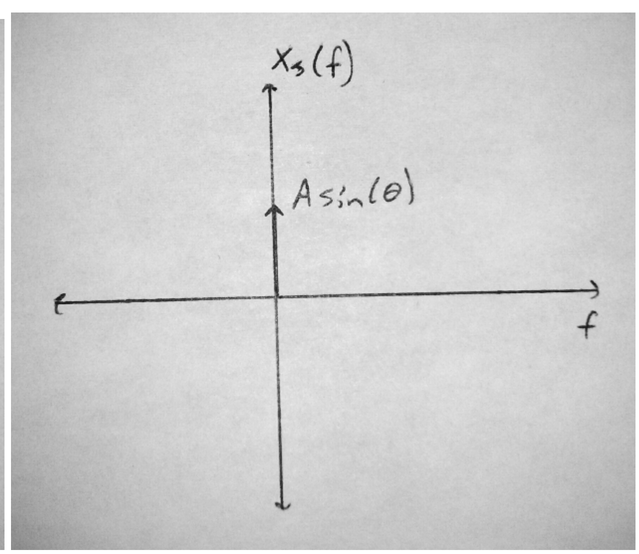
Thus we see:

$$x_c(t) = A\cos(\theta)$$
$$x_s(t) = A\sin(\theta)$$

Equivalently, in the frequency domain:

$$X_c(f) = A\cos(\theta)\delta(f)$$
$$X_s(f) = A\sin(\theta)\delta(f)$$





Problem 6

a.	False
b.	True
c.	False
d.	True
e.	False
f.	False
g.	True
h.	True
i.	True
j.	False