1. **a.** $P_2 = 1 - q^4 - 4q^3(1-q)$.

   b. $P_3 = 1 - q^3 - 3q^2(1-q)$.

   c. $P_2 > P_3 \Leftrightarrow 1-q^4-4q^3(1-q) > 1-q^3-3q^2(1-q) \Leftrightarrow q((4-3q) < (3-2q) \Leftrightarrow 4q-3q^2 < 3-2q \Leftrightarrow -3q^2+6q < 3 \Leftrightarrow -q^2+2Q < 1 \Leftrightarrow 0 < 1-2q+q^3 \Leftrightarrow 0 < (1-q)^2$.

2. We must have $\int_{-\infty}^{\infty} f(x), dx = 1$. Thus,

$$\int_{-\infty}^{\infty} f(x)\,dx \;=\; \int_{-\infty}^{0} b\,\exp(\beta\,x)\,dx \;+\; \int_{0}^{\infty} a\,\exp(-\alpha\,x)\,dx$$

$$=\;(b/\beta)\exp(\beta\,x)|\begin{matrix}0\\-\infty\end{matrix}\;+\;(a/-\alpha)\exp(-\alpha\,x)|\begin{matrix}\infty\\0\end{matrix}\;=\;\frac{b}{\beta}+\frac{a}{\alpha}\;=\;1\,.$$

   Thus, for $f(x)$ to be a valid pdf, we must have $(b/\beta)+(a/\alpha)=1$.

3. **a.** $P(B) = 2/3$.

   b. $P(A) = P(A|B)P(B) + P(A|B^c)P(B^c) = (4/5)(2/3) + (19/20)(1/3) = 51/60$.

   c. $P(B|A^c)$.

   d.

$$P(B|A^c) = \frac{P(B\cap A^c)}{P(A^c)} = \frac{P(A^c|B)P(B)}{P(A^c)} = \frac{(1/5)(2/3)}{1-(51/60)} = 8/9\,.$$

4. $A = \{TTT\,, HHH\}$. Thus, $P(A) = 1/4$. $B = \{TTT, HTT, THT, TTH\}$. Thus, $P(B) = 1/2$. $A\cap B = \{TTT\}$. Thus, $P(A\cap B) = 1/8 = P(A)P(B) = (1/4)(1/2)$. Thus, event A and event B are independent.