EE 131A **Midterm Exam** **Winter 2013**

(60 minutes/25 pts - An 8.5"x11" sheet(two-sided) is allowed) **K. Yao**

1. A bakery makes 80 loaves of bread daily. Ten of them are underweight. An inspector weighs 5 loaves at random. What is the probability that an underweight loaf will be discovered? (You can leave your answers in terms of C_r^n. No need to obtain numerical numbers). (5 pts)

2. One per thousand of a population is subject to certain kinds of accident each year. Given that an insurance company has insured 5,000 persons from the population, find the probability that at most 2 persons will incur this accident. (Use the Poisson approximation and give your answer as a real-valued number.) (6 pts)

3. A person goes to work following one of three routes denoted by A, B, and C. His choice of the route is the same regardless of the weather. The probabilities of being late, if it is sunny and if the person is using the A, B, and C routes are 0.05, 0.10, and 0.15, respectively. Assume that, on average, one in every four days is rainy. Let L denote the event that the person arrives late, and let SU denote the event that the day is sunny. Given that it is a sunny day and that person arrives late, what is the probability that the person took route C? Hint: What are $P(A|SU)$, $P(B|SU)$, $P(C|SU)$, $P(L|A \cap SU)$, $P(L|B \cap SU)$, and $P(L|C \cap SU)$? (8 pts)

4. The height of men is normally (i.e., Gaussianly) distributed with a mean of $\mu = 167$ cm and a standard deviation of $\sigma = 3$ cm. What is the probabilities of the population of men that have height:

   a. Greater than 167 cm? (2 pts)

   b. Greater than 170 cm? (1 pt)

   c. Between 161 cm and 173 cm? (3 pts)

   Express your answers as real-valued numbers. Note: $\Phi(1) = 0.8413$ and $\Phi(2) = 0.9772$.

1.

$$P(\text{An underweight loaf will be discovered}) = 1 - \frac{C_0^{10} C_5^{70}}{C_5^{80}} = 1 - \frac{C_5^{70}}{C_5^{80}}.$$

2. Let X be the number of persons that incurs an accident in a year. Then X obeys a binomial law with $n = 5,000$ and $p = 1/1,000$. Since $\lambda = np = 5$, then the Poisson approximation to the binomial distribution holds. Thus,

$$P(X \leq 2) = exp(-5)(\frac{5^0}{0!} + \frac{5}{1!} + \frac{5^2}{2!}) = exp(-5) * (18.5) = 0.12465\,.$$

3. We know that $P(A|SU) = P(B|SU) = P(C|SU) = 1/3$. We also know $P(L|A \cap SU) = 0.05$, $P(L|B \cap SI) = 0.10$, and $P(L|C \cap SU) = 0.15$. Then by Bayes Rule and Total Theorem of Probability we have

$$P(C|L \cap SU) = \frac{P(C \cap L \cap SU)}{P(L \cap SU)} = \frac{P(SU)P(L \cap C|SU)}{P(SU)P(L|SU)}$$

$$= \frac{P(C|SU)P(L|C \cap SU)}{P(A|SU)P(L|A \cap SU) + P(B|SU)P(L|B \cap SU) + P(C|SU)P(L|C \cap SU)}$$

$$= \frac{(1/3)(0.15)}{(1/3)(0.05) + (1/3)(0.10) + (1/3)(0.15)} = \frac{0.15}{0.30} = 0.50\ .$$

4. **a.** Since $X \sim N(167, 9)$, then

$$P(X > 167) = P\left(\frac{X - 167}{3} > \frac{167 - 167}{3}\right) = P(Z > 0) = 0.5\,,$$

where $Z \sim N(0, 1)$.

b.

$$P(X > 170) = P\left(\frac{X - 167}{3} > \frac{170 - 167}{3}\right) = P(Z > 1)$$

$$= 1 - P(Z < 1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587 \simeq 0.16\,.$$

c.

$$P(161 < X < 173) = P\left(\frac{161 - 167}{3} < \frac{X - 167}{3} > \frac{173 - 167}{3}\right)$$

$$= P(-2 < Z < 2) = \Phi(2) - \Phi(-2) = \Phi(2) - (1 - \Phi(2))$$

$$= 2\Phi(2) - 1 = 2 * 0.9772 - 1 = 0.9544 \simeq 0.95\,.$$