EE 131A **Midterm Exam (25 pts; 60 min.)** **Winter 2006**
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1. **a.** In how many ways can four men and four women be seated in a row if men and women must occupy alternate seats? (2 pts)

   b. How many ways can they be seated if only a man is required to sit in the first seat? (2 pts)
   Express both answers as integers.

2. Suppose there are three boxes numbered 1, 2, and 3 (denoted by B_1, B_2, and B_3, respectively), with different number of colored marbles in each box as shown below in the second and third columns.

	No. of marbles	
Box No.	Red	White
1	5	5
2	5	10
3	10	5

   A box is selected depending on the outcomes of the toss of two fair coins. Box 1 is selected if two heads appear, Box 2 is selected if two tails appear, otherwise Box 3 is selected.

   a. What are the probabilities of $P(B_i)$, $i = 1, 2, 3$? (1 pt)

   b. Let A denote the event the marble is white. What are the conditional probabilities of $P(A|B_1)$, $i = 1, 2, 3$? (1 pt)

   c. What is the probability of $P(A)$? (Express your result as a fraction.) (2 pts.)

   d. What is the conditional probability of $P(B_1|A)$? (3 pts.)

3. A wheel used in gambling can stop in 18 equally likely positions numbered from 1 to 18. A person places bets on all the positions divisible by 3.

   a. What is the probability that the person will win exactly 3 times in 8 attempts? (2 pts)

   b. What is the probability that the person will win 3 or more times in 8 attempts? (4 pts)

   Express both of your solutions as three significant digit decimals.

4. Consider a discrete r.v. X with $P(X = x) = p(x) = 1/2^n$, for all positive integers $n = 1, 2, \ldots$.

   a. Write the cdf $F_X(x), -\infty < x < \infty$, of this r.v. (2 pts)

   b. Sketch $F_X(x), -\infty < x < \infty$, and label all points of interest. (1 pt)

   c. Write the pdf $f_X(x), -\infty < x < \infty$, of this r.v. (2 pts)

   d. Sketch $f_X(x), -\infty < x < \infty$, and label all points of interest. (1 pt)

   e. What is the smallest x satisfying $P(X \leq x) = 15/16$? (1 pt)

   f. What is the largest x satisfying $P(X \leq x) = 15/16$? (1 pt)

1. **a.** $2x4x4x3x3x2x2x1x1 = 1152$.

   b. $4x7! = 2880$.

2. **a.** $P(B_1) = 1/4$, $P(B_2) = 1/4$, $P(B_3) = 2/4 = 1/2$.

   b. $P(A|B_1) = 5/10 = 1/2$, $P(A|B_2) = 10/15 = 2/3$, $P(A|B_3) = 5/15 = 1/3$.

   c. Use the Theory of Total Probability. $P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) = (1/2)x(1/4) + (2/3)x(1/4) + (1/3)x(1/2) = 11/24$.

   d. Use Bayes Rule.

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_1)} = \frac{(1/2)x(1/4)}{(11/24)} = 3/11.$$

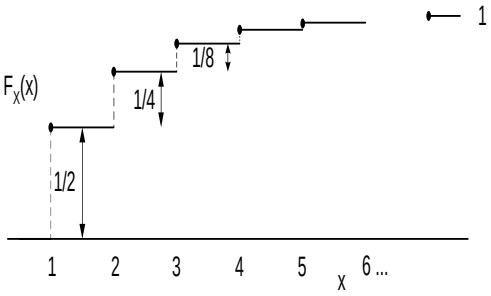
3. **a.** Since there are 6 possible number from 1 to 18 divisible by 3, and each one is equally likely, then the probability of winning at each attempt is $p = 6/18 = 1/3$. The probability of losing at each attempt is $q = 2/3$. Since this is a Bernoulli trail of length $n = 8$, then $p(3) = C_3^8(1/3)^3(2/3)^5 = 1792/6561 = 0.273$.

   b.

$$\sum_{k=3}^{8} p(k) = 1 - \sum_{k=0}^{2} p(k) = 1 - C_0^8 p^0 q^8 - C_1^8 p^1 q^7 - C_2^8 p^2 q^8 = \frac{3489}{6561} = 0.532$$

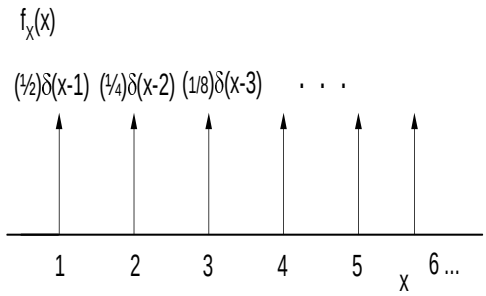
4. **a.** $F_X(x) = \sum_{n=1}^{\infty}(1/2^n)U(x-n)$, $\infty < x < \infty$.

   b. The cdf $F_X(x)$ is given below.



   c. $f_X(x) = \sum_{n=1}^{\infty}(1/2^n)\delta(x-n)$, $\infty < x < \infty$.

   d. The pdf $f_X(x)$ is given below.



   e. Since $(1/2) + (1/4) + (1/8) + (1/16) = (15/16)$, the smallest x satisfying $P(X \leq x) = (15/16)$ is 4, since $P(X \leq 3.99999...) = (14/16)$.

   f. Since $P(X \leq 4.99999...) = (15/16)$ but $P(X \leq 5) = (31/32)$, thus the largest x satisfying $P(X \leq x) = (15/16)$ is $4.99999... = 5^-$.