EE 131A Midterm Exam

Wednesday February 17, 2010

DO NOT OPEN UNTIL YOU ARE TOLD TO DO SO

You have 1 hr. and 45 min.

Only this booklet and TWO sheets of notes should be on your desk
Lecture Notes, Homework solutions, and books are not allowed

Write your answer to each question in the space provided

Read the problem statements carefully
Show your work and reasoning clearly

For partial credit justify your results:
Simply writing down one line with the correct answer is not adequate

Your Name and Student Id#: ______________________________

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Name and Student Id: ______________________

Problem 1. *(20pts)* Let $P[A|B]=0.5$, $P[B]=0.2$, and $P[A \cup B]=0.80$.

(a) (10 pts) Find $P[A \cap B]$ and $P[A]$.

(b) (5 pts) Are events A and B independent? Are they mutually exclusive?

(c) (5pts) Find $P[(A \cup B)^c]$ and $P[B^c|A^c]$.

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Problem 2. *(20 pts.)* Suppose that an insurance company classifies people into one of three classes – **good** risks, **average** risks, and **bad** risks. Their records indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are, 0.05, 0.15, and 0.30, respectively.

a) If 20% of the population are good risks, 50% are average risks, and 30% are bad risks, what proportion of people have accidents in a fixed year? **[Total Probability]**

b) If a certain policy holder, Ms. X, does not have an accident in the first year, then what is the probability she is a good risk? **[Baye's Theorem]**

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Problem 3. *(20pts)* In the 2003 world series, suppose that an analyst had argued that the Yankees and the Marlins were evenly balanced, i.e., the probability of each team winning a game was 50% and that every game was independent of the others.

a) *(10 pts)* Given such an analysis, what is the probability that the championship would be **decided in four games**?
b) *(10 pts)* Given such an analysis, what is the probability that the championship would be **decided in five games**?

Hint: Recall that the **winner is the first team to win four games,** and a **maximum of seven games are played**.

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Problem 4. *(20 pts.)*

(a) *(10 pts)* In a game of bridge involving four players, identified as **North, South, East** and **West**, calculate the probability that North and South together get 8 spades **and** West gets 4 spades.

(b) *(10 pts)* A box has **9 unused** and **6 used** tennis balls. A player randomly picks **three** of the 15 balls and uses the selected balls to play with, and **puts them all back** in the box after he is done playing. Later, a second player picks **three balls** at random from the box. What is the probability that **all the three balls** picked by the second player **are unused**?
Hint: *The number of unused balls in the box when the second player arrives depends on the random pick made by the first player. For example, if only one of the three balls picked by the first player is unused, then for the second player, there will be 8 unused balls and 7 used ones.*

Problem 5. *(20 pts.)* An audio player uses a low-quality hard drive. The hard drive fails after each month of use with probability 1/12. The cost to repair the hard drive is \$20. If a 1-year warranty is offered, how much should the manufacturer charge so that the probability of losing money on a player is 1% or less? What is the average cost per player?

Hint: *The number of times the hard drive fails in a twelve month period is a Binomial Random variable with $n=12$, and $p=1/12$.*

EE131A (Winter 2010) Mid-term solution

1. **(a)** $P[A\cap B]= P[A|B]\ P[B]=(0.5)(0.2)=0.1$

$P[A]= P[A\cup B]-P[B]+P[A\cap B]=0.8-0.2+0.1=0.7$

(b) $P[A]P[B]=(0.7)(0.2)=0.14\neq P[A\cap B]=0.1$

So, they are not independent.

Since $P[A\cap B]\neq 0$, they are not mutually exclusive.

(c) $P[(A\cup B)^c]=1-P[A\cup B]=1-0.8=0.2$

$P[B^c|A^c]=P[B^c\cap A^c]/P[A^c]=P[(A\cup B)^c]/P[A^c]=0.2/(1-0.7)=0.67$

2. (a) *P[Accident]=P[**Accident|| Good Risks**]P[**Good Risks**]+P[**Accident | Average Risks**]P[**Average Risks**]+P[**Accident | Bad Risk**]P[**Bad Risk**]*

$= (0.05)(0.2)+(0.15)(0.5)+(0.3)(0.3)=0.175$

(b)
$$P(G\,|\,A^c)=P(A^c\,|\,G)P(G)/P(A^c)$$
$$=(0.95)(0.2)/(1-0.175)=0.230$$

3. (a)
$$P[\text{World series is decided in 4 games }]=P(\text{Yankees win the first four}: WWWW) + P(\text{Yankees lose first four}: LLLL)=2(\frac{1}{2})^4=\frac{1}{8}$$

(b) The outcomes where the Yankees win in 5 games are:

$$\underbrace{\text{Yankees win exactly 3 of the first four games}}\ \underbrace{\text{Yankees win game 5}}$$

Similarly, the outcomes where the Yankees lose in 5 games are:

$$\underbrace{\text{Yankees lose exactly 3 of the first four games}}\ \underbrace{\text{Yankees lose game 5}}$$

$$P[\text{world series is decided in 5 games }]=2\binom{4}{3}(\frac{1}{2})^5=\frac{1}{4}$$

4. (a) There are many equivalent ways of calculating the probability and will spell out four of them. They all give the same answer (verify! If you do not believe ☺). Any one of such correct answers will suffice.

$$P=\frac{\begin{pmatrix}\text{No. of ways to distribute 8 spades}\\ \text{and 18 non-spades to N-S and 4}\\ \text{spades and 9 non-spades to W}\end{pmatrix}}{\begin{pmatrix}\text{No. of ways to distribute 26 cards to}\\ \text{N-S and 13 cards each to E and W}\end{pmatrix}}=\frac{\underbrace{\binom{13}{8}\binom{39}{18}}_{\text{For N-S together}}\times\underbrace{\binom{5}{4}\binom{21}{9}}_{\text{For w; the rest for E}}}{\binom{52}{26,13,13}}=\frac{\frac{13!}{8!5!}\times\frac{39!}{18!21!}\times\frac{5!}{4!}\times\frac{21!}{9!12!}}{\frac{52!}{26!13!13!}}$$

$$P=\frac{\begin{pmatrix}\text{No. of ways to distribute}\\ \text{13 spades such that W gets 4, E gets 1}\\ \text{and N-S gets 8 together; AND distribute}\\ \text{39 non-spades such that W gets 9, E gets 12,}\\ \text{and N-S together get 18}\end{pmatrix}}{\begin{pmatrix}\text{No. of ways to distribute 26 cards to}\\ \text{N-S and 13 cards each to E and W}\end{pmatrix}}=\frac{\underbrace{\binom{13}{4,1,8}}_{\text{Distributing 13 spades among W, E, and N-S}}\times\underbrace{\binom{39}{9,12,18}}_{\text{Distributing non-spades among W, E, and N-S}}}{\binom{52}{26,13,13}}$$

$$P=\frac{\begin{pmatrix}\text{No. of ways to distribute 13 cards}\\ \text{each to N and S, such that they have}\\ \text{8 spades and 18 non-spades, and 4}\\ \text{spades and 9 non-spades to W}\end{pmatrix}}{\begin{pmatrix}\text{No. of ways to distribute 13 cards}\\ \text{each to N, S, E and W}\end{pmatrix}}=\frac{\underbrace{\binom{13}{8}\binom{39}{18}}_{\text{26 cards For N-S together}}\times\underbrace{\binom{26}{13}}_{\text{13 cards each to N-S}}\times\underbrace{\binom{5}{4}\binom{21}{9}}_{\text{13 cards For w (with 4 spades), the rest for E}}}{\binom{52}{13,13,13,13}}$$

$$P=\frac{\begin{pmatrix}\text{No. of ways to distribute}\\ \text{4 spades to W, 9 non-spades to W}\\ \text{1 spade to E, 12 non-spades to E}\\ \text{Split remianing 26 cards between N and S}\end{pmatrix}}{\begin{pmatrix}\text{No. of ways to distribute 13 cards}\\ \text{each to N, S, E and W}\end{pmatrix}}=\frac{\underbrace{\binom{13}{4}\binom{39}{9}}_{\text{4 spades and 9 non-spades for W}}\times\underbrace{\binom{9}{1}\binom{30}{12}}_{\text{1 spade and 12 non-spades for E}}\times\underbrace{\binom{26}{13}}_{\text{Splitting the remianing cards between N and}}}{\binom{52}{13,13,13,13}}$$

(b) Let A_k denote that event that there are *k* unused ball picked up by the first player. Let B be the event that all three ball picked by the second player are unused.

$$P(B)=P(B\mid A_0)P(A_0)+P(B\mid A_1)P(A_1)+P(B\mid A_2)P(A_2)+P(B\mid A_3)P(A_3)$$

$$=\frac{\binom{9}{3}}{\binom{15}{3}}\frac{\binom{6}{3}}{\binom{15}{3}}+\frac{\binom{8}{3}}{\binom{15}{3}}\frac{\binom{9}{1}\binom{6}{2}}{\binom{15}{3}}+\frac{\binom{7}{3}}{\binom{15}{3}}\frac{\binom{9}{2}\binom{6}{1}}{\binom{15}{3}}+\frac{\binom{6}{3}}{\binom{15}{3}}\frac{\binom{9}{3}}{\binom{15}{3}}$$

$$=\frac{18480}{207025}=0.0506$$

Problem 5:

Let N be the number of failures in 12 months. N is a binomial random variable with n = 12 and p = 1/12.

$$P[N=k]=\binom{n}{k}p^k(1-p)^{n-k}=\binom{12}{k}\frac{1}{12}^{k}\frac{11}{12}^{12-k}.$$

k	P[N=k]	P[N<=k]	P[N>k]
0	0.352	0.352	0.648 > 1%
1	0.384	0.736	0.264 > 1%
2	0.192	0.928	0.072 > 1%
3	0.058	0.986	0.014 > 1%
4	0.012	0.998	0.002 < 1%

Hence,

$$P[\text{charge} < \text{repair cost}] = P[\text{charge} < 20N] < 1\%$$
$$\Rightarrow P[N > \text{charge}/20] < 1\%$$
$$\Rightarrow \text{charge/20=4}$$
$$\Rightarrow \text{charge=\$80}$$

The average repair cost on one player is $E[20N]=20E[N]=20np=\$20.$