EE 121B Mid-term Examination
Winter 2017

1) 24

2) 50

3) 24

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Name

Open Book
Assume T=300K and the substrate is silicon

$n_i = 1.5 \times 10^{10}\ cm^{-3}$

1. (a) Consider a *pn* junction with uniformly doped *n* and *p* region as shown below. What are the built-in potential and the depletion with at 300K? (12 points)

Short base

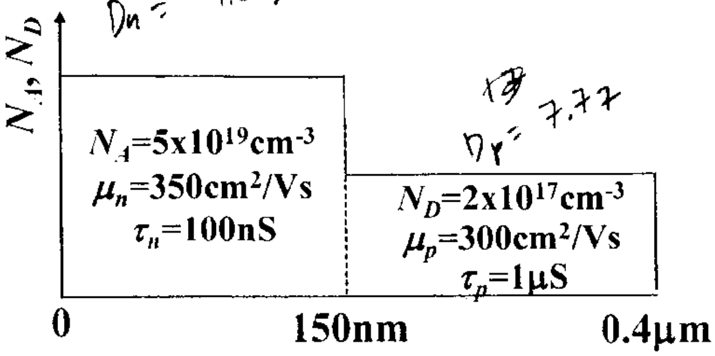
$\sqrt{D_n \tau_n} = 9.5\times10^{-4}$

L_p ?? W_p

$L_n = \sqrt{D_p \tau_p}$?? W_n

$D_n = 9.065$

$D_p = 7.77$



$V_{bi} = \frac{KT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.0259\ V \ln\left(\frac{5\times10^{19}\ cm^{-3} \cdot 2\times10^{17}\ cm^{-3}}{1.5\times10^{10}\ cm^{-3}}\right)$

$V_{bi} = 0.993\ V$

$W_D = \sqrt{\frac{2\varepsilon V_{bi}}{q\left(\frac{N_A N_D}{N_A+N_D}\right)}} = \sqrt{\frac{2 \cdot 11.7 \times 8.85\times10^{-14}\ F/cm \cdot 0.993\ V}{1.6\times10^{-19}\ C \left(\frac{5\times10^{19} \cdot 2\times10^{17}}{5\times10^{19}+2\times10^{17}}\right)}} = 8.03\times10^{-6}\ cm$

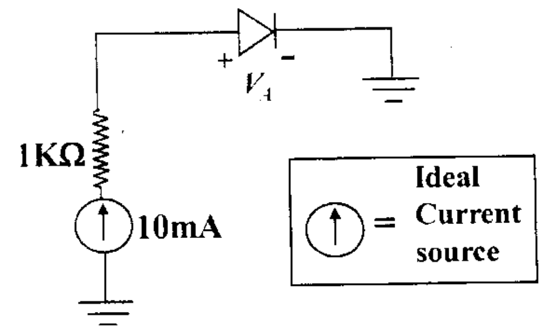
$= 80.3\ nm$

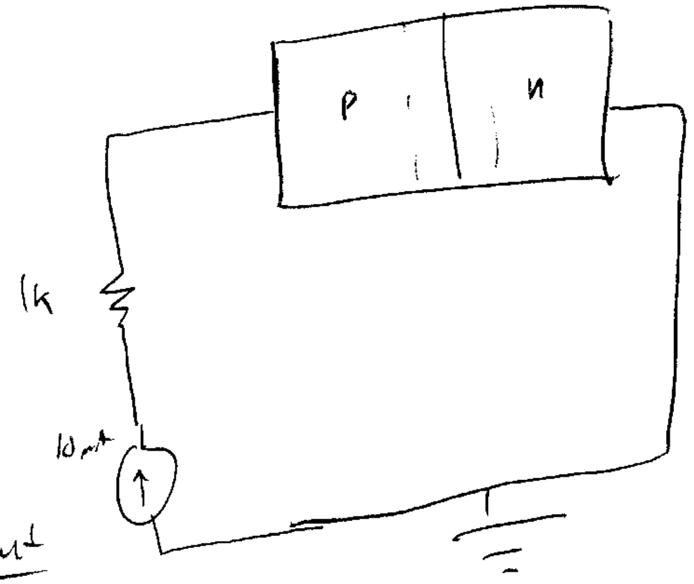
$= W_D$

$W_{Dp} = W_D\left(\frac{N_D}{N_D+N_A}\right) = 3.2\times10^{-8}\ cm$

$W_{Dn} = W_D\left(\frac{N_A}{N_D+N_A}\right) = 7.998\times10^{-6}\ cm$

(b) **If the size (cross-section) of the *pn* diode, $A=10^{-2}cm^2$, and is put into the following circuit, what is V_A at 300K? (12 points)**





- find IV relation at

$I = 10mA$

$W_A = 150nm - W_{Dp} \approx 150nm$

$W_{N_D} = 250nm - W_{Dn} = 250nm - 80nm$

$= 170nm$

$= 170\times10^{-7} cm$

Short based current

$$I = A\left(\frac{qD_n n_i^2}{W_A N_A} + \frac{qD_p n_i^2}{W_{N_D} N_D}\right)\left(e^{\frac{qV_a}{kT}}-1\right) = 10mA$$

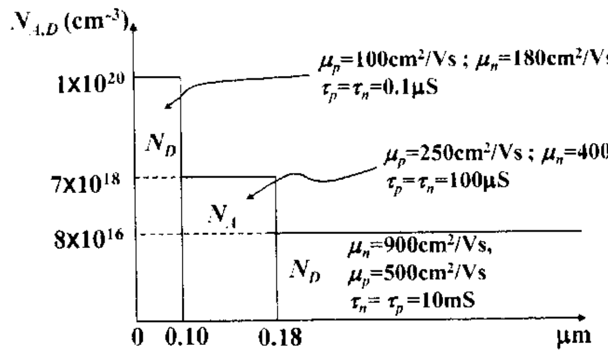
$$10^{-2}cm^2\left(\frac{1.6\times10^{-19}C\cdot 9.07cm^2/s\,(1.5\times10^{10}cm^{-3})^2}{150\times10^{-7}cm\cdot 5\times10^{17}cm^{-3}} + \frac{1.6\times10^{-19}\cdot 7.77\cdot(1.5\times10^{10})^2}{170\cdot10^{-7}cm\cdot 2\times10^{17}cm^{-3}}\right)\left(e^{\frac{V_a}{V_t}}-1\right) = 10mA$$

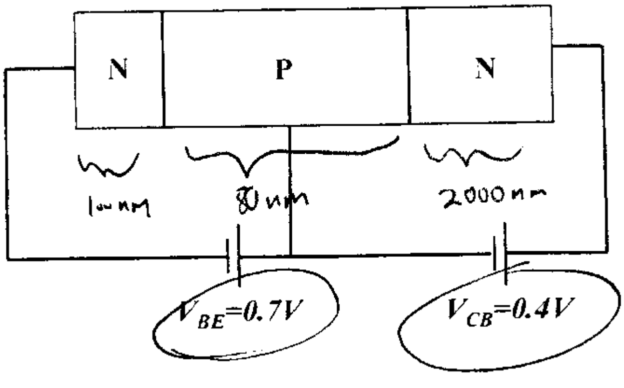
$$10^{-2}\left(4.35\times10^{-13} + 8.23\times10^{-11}\right)\left(e^{\frac{V_a}{V_t}}-1\right) = 10mA$$

$$e^{\frac{V_a}{V_t}}-1 = 1.209\times10^{10} \rightarrow e^{\frac{V_a}{V_t}} = 1.209\times10^{10},\ \frac{V_a}{V_t} = 23.2$$

$V_a = 0.601\ V$

2) **Consider the following bipolar transistor (the collector length is 2μm)**





A=100μm²

a) **What is the neutral base width of the device with the DC biases as indicated? (12 points)**

$N_{DE} = 10^{20}$ $N_{AB} = 7\times10^{18}$ $N_{DC} = 8\times10^{16}$

$D_{pE} = \mu_p V_t = 2.59\ cm^2/s$ $D_{nB} = \mu_n V_t = 10.36\ cm^2/s$ $D_{pC} = \mu_p V_t = 12.95\ cm^2/s$

Short base, short emitter:
$L_{Ep} = \sqrt{D_{E} \cdot \tau_{pE}} = \sqrt{2.59 \cdot 10^{-7} s} = 5.09\times10^{-4} cm >> W_{EM}$

$L_{Bn} = \sqrt{D_{nB}\,\tau_{nB}} = \sqrt{10.36 \cdot 10^{-4} s} = 3.2\times10^{-2} cm >> W_{BM}$

$V_{bibe} = V_t \ln\left(\frac{N_{DE} N_{AB}}{n_i^2}\right) = 1.103\ V$ V_{bibe}

$V_{bibc} = V_t \ln\left(\frac{N_{DC} N_{AB}}{n_i^2}\right) = 0.0259 \ln\left(\frac{7\times10^{18} \cdot 8\times10^{16}}{(1.5\times10^{10})^2}\right) = 0.918\ V$ V_{bibc}

$W_{DOE} = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{V_{bibe} - V_{be}}{\frac{N_{DE} N_{AB}}{N_{DE}+N_{AB}}}\right)} = \sqrt{\frac{2\varepsilon_s}{q}\frac{(1.103-0.7)}{(\cdots)}} = 8.93\ nm$

$W_{DBEp} = W_{DBE}\left(\frac{N_{DE}}{N_{DE}+N_{AB}}\right) = 8.93\times10^{-7} cm \left(\frac{10^{20}}{10^{20}+7\times10^{18}}\right) = 8.35\ nm$

$W_{DBC} = \sqrt{\frac{2\varepsilon_{si}(V_{bibc}+V_{bc})}{q\left(\frac{N_{DC}N_{AB}}{N_{DC}+N_{AB}}\right)}} = \sqrt{\frac{2\varepsilon_s (0.918+0.4v)}{q(\cdots)}} = 147\ nm$

$W_{DBCp} = W_{DBC}\left(\frac{N_{DC}}{N_{DC}+N_{AB}}\right) = 147\times10^{-7} cm \left(\frac{8\times10^{16}}{8\times10^{16}+7\times10^{18}}\right) = 1.66\ nm$

$W_B = W_{BM} - W_{DEp} - W_{DCp} = 80nm - 8.35\ nm - 1.66\ nm = 70.0\ nm = W_B$

$1\,\mu m^2 = 10^{-12}\,m^2$

$D_{Bn} = 10.36$ $\quad 1\,\mu m^2 = 10^{-8}\,cm^2$ $\quad A = 100\,\mu m^2 \left(\frac{10^{-4}\,cm}{\mu m}\right)\left(\frac{10^{-4}\,cm}{\mu m}\right) =$

$= 10^{-10}\,m^2$

b) What are the values of I_C, I_E, and I_B? (14 points) $A = 10^{-6}\,cm^2$

$$I_C \approx \frac{A q D_{Bn} n_i^2}{W_B N_{AB}} e^{\frac{V_{BE}}{V_t}} = \frac{10^{-6}\,cm^2 \cdot 1.6\times10^{-19}\,C \cdot 10.36\,cm^2/s \cdot (1.5\times10^{10}\,cm^{-3})^2}{70\cdot10^{-7}\,cm \cdot 7\times10^{18}\,cm^{-3}} e^{\frac{0.7}{0.0259}}$$

$$= \frac{3.73\times10^{-4}}{4.9\times10^{13}} e^{27.03} = \boxed{4.16\times10^{-6}\,A = I_C}$$

$$I_B = \frac{A q D_{Ep} n_i^2}{W_E N_{DE}} e^{\frac{V_{BE}}{V_t}}, \quad W_E = W_{EM} - W_{DDEn} = 100\,nm - (8.93\,nm - 8.33\,nm) = 99.4\,nm$$

$$= \frac{10^{-6}\,cm^2 \cdot 1.6\times10^{-19}\,C \cdot 2.59\,cm^2/s\,(1.5\times10^{10})^2}{99.4\times10^{-7}\,cm \cdot 10^{20}\,cm^{-3}} e^{\frac{0.7}{0.0259}} = \boxed{5.13\times10^{-8}\,A = I_B}$$

$$\boxed{I_E = I_C + I_B = 4.21\times10^{-6}\,A}$$

c) What are the common base current gain, α_o and the common emitter current gain, β_o at these biases? (12 points)

$$\gamma \approx \frac{1}{1+\frac{D_E W_B N_{AB}}{D_B W_E N_{DE}}} = \frac{1}{1+\frac{2.59 \cdot 70\times10^{-7}\,cm \cdot 7\times10^{18}\,cm^{-3}}{10.36 \cdot 99.4\times10^{-7}\,cm \cdot 10^{22}\,cm^{-3}}} \quad 0.988$$

$$L_B = \sqrt{D_B \tau_B} = \sqrt{10.36\,cm^2/s \cdot 10^{-4}\,s} = 3.22\times10^{-2}\,cm$$

$$\alpha_T \approx 1 - \frac{W_B^2}{2L_B^2} = 1 - \frac{(70\times10^{-7}\,cm)^2}{2\cdot(3.22\times10^{-2}\,cm)^2} = 0.9999999\ldots$$

$$\alpha_o = \gamma \alpha_T M, \; M \approx 1 \rightarrow \alpha_o = \gamma \alpha_T \Rightarrow \alpha_o = 0.988$$

$$\beta = \frac{\alpha_o}{1-\alpha_o} = 82.3 = \beta$$

(e) **What are the Early Voltage and the base transit time of this transistor at these biases? (12 points)**

$$V_a = \frac{q\, W_B N_{AB}}{C_{DBC}} = \qquad C_{DBC} = \frac{\varepsilon}{W_{DBC}} = \frac{11.7 \cdot 8.85 \times 10^{-14}\ F/cm}{147 \times 10^{-7}\ cm} = 7.04 \times 10^{-8}\ F/cm^2$$

$$V_a = \frac{1.6 \times 10^{-19}\ C \cdot 70 \times 10^{-7}\ cm \cdot 7 \times 10^{18}\ cm^{-3}}{7.04 \times 10^{-8}\ F/cm^2} = \boxed{111.4\ V = V_a}$$

$$\tau_B = \frac{W_D^2}{2 D_n} = \frac{(70 \times 10^{-7}\ cm)^2}{2 \cdot 10.36\ cm^2/s} \therefore \boxed{\tau_B = 2.36 \times 10^{-12}\ s}$$

3 (a) For the transistor in (2) if we want to while maintaining the same current gain, β_o, but with the base width (metallurgical) reduced to 0.06μm, what is the new base doping concentration assuming the same minority carrier (i.e. electron) mobility? (same biases as in (2)) (14 points)

$$\beta = \frac{\alpha_0}{1-\alpha_0} = 82.3 \rightarrow \alpha_0 = 0.988 \rightarrow \text{since } \alpha_T \approx 1, \quad \gamma = 0.988$$

$$\gamma = 0.988 = \left(1 + \frac{D_{Ep} W_B' N_{AB}'}{D_{Bn} W_E N_{DE}}\right)^{-1}$$

N_{AB}', W_B' : new Base width & Doping

Same → $\frac{D_{Ep} W_B N_{AB}}{D_{Bn} W_E N_{DE}} = 0.01215$ ← Same

Same ↑ W_E will be approximately the same bc Base will still be doped much less

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$$W_B N_{AB} = 0.01215 \cdot \frac{D_{Bn} W_E N_{DE}}{D_{Ep}} = 4.83 \times 10^{13}$$

The product of W_B and N_{AB} must maintain this value. If we approximate that W_B will shorten by as much as W_{BM}, then $W_B' = 50 \times 10^{-7}$ cm. In this case, $N_{AB}' = 4.83 \times 10^{13} / 50 \times 10^{-7}$ cm $= 9.66 \times 10^{18}$, which is not that much greater than N_{AB}. Thus, we justify our approximation because now assuming N_{AB}' is valid, we look at depletion widths and they will not change very much.

So, $\boxed{N_{AB}' \approx 9.66 \times 10^{18}\ \text{cm}^{-3}}$

(b) What are the Early Voltage and the base transit time for the case in 3(a)? Discuss. (12 points)

$$V_a = \frac{2 w_B N_{AB}}{C_{DBC}}$$

By definition $W_B N_{AB}$ = same as for previous case.

4.9×10^{13}

$$C_{DBC} = \frac{\varepsilon}{W_{DBC}}, \quad W_{DBC} = \sqrt{\frac{2\varepsilon(V_{bi\,bc} + V_{bc})}{q\left(\frac{N_{DC} N_{AB}}{N_{DC} + N_{AB}}\right)}} = 1.471 \times 10^{-5}\ \text{cm}$$

$V_{bi\,bc} + V_{bc} = 1.318\ V$

$$C_{DBC} = 7.04 \times 10^{-8}$$

$$V_a = \frac{q \cdot 4.9 \times 10^{13}}{7.04 \times 10^{-8}}$$

$111.4\ V = V_a$

$$\tau_B = \frac{W_B^2}{2 D_{Bn}} \rightarrow \text{approximately } W_B \approx 50 \times 10^{-7}\ \text{cm as prev discussed}$$

$\tau_B = 1.21 \times 10^{-12}\ s$

Since V_a depends on $W_B N_{AB}$, that part will not change. C_{DBC} depends on the depletion width of BC junction, as prev discussed, this does not change much. The result is V_a is very similar to previous V_a, which makes sense because current gain β remains constant.

τ_B becomes smaller because the base width shrinks, meaning carriers can travel across it much faster bc of shorter travel distance.