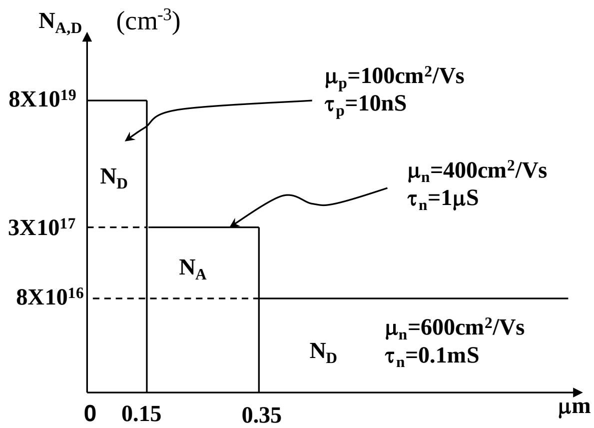
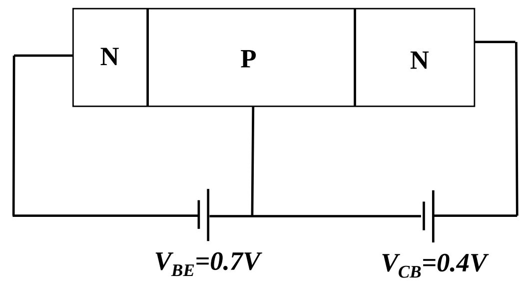
# EE 121B Mid-term Examination
## Spring 2014

**Name** ______________________________________________

**Open Book**
**Assume T=300K and the substrate is silicon**

## 1) Consider the following bipolar transistor




$A = 100\mu m^2$

**a) What is the neutral base width of the device with the DC biases as indicated? (5 points)**

Sol: Calculate the depletion inside the base region

$$x_p \approx \sqrt{\frac{2\varepsilon_{Si}\left(V_{bi,BE} - V_{BE}\right) \cdot N_{DE}}{qN_{AB}(N_{AB} + N_{DE})}} \approx 37nm$$

$$x_p' \approx \sqrt{\frac{2\varepsilon_{Si}\left(V_{bi,BC} + V_{CB}\right) \cdot N_{DC}}{qN_{AB}(N_{AB} + N_{DC})}} \approx 33.5nm$$

We have the neutral base width

$$W_B = L_B - x_p - x_p' \approx 130nm$$

$x_p, x_p'$ (3') $W_B$ (2')

**b) What is the values of $I_C$, $I_E$, and $I_B$? (15 points)**

As the diffusion length

$$L_{nB} = \sqrt{D_n \tau_n} \approx 32\mu m \gg L_B > W_B$$
$$L_{pE} = \sqrt{D_p \tau_p} \approx 1.6\mu m \gg L_E > W_E$$

By ignoring recombination current, the electron and hole current can be calculated by

$$I_n(0) = \frac{AqD_n n_i^2}{N_{AB} W_B}\left(\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right) \approx 52.3\mu A$$
$$I_p(-W_{DBE}) = \frac{AqD_p n_i^2}{N_{DE} W_E}\left(\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right) \approx 4.25 \times 10^{-8} A$$

So,

$$I_B \approx I_p(-W_{DBE}) \approx 4.25 \times 10^{-8} A$$
$$I_C \approx I_n(0) \approx 52.3\mu A$$
$$I_E \approx I_p(-W_{DBE}) + I_n(0) \approx 52.3\mu A$$

$L_{nB}$, $L_{pE}$ (1')

$I_n(0)$, $I_p(-W_{DBE})$ (5')

$I_B$, $I_C$, $I_E$ (10)

Or

$I_B$, $I_C$, $I_E$ (5,5,5)
Wrong in numbers (-2' for each)

For each part, if wrong answer caused by $W_B$, (-1' each)

**c) What are the common base current gain, $\alpha_o$ and the common emitter current gain, $\beta_o$? (20 points)**

Sol: As we ignore the recombination current,

$$\beta_0 \approx \frac{D_n W_E N_{DE}}{D_p W_B N_{AB}} \approx 1231$$

And

$$\gamma \approx \frac{1}{1+\frac{1}{\beta_0}} \approx 0.99919$$

while $W_B \ll L_{nB}$

$$\alpha_T \approx 1-\left(\frac{W_B}{L_{nB}}\right)^2 \approx 0.99998$$

Taking $M \approx 1$, as we ignore the reversed hole current and generation in BC junction,

$$\alpha_0 \approx \gamma\alpha_T M \approx 0.99917$$

using DC definition (5')
$\beta_0$ (8')
Wrong in numbers (-2')
Mistake Caused by Wrong $W_B$ (-1')
$\gamma$(4')
Wrong in numbers (-2')
Mistake Caused by Wrong $W_B$ (-1')
$\alpha_T$(4')
If no $\alpha_T$ appears at all (-4')
if directly consider $\alpha_T = 1$, (-2')
if $W_B \ll L_B$ noted or estimation of $\alpha_T$, full points
$\alpha_0$(4')
Wrong in numbers (-2')
Mistake Caused by Wrong $W_B$ (-1')

**2 (a) For the transistor in (1) if we want to increase the common emitter current gain, $\beta_o$ by 50% by reducing the base doping, what is the new base doping concentration, assuming the minority carrier (i.e. electron) mobility is $\propto (N_A)^{-0.33}$? (20 points)**

Sol:

As $\beta \approx \frac{D_n W_E N_{DE}}{D_p W_B N_{AB}}$, we ignore the change in the neutral base width at first (neutral emitter width barely changes),

$$\frac{\beta}{\beta_0} = \frac{D_n/D_{n0}}{N_{AB}/N_{AB0}} \approx 1.5$$

while $\frac{D_n}{D_{n_0}} = \left(\frac{N_{AB}}{N_{AB0}}\right)^{-0.33}$, we have

$$\left(\frac{N_{AB}^{(0)}}{N_{AB0}}\right)^{-1.33} \approx 1.5$$

which gives us the initial guess for $N_{AB}$

$$N_{AB}^{(0)} \approx 0.737 N_{AB0} \approx 2.21 \times 10^{17} cm^{-3}$$

Then we consider the perturbation of $W_B$ caused by the change of $N_{AB}$

$$W_B^{(0)} \approx L_B - x_p - x_p' \approx 200 - 42.4 - 43.7\, nm \approx 113.9\, nm \approx 0.88 W_{B0}$$

So, to find out the new $N_{AB}$,

$$\left(\frac{N_{AB}^{(1)}}{N_{AB0}}\right)^{-1.33} \left(\frac{W_B^{(0)}}{W_{B0}}\right)^{-1} \approx 1.5$$

$N_{AB}^{(1)} \approx 2.43 \times 10^{17} cm$ and $W_B^{(2)} \approx 119nm \approx 0.916 W_{B0}$

again iterate for $N_{AB}^{(2)} \approx 2.36 \times 10^{17} cm$ and $W_B^{(2)} \approx 117.6nm \approx 0.905 W_{B0}$

$N_{AB}^{(3)} \approx 2.38 \times 10^{17} cm$ and $W_B^{(3)} \approx 117.6nm$

$\beta$ 5'

$\frac{\beta}{\beta_0}$ 5'

Reasonable $N_{AB}^{(0)}$ 3' and $W_B^{(0)}$ 3'

Iteration 4'

**(b)What are the Early voltages in the two cases? (15 points)**

Sol: The Early voltage is given by

$$V_A = \frac{qN_{AB}W_B}{C_{DBC}} = \frac{qN_{AB}W_B}{\varepsilon_{Si}} W_{DBC}$$

5'

In case 1)

$$V_A \approx \begin{cases} 9.5\,V\ @V_{CB} = 0.4V\ (W_{DBC} \approx 159nm) \\ 7.8V\ @V_{CB} = 0V\ (W_{DBC} \approx 131nm) \end{cases}$$

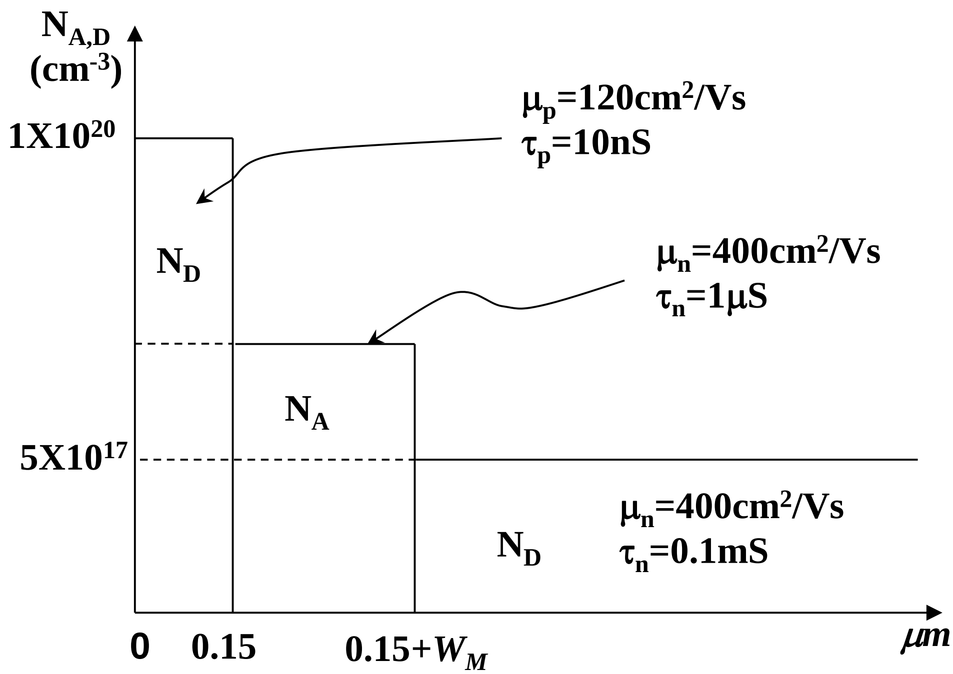
5'

In case 2)

$$V_A \approx \begin{cases} 7.0\,V\ @V_{CB} = 0.4V\ \ (W_{DBC} \approx 163nm) \\ 5.8V\ @V_{CB} = 0V\ \ (W_{DBC} \approx 134nm) \end{cases}$$

5'

**(3) For the bipolar transistor as shown below**




**If the common emitter current gain, $\beta_o$, measured with $V_{BE}$=0.7V and $V_{BC}$=0V, is 80, and the Early voltage is 50V, what are $N_A$ and $W_M$? (25 points)**

Sol: Ignoring the recombination, we have

$$\beta_0 \approx \frac{D_n W_E N_{DE}}{D_p W_B N_{AB}} \approx 80$$

5'

as the depletion in the emitter is negligible (highly doped emitter)

$$W_B N_{AB} \approx \frac{\mu_n W_E N_{DE}}{\mu_p \beta_0} = 6.25 \times 10^{13} cm^{-2}$$

5'

Also, we can have Early voltage

$$V_A = \frac{q N_{AB} W_B}{\varepsilon_{Si}} W_{DBC}$$

5'

i.e.,

$$W_{DBC} = \frac{V_A \varepsilon_{Si}}{q N_{AB} W_B} \approx 52.2nm = \sqrt{\frac{2\varepsilon_{Si} V_{bi}}{q}\left(\frac{1}{N_{AB}} + \frac{1}{N_{DC}}\right)}$$

5’

Iterate for $N_{AB}$ and $V_{bi}$:

$$N_{AB} = \left(\frac{q W_{DBC}^2}{2\varepsilon_{Si} V_{bi}} - \frac{1}{N_{DC}}\right)^{-1}$$

with initial value $V_{bi}^{(0)} = 0.9V$,

$$N_{AB}^{(0)} \approx 3.3 \times 10^{18} cm^{-3}$$

and after a few iteration

$$V_{bi}^{(1)} = 0.95V, N_{AB}^{(1)} \approx 5.2 \times 10^{18} cm^{-3}$$
$$V_{bi}^{(2)} = 0.96V, N_{AB}^{(2)} \approx 6.1 \times 10^{18} cm^{-3}$$
$$V_{bi}^{(3)} = 0.964V, N_{AB}^{(3)} \approx 6.5 \times 10^{18} cm^{-3}$$

$$V_{bi}^{(n)} = 0.964V, N_{AB}^{(n)} \approx 6.7 \times 10^{18} cm^{-3}$$

Therefore,

$$W_M \approx W_B + x_p' + x_p \approx 93.7 + 3.6 + 8.5\ nm \approx 105.8nm$$

5’