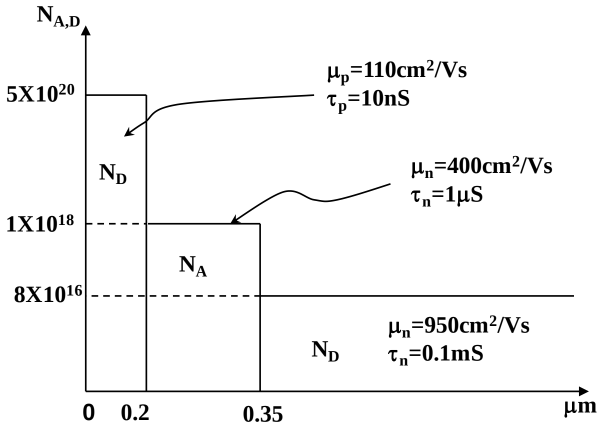
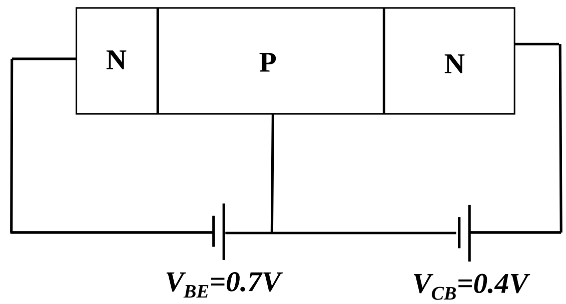
EE 121B Mid-term Examination
Spring 2013

Name ____

Open Book
Assume T=300K and the substrate is silicon
and $n_i = 1.5 \times 10^{10} cm^{-3}$

1) Consider the following bipolar transistor





A=100μm²

a) What is the neutral base width of the device with the DC biases as indicated? (9 points)

$$V_{bi,BE} \approx \frac{kT}{q}\ln\frac{N_{AB}N_{DE}}{n_i^2} \approx 1.09V$$

$$V_{bi,BC} \approx \frac{kT}{q}\ln\frac{N_{AB}N_{DC}}{n_i^2} \approx 0.868V$$

Thus,

$$x_p \approx \sqrt{\frac{2\epsilon_{Si}N_{DE}\left(V_{bi,BE} - V_{BE}\right)}{qN_{AB}(N_{AB}+N_{DE})}} \approx 22.4nm$$

$$x_p' \approx \sqrt{\frac{2\epsilon_{Si}N_{DC}\left(V_{bi,BE} + V_{CB}\right)}{qN_{AB}(N_{AB}+N_{DC})}} \approx 11nm$$

(6')

And,

$$W_B \approx L_B - x_p - x_p' = 116.6nm$$

(3')

b) What is the value of I_C, I_E, and I_B? (17 points)

Using Einstein relation, we have diffusion constant and diffusion length

$$D_{nB} = \frac{kT}{q}\mu_{nB} \approx 10.4\, cm^2/s \text{ and } L_{nB} = \sqrt{D_{nB}\tau_{nB}} \approx 32\mu m$$

$$D_{pE} = \frac{kT}{q}\mu_{pE} \approx 2.85\, cm^2/s \text{ and } L_{pE} = \sqrt{D_{pB}\tau_{pB}} \approx 1.7\mu m$$

$$D_{pC} = \frac{kT}{q}\mu_{pC} \approx 24.6\, cm^2/s$$

Using charge neutral, $x_n \approx x_p \frac{N_{AB}}{N_{DE}} \approx 0.04nm$, so we can approximate $W_E \approx L_E$.

Calculate ideal current for BE junction,

$$I_n(0) \approx \frac{qAD_{nB}n_i^2}{N_{AB}W_B}\left[\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right] \approx 1.8 \times 10^{-5}A$$
$$I_p(-W_D) \approx \frac{qAD_{nE}n_i^2}{N_{DE}W_E}\left[\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right] \approx 5.6 \times 10^{-9}\text{A}$$

while the recombination current is given by

$$I_{REC} \approx \frac{qAn_i\overline{W_{DBE}}\left(\exp\frac{qV_{BE}}{nkT} - 1\right)}{\tau_{REC}} \approx \frac{qAn_ix_p\left(\exp\frac{qV_{BE}}{2kT} - 1\right)}{10\tau_{REC}}$$
$$\approx 4.0 \times 10^{-10}A$$

(2')

So,

$$I_B \approx I_p(-W_D) + I_{REC} \approx 6.0 \times 10^{-9}A$$

$$I_E \approx I_n(0) + I_p(-W_D) + I_{REC} \approx 1.8 \times 10^{-5}A$$

and **(5' for I_C+5' for I_E+5' for I_B)**

$$I_C \approx I_E \approx 1.8 \times 10^{-5}A$$

c) What is the common emitter current gain, β_o? (10 points)

$$\beta \approx \left[\frac{D_{pE}N_{AB}W_B}{D_{nB}N_{DE}W_E} + \frac{\overline{W_{DBE}}W_B N_{AB}\exp\left[-\frac{qV_{BE}}{kT}\left(\frac{n-1}{n}\right)\right]}{D_{nB}n_i\tau_{REC}}\right]^{-1}$$
$$\approx [3.2\times10^{-4} + 2.2\times10^{-5}]^{-1} \approx (3.2\times10^{-4})^{-1} \approx 3000$$

(In this case, we ignore the contribution of the recombination current for simplicity)

(5' for equation, 5' for calculation)

(e) What is the Early voltage? (10 points)

$$V_A \approx \frac{qW_B N_{AB}}{C_{DBC}} = \frac{qW_B N_{AB} W_D'}{\epsilon_{Si}} \approx 27V$$

where $W_D' \approx \sqrt{\frac{2\epsilon_{Si}(V_{CB}+V_{bi,BC})(N_{DC}+N_{AB})}{qN_{DC}N_{AB}}} \approx 149\text{nm}$

(5’ for equation, 5’ for calculation)

(f) What is base transit time τ_B? (8 points)

Base transit time $\tau_B = \frac{W_B^2}{2D_{nB}} \approx 6.5\text{ps}$

(5' for equation, 5' for calculation)

2 (a) If we want to increase β_0 by 25% by reducing the base metallurgical width (which was originally 150nm), what is the new base metallurgical width? (10 points)

Again, if we ignore the recombination current,

$$\beta_0 \approx \frac{D_{nB} N_{DE} W_E}{D_{pE} N_{AB} W_B}$$

To increase β_0 25% by changing W_B,

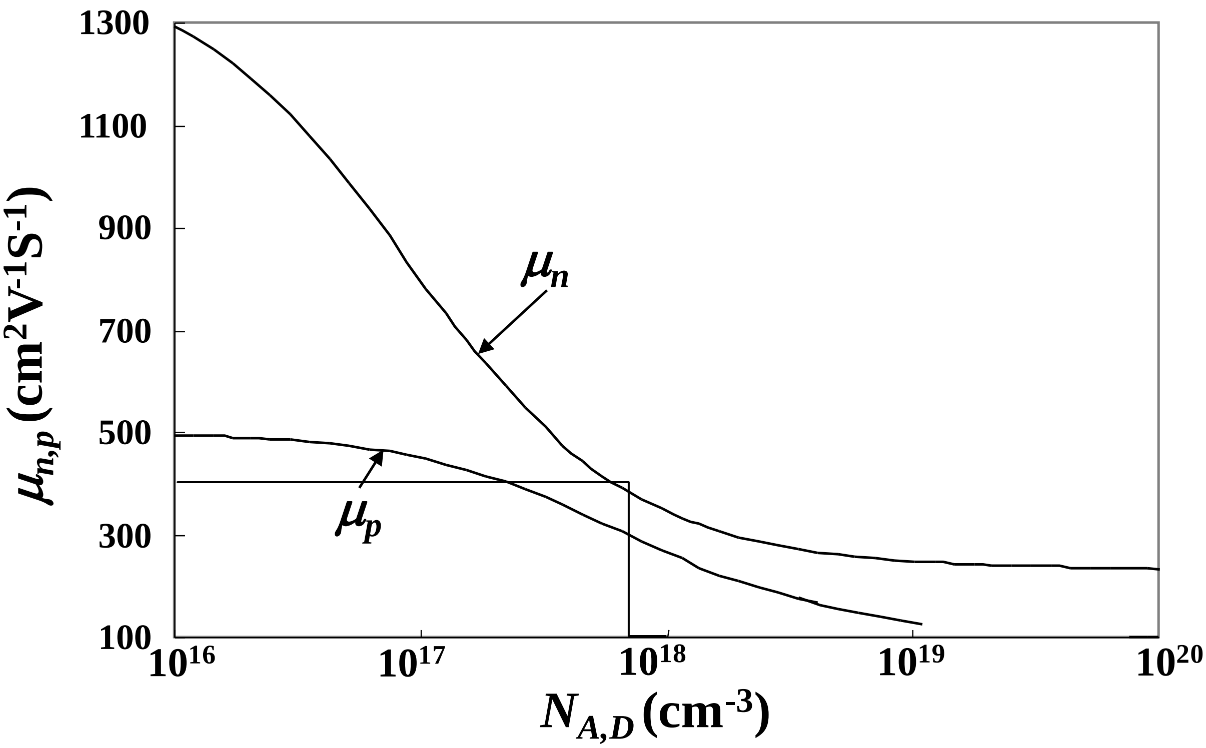
$$W_B \approx 0.8 W_{B0}$$

So the new metallurgical width can be estimated by

$$L_B \approx 0.8 W_{B0} + x_p + x_p' \approx 127nm$$

(5' for equation, 5' for results)

(b) If instead, we want to increase β_o by 25% by reducing the base doping concentration, what is the new base doping? Use the graph below for mobility dependence on concentration. (12 points)



Again, if we ignore the recombination current,

$$\beta_0 \approx \frac{D_{nB} N_{DE} W_E}{D_{pE} N_{AB} W_B}$$

To increase β_0 by 25%, in first order, we can estimate

$$N_{AB} \approx 0.8 N_{AB0} \approx 8 \times 10^{17} cm^{-3}$$

However, since D_{nB} and W_B also change with N_{AB}, we need to iterate to get N_{AB}. As D_{nB} and W_B change slowly with N_{AB} (which is about $410\,\mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$ and $W_B \approx L_B - x_p - x_p' \approx L_B - x_{p0} \cdot \sqrt{N_{AB0}/N_{AB}} - x_p' \cdot N_{AB0}/N_{AB} \approx 111nm$), iterate for N_{AB}

$$N_{AB} \approx 0.8 N_{AB0} \cdot \frac{W_{B0}}{W_B} \cdot \frac{D_{nB}}{D_{nB_0}} \approx 8.6 \times 10^{17} cm^{-3}$$

(6' for formula/equations, 3' for iteration, 3' for answer)

(c) What are the Early voltages for (a) and (b)? Discuss. (12 points)

$$V_A \approx \frac{qW_B N_{AB}}{C_{DBC}} = \frac{qW_B N_{AB} W_D'}{\epsilon_{Si}}$$

In (a) $W_B = 0.8W_{B0}$, so

$$V_A \approx 0.8V_{A0} \approx 21.6V$$

(4')

In (b) $N_{AB} \approx 0.86N_{AB0}$, $W_B/W_{B0} \approx 0.95$, and $C_{DBC}/C_{DBC0} = W_{D0}'/W_D' \approx 1$,

$$V_A \approx 0.86 \cdot 0.95V_{A0} \approx 0.81V_A \approx 22V$$

(4')

In two cases, the early voltage reduces as the total charge of the base $W_B N_{AB}$ decreases. And in both cases, V_A will be very similar as the change of C_{DBC} can be ignored. the only difference is caused by the mobility improvement when lowering the base doping (which causes $W_B N_{AB}$ to be different in two cases)

(4')

(d) What are the base transit time for (a) and (b)? Discuss. (12 points)
The base transit time is given by

$$\tau_B = \frac{W_B^2}{2D_{nB}}$$

In case (a), $W_B = 0.8W_{B0}$, so

$$\tau_B = \frac{W_B^2}{2D_{nB}} = 0.64\tau_{B0} \approx 4.2ps$$

(4')
In case (b), $W_B = 0.95W_{B0}$ and $D_{nB} \approx 1.03D_{nB0}$

$$\tau_B = \frac{W_B^2}{2D_{nB}} \approx 0.88\tau_{B0} \approx 5.7ps$$

(4')

Apparently, in case (a), τ_B is much smaller than the one in case (b), due to the reason that τ_B is proportional to the square of the neutral base width. Thus, given D_{nB} is somewhat a constant, the way to increase β by reducing W_B more will greatly improve the transit time, resulting in a better high frequency performance.
(4')