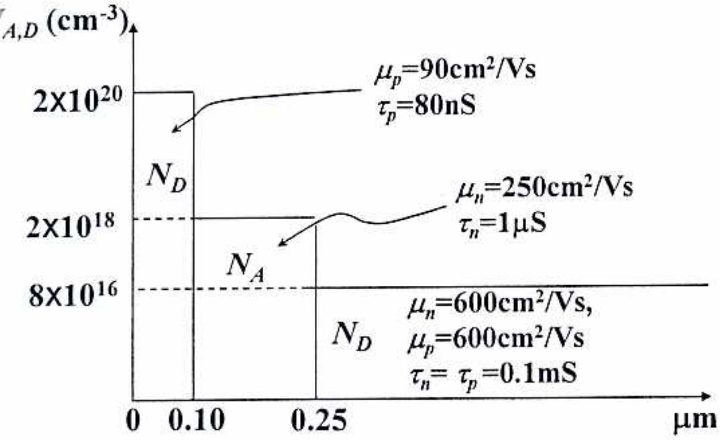
EE 121B Mid-term Examination
Spring 2012

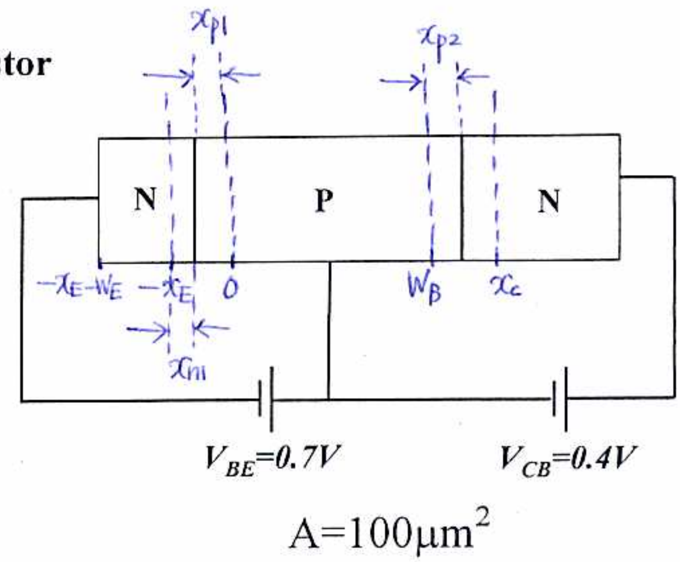
$\boxed{\text{Midterm Solution}}$

Name ____

Assume T=300K and the substrate is silicon

1) **Consider the following bipolar transistor**





a) **What is the neutral base width of the device with the DC biases as indicated? (10 points)**

$(n_i = 1.5\times10^{10}\,cm^{-3})$

Metallurgical base width = 0.15 µm = 150 nm

$$W_B = W_{BM} - (x_{p1} + x_{p2}) \quad 2$$

where
$$x_{p1} = \sqrt{\frac{2\epsilon_{si}}{q}\cdot\frac{N_{DE}(V_{bi,BE} - V_F)}{N_{AB}(N_{AB}+N_{DE})}} \quad \left(V_{bi,BE} = \frac{kT}{q}\ln\left(\frac{N_{AB}N_{DE}}{n_i^2}\right) = 1.09V\right)$$

$$= \sqrt{\frac{2\times11.8\times(8.85\times10^{-14}\,F/cm)}{(1.6\times10^{-19}C)}\cdot\frac{(2\times10^{20})(1.09-0.7)V}{(2\times10^{18})(2\times10^{18}+2\times10^{20})\,cm^{-3}}} \quad 3$$

$$= 1.588\times10^{-6}\,cm = 15.88\,nm$$

$$x_{p2} = \sqrt{\frac{2\epsilon_{si}}{q}\cdot\frac{N_{DC}(V_{bi,BC}+V_R)}{N_{AB}(N_{AB}+N_{DC})}} \quad \left(V_{bi,BC} = \frac{kT}{q}\ln\left(\frac{N_{AB}N_{DC}}{n_i^2}\right) = 0.89V\right)$$

$$= \sqrt{\frac{2\times11.8\times(8.85\times10^{-14}\,F/cm)}{(1.6\times10^{-19}\,C)}\cdot\frac{(8\times10^{16})(0.89+0.4)}{(2\times10^{18})(2\times10^{18}+8\times10^{16})}} \quad 3$$

$$= 5.691\times10^{-7}\,cm = 5.69\,nm$$

$$\therefore W_B = 150\,nm - (15.88 + 5.69)\,nm = 128.43\,nm \quad 2$$

b) What are the values of I_C, I_E, and I_B? (10 points)

(1) $I_c = AJ_c \cong A\left[-J_n(W_B) - J_p(x_c)\right]$

3

$$= -qAD_B \frac{\partial \Delta n}{\partial x}\Big|_{x=W_B} + qAD_c \frac{\partial \Delta p}{\partial x}\Big|_{x=x_c}$$

$$= \frac{qAD_B n_{B0}}{L_B}\left[\mathrm{csch}\left(\frac{W_B}{L_B}\right)\left(\exp\left(\frac{qV_{BE}}{kT}\right)-1\right) - \underbrace{\coth\left(\frac{W_B}{L_B}\right)\left(\exp\left(\frac{qV_{BC}}{kT}\right)-1\right)}_{\simeq 0}\right]$$

$$- \underbrace{\frac{qAD_c p_{c0}}{L_c}\left(\exp\left(\frac{qV_{BC}}{kT}\right)-1\right)}_{\simeq 0}$$

$$\simeq \frac{qAD_B n_{B0}}{L_B}\,\mathrm{csch}\left(\frac{W_B}{L_B}\right)\left(\exp\left(\frac{qV_{BE}}{kT}\right)-1\right)$$

$L_B = \sqrt{D_B \tau_B} = \sqrt{6.48\times10^{-6}} = 0.0025\ \text{cm} = 25\ \mu\text{m} \Rightarrow W_B \ll L_B$

$\left(\frac{D_B}{\mu_B} = \frac{kT}{q} \rightarrow D_B = \mu_B\frac{kT}{q} = (250\ \text{cm}^2/V\cdot s)(0.0259\ V) = 6.48\ \text{cm}^2/s,\ \tau_B = 10^{-6}s,\right.$

$\left. A = 10^{-6}\ \text{cm}^2\right)$

$\therefore I_c \cong \frac{qAD_B n_{B0}}{\cancel{L_B}}\left(\frac{\cancel{L_B}}{W_B}\right)\left(\exp\left(\frac{qV_{BE}}{kT}\right)-1\right) = \frac{qAD_B n_i^2}{W_B N_{AB}}\left(\exp\left(\frac{qV_{BE}}{kT}\right)-1\right) = 4.964\ \mu A$

(If we assume $n_i \cong 10^{10}\ \text{cm}^{-3}$, then $I_c \simeq 2.206\ \mu A$)

(2) $I_E \simeq \frac{qAD_B n_i^2}{W_B N_{AB}}\left(\exp\left(\frac{qV_{BE}}{kT}\right)-1\right) + \frac{qAD_E n_i^2}{W_E N_{DE}}\left(\exp\left(\frac{qV_{BE}}{kT}\right)-1\right)$ $\quad (L_E = \sqrt{D_E\tau_E} = 4.32\ \mu m)$

3

$x_{n1} = \sqrt{\frac{2\epsilon_{Si}}{q}\cdot\frac{N_{AB}(V_{bi,BE}-V_F)}{N_{DE}(N_{DE}+N_{AB})}} = 0.16\ \text{nm} \rightarrow W_E = 100\text{nm} - 0.16\text{nm} = 99.84\text{nm}$

$\Rightarrow W_E \ll L_E$

$\therefore I_E = qAn_i^2\left[\frac{D_B}{W_B N_{AB}} + \frac{D_E}{W_E N_{DE}}\right]\left(\exp\left(\frac{qV_{BE}}{kT}\right)-1\right) = 4.987\ \mu A$

(If we assume $n_i \simeq 10^{10}\ \text{cm}^{-3}$, then $I_E \simeq 2.217\ \mu A$)

(3) $I_B = -I_c + I_E = -4.964 + 4.987 = 0.023\ \mu A = 2.3\times10^{-8}\ A$

3 (for $n_i \simeq 10^{10}\ \text{cm}^{-3}$, $I_B = 0.011 = 1.1\times10^{-8}\ A$)

cf) $I_{rec} \simeq 3.088\times10^{-9}\ A$

$I_{gen} \simeq 3.517\times10^{-10}\ A$

c) What are the common base current gain, α_o and the common emitter current gain, β_o? (12 points)

(1) $\alpha_o = \dfrac{\partial I_C}{\partial I_E} \Rightarrow \alpha_o \simeq \gamma \cong \dfrac{1}{1+\left(\dfrac{D_E W_B N_{AB}}{D_B W_E N_{DE}}\right)} = \dfrac{1}{1+0.0046}$

$= \alpha_T \cdot \gamma \cdot M$

$= 0.9954$

$\left(\alpha_o = \underbrace{\dfrac{\partial I_{nE}}{\partial I_E}}_{\gamma} \cdot \underbrace{\dfrac{\partial I_{nC}}{\partial I_{nE}}}_{\alpha_T} \cdot \underbrace{\dfrac{\partial I_C}{\partial I_{nC}}}_{M}\right)$

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(2) $\beta_o = \dfrac{\partial I_C}{\partial I_B} = \dfrac{\alpha_o}{1-\alpha_o} \simeq \dfrac{\gamma}{1-\gamma} = \dfrac{0.9954}{1-0.9954}$ 6

$= 216.4$

→ A bit more detailed explanation on these approximations needs to be specified.

d) **What is the Early Voltage of this transistor at these biases? (10 points)**

$$V_A = \frac{q W_B N_{AB}}{C_{DBC}}$$

Where $C_{DBC} = \frac{\epsilon_{si}}{W_{DBC}} = \frac{\epsilon_{si}}{\sqrt{\frac{2\epsilon_{si}(V_{bi,BC}+V_R)}{q\left(\frac{N_{AB}N_{DC}}{N_{AB}+N_{DC}}\right)}}}$

$$= \frac{1.04\times10^{-12}\ F/cm}{1.48\times10^{-5}\ cm}$$

2

$$= 7.027\times10^{-8}\ F/cm^2$$

$$\therefore V_A = \frac{(1.6\times10^{-19}C)(128.43\times10^{-7}\ cm)(2\times10^{18}\ cm^{-3})}{(7.027\times10^{-8}\ F/cm^2)}$$

$$= 58.5\ V$$

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2 (a) **If we want to increase the β_o of the transistor in (1) by 50% by increasing the emitter width, while keeping everything else the same, what is the new (metallurgical) emitter width? (12 points)**

$\beta_o' = 1.5\beta_o$ 2 , where $\beta_o = \dfrac{D_B W_E N_{DE}}{D_E W_B N_{AB}}$ 2

$$\frac{\cancel{D_B} W_E' \cancel{N_{DE}}}{\cancel{D_E} \cancel{W_B} \cancel{N_{AB}}} = 1.5 \frac{\cancel{D_B} W_E \cancel{N_{DE}}}{\cancel{D_E} \cancel{W_B} \cancel{N_{AB}}}$$

2 $W_E' = 1.5 W_E \longrightarrow \dfrac{W_E'}{W_E} = 1.5$ since change in x_{n1} is very small

$\therefore W_E' = 1.5 \times (99.84 nm) = 149.76 nm$ 6

$\simeq W_{EM}'$

(b) For the transistor in (1) if we want to increase the Early voltage by 25% while maintaining the same current gain, β_o, what is the new base doping concentration and the base width assuming the same minority carrier (i.e. electron) mobility? (12 points)

2 $W_B' N_{AB}' = W_B N_{AB}$ where W_B' & N_{AB}' are the new neutral base width & base doping, respectively.

2 $V_A' = 1.25\, V_A$

1 where $V_A = \dfrac{q W_B N_{AB}}{C_{DBC}}$ and $C_{DBC} = \dfrac{\epsilon_{si}}{W_{DBC}}$ 1

$$V_A' = \frac{q W_B' N_{AB}'}{C_{DBC}'} = \frac{\cancel{qW_BN_{AB}}}{(\cancel{\epsilon_{si}}/W_{DBC}')} = \frac{\cancel{qW_BN_{AB}}}{(\cancel{\epsilon_{si}}/W_{DBC})} \times (1.25)$$

2 $\therefore W_{DBC}' = 1.25\, W_{DBC}$

$$\Rightarrow \sqrt{\frac{\cancel{2\epsilon_{si}(V_{bi,BC}'+V_R)}}{\cancel{q}\left(\frac{N_{AB}'\cancel{N_{DC}}}{N_{AB}'+N_{DC}}\right)}} = 1.25 \sqrt{\frac{\cancel{2\epsilon_{si}(V_{bi,BC}+V_R)}}{\cancel{q}\left(\frac{N_{AB}\cancel{N_{DC}}}{N_{AB}+N_{DC}}\right)}}$$

Since $V_{bi} = \frac{kT}{q}\ln\left(\frac{N_{AB}N_{DC}}{n_i^2}\right)$, V_{bi} change due to N_{AB} change is negligible compared to the change in N_{AB} in the denominator.

$$\frac{N_{AB}'+N_{DC}}{N_{AB}'} = 1.56\,\frac{N_{AB}+N_{DC}}{N_{AB}} = 1.56 \quad (\because N_{AB} \gg N_{DC})$$

$$N_{AB}' + N_{DC} = 1.56\, N_{AB}' \longrightarrow 0.56\, N_{AB}' = 8\times10^{16}$$

$$\therefore N_{AB}' = 1.43\times10^{17}\ cm^{-3}$$ 2

Since $W_B' = \dfrac{W_B N_{AB}}{N_{AB}'} = \dfrac{(128.43\times10^{-7}cm)(2\times10^{18}cm^{-3})}{(1.43\times10^{17}cm^{-3})}$

$$= 1.8\times10^{-4}\ cm$$

$$\therefore W_B' = 1.8\ \mu m$$ 2

(c) **Do the same as in (b) if the minority carrier (i.e. electron) mobility is** $\propto \dfrac{1}{1+\left(\dfrac{N_A}{1\times 10^{16}\,cm^{-3}}\right)^{0.25}}$**? (12 points)**

$$\beta_o = \underbrace{\frac{D_B W_E N_{DE}}{D_E W_B N_{AB}}}_{\text{old}} = \underbrace{\frac{D_B' W_E N_{DE}}{D_E W_B' N_{AB}'}}_{\text{new}} \longrightarrow \frac{D_B}{W_B N_{AB}} = \frac{D_B'}{W_B' N_{AB}'} \qquad 3$$

$$\longrightarrow \frac{W_B' N_{AB}'}{W_B N_{AB}} = \frac{D_B'}{D_B} \qquad \text{where } D_B = \frac{kT}{q}\mu_B \text{ by Einstein relationship}$$

$$\frac{W_B' N_{AB}'}{W_B N_{AB}} = \frac{\dfrac{1}{1+\left(\frac{N_{AB}'}{1e16}\right)^{0.25}}}{\dfrac{1}{1+\left(\frac{N_{AB}}{1e16}\right)^{0.25}}} = \frac{1+\left(\frac{N_{AB}}{1e16}\right)^{0.25}}{1+\left(\frac{N_{AB}'}{1e16}\right)^{0.25}} \qquad 2$$

$$V_A' = \frac{qN_{AB}'W_B'}{C_{DBC}'} = 1.25\,\frac{qN_{AB}W_B}{C_{DBC}}$$

$$\rightarrow \frac{W_B' N_{AB}'}{W_B N_{AB}} = 1.25\,\frac{C_{DBC}'}{C_{DBC}} = 1.25\,\frac{W_{DBC}}{W_{DBC}'} \qquad \text{where } C_{DBC} = \frac{\varepsilon_{si}}{W_{DBC}}$$

$$\therefore \frac{W_B' N_{AB}'}{W_B N_{AB}} = \frac{1+\left(\frac{N_{AB}}{1e16}\right)^{0.25}}{1+\left(\frac{N_{AB}'}{1e16}\right)^{0.25}} = 1.25\,\frac{W_{DBC}}{W_{DBC}'} = 1.25\,\frac{\sqrt{\cancel{\frac{2\varepsilon_{si}}{q}}\left(\frac{N_{AB}+N_{DC}}{N_{AB}N_{DC}}\right)\cancel{(V_{bi}+V_R)}}}{\sqrt{\cancel{\frac{2\varepsilon_{si}}{q}}\left(\frac{N_{AB}'+N_{DC}}{N_{AB}'N_{DC}}\right)\cancel{(V_{bi}'+V_R)}}}$$

$$= 1.25\,\frac{1}{\sqrt{\dfrac{N_{AB}'+N_{DC}}{N_{AB}'}}}$$

$$= 1.25\sqrt{\frac{N_{AB}'}{N_{AB}'+N_{DC}}}$$

$$\frac{1+\left(\frac{N_{AB}}{1e16}\right)^{0.25}}{1.25} = \left(1+\left(\frac{N_{AB}'}{1e16}\right)^{0.25}\right)\sqrt{\frac{N_{AB}'}{N_{AB}'+N_{DC}}}$$

$$\Rightarrow \frac{1+\left(\frac{2e18}{1e16}\right)^{0.25}}{1.25} = 3.808 = \left[1+\left(\frac{N_{AB}'}{1e16}\right)^{0.25}\right]\sqrt{\frac{\frac{N_{AB}'}{1e16}}{\frac{N_{AB}'}{1e16}+\frac{N_{DC}}{1e16}}} \quad \frac{N_{DC}}{1e16} \rightarrow 8 \; (N_{DC} = 8e16\,cm^{-3})$$

Let $x = \frac{N'_{AB}}{1e16}$,

$$3.808 = \left[1 + x^{0.25}\right]\sqrt{\frac{x}{x+8}}$$

$$3.808\sqrt{x+8} = (1 + x^{0.25})x^{\frac{1}{2}} = x^{\frac{1}{2}} + x^{0.75}$$

$$14.501\,(x+8) = (x^{\frac{1}{2}} + x^{0.75})^2$$

$$14.501x + 116 = x + 2x^{1.25} + x^{1.5}$$

$$\Rightarrow x^{1.5} + 2x^{1.25} - 13.501x - 116 = 0$$

$$x^{1.5} = 116 + 13.501x - 2x^{1.25}$$

$$\therefore x = \left[116 + 13.501x - 2x^{1.25}\right]^{\frac{2}{3}}$$

$\therefore x = 80.274$ by "iteration"

$$\boxed{\therefore N'_{AB} = 8.027 \times 10^{17}\ cm^{-3}}$$ 4

$$W'_B N'_{AB} = \frac{D'_B W_B N_{AB}}{D_B} = 1.1922 \times W_B N_{AB}$$
$$= 3.0623 \times 10^{13}$$

$$\therefore W'_B = \frac{3.0623 \times 10^{13}}{N'_{AB}} = 3.815 \times 10^{-5}\ cm$$
$$= 382\ nm$$

$$\therefore \boxed{W'_B = 382 nm}$$ 3

cf) If we don't assume $N_{AB} \gg N_{DC}$ and take both values into account,

$$\therefore \begin{cases} N'_{AB} = 7.37 \times 10^{17}\ cm^{-3} \\ W'_B = 423\ nm \end{cases}$$

(d) **How does the the cutoff frequency for the transistor in 2(c) compared to that in (1) (12 points)**

$$\frac{1}{2\pi f_T} = \frac{kT}{qI_c}\left(C_{b'e\,dep} + C_{b'c}\right) + \tau_B + \underbrace{\frac{x_c}{2v_{sat}}}_{\tau_c} \quad 2$$

$$\text{where } C_{b'c} = \frac{A\epsilon_{si}}{W_{DBC}}, \quad \tau_B = \frac{W_B^2}{2D_B}, \quad \tau_c = \frac{W_{DBC}}{2v_{sat}} \quad 2$$

From the transistor in (1) to 2(c),

W_{DBC} is increased by 1.25 times & W_B by 3 times & D_B by 1.2 times.

$$\therefore \tau_B' = \frac{W_B'^2}{2D_B'} \rightarrow \frac{\tau_B'}{\tau_B} = \frac{W_B'^2/2D_B'}{W_B^2/2D_B} = \frac{9W_B^2/2.4D_B}{W_B^2/2D_B} = 7.5$$

$$\tau_c' = \frac{W_{DBC}'}{2v_{sat}} \rightarrow \frac{\tau_c'}{\tau_c} = \frac{1.25W_{DBC}/2v_{sat}}{W_{DBC}/2v_{sat}} = 1.25 \quad 2$$

There is a significant increase in $(\tau_B + \tau_c)$ which lowers the cutoff frequency.

$$\frac{1}{2\pi f_T} \propto \left(\frac{kT}{qI_c}C_{b'c} + \tau_B + \tau_c\right) \simeq 3.8\times10^{-10}$$

$$\frac{1}{2\pi f_T'} \propto \left(\frac{kT}{qI_c}C_{b'c}' + \tau_B' + \tau_c'\right) = \left(\frac{kT}{qI_c}(1.25)^{-1}C_{b'c} + 7.5\tau_B + 1.25\tau_c\right)$$

$$\simeq 3.9\times10^{-10}$$

$$\left[\begin{array}{l} f_T \simeq 4.2\times10^8\ Hz \\ f_T' \simeq 4\times10^8\ Hz \end{array}\right. \quad 3$$

If we neglect the junction capacitances, $f_T \simeq 11.8\,GHz$ & $f_T' \simeq 1.65\,GHz$

$\therefore$ Not a good idea to change the base doping when you want to increase the Early Voltage. 3