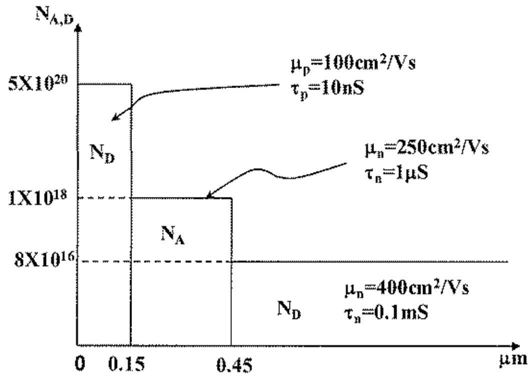
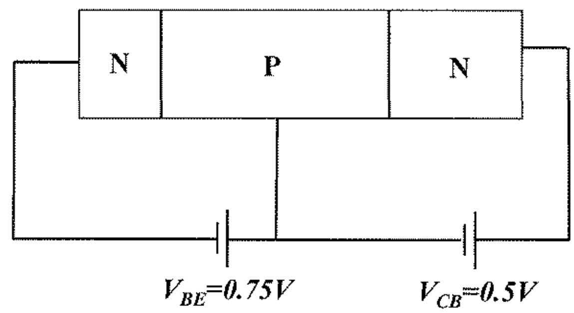
EE 121B Mid-term Examination
Spring 2010

Name ____

Open Book
Assume T=300K and the substrate is silicon

1) Consider the following bipolar transistor





$A = 100\mu m^2$

a) What is the neutral base width of the device with the DC biases as indicated? (5 points)

$N_{DE} = 5\times10^{20}/cm^3, \quad N_{AB} = 1\times10^{18}/cm^3, \quad N_{DC} = 8\times10^{16}/cm^3, \quad V_{BE} = 0.75V, \quad V_{CB} = 0.5V$

$$V_{bi-BE} = \frac{kT}{q}\ln\frac{N_{DE}N_{AB}}{n_i^2} = 0.026\times\ln\frac{5\times10^{20}\times1\times10^{18}}{1\times10^{20}} \approx 1.12V$$

$$V_{bi-BC} = \frac{kT}{q}\ln\frac{N_{DC}N_{AB}}{n_i^2} = 0.026\times\ln\frac{8\times10^{16}\times1\times10^{18}}{1\times10^{20}} \approx 0.89V$$

$$x_{p1} = \sqrt{\frac{2\varepsilon}{q}\frac{N_{DE}}{N_{AB}(N_{DE}+N_{AB})}\cdot(V_{bi-BE}-V_{BE})}$$

$$= \sqrt{\frac{2\times11.7\times8.85\times10^{-14}}{1.6\times10^{-19}}\cdot\frac{5\times10^{20}}{1\times10^{18}(5\times10^{20}+1\times10^{18})}\cdot(1.12-0.75)} \approx 2.19\times10^{-6}cm = 21.9\,nm$$

$$x_{p2} = \sqrt{\frac{2\varepsilon}{q}\cdot\frac{N_{DC}}{N_{AB}(N_{DC}+N_{AB})}\cdot(V_{bi-BC}+V_{CB})}$$

$$= \sqrt{\frac{2\times11.7\times8.85\times10^{-14}}{1.6\times10^{-19}}\cdot\frac{8\times10^{16}}{1\times10^{18}(8\times10^{16}+1\times10^{18})}\cdot(0.89+0.5)} \approx 1.15\times10^{-6}cm = 11.5\,nm$$

$$W_{BM} = 0.45 - 0.15 = 0.3\mu m$$

$$\therefore W_B = W_{BM} - x_{p1} - x_{p2}$$

$$= 0.3\mu m - 21.9nm - 11.5nm$$

$$= 266.6nm$$

b) What is the values of I_C, I_E, and I_B? (15 points)

$$I_E \approx \frac{qAD_{nB}}{W_B} \cdot \frac{n_i^2}{N_{AB}} \cdot \left(e^{qV_{BE}/kT} - 1\right) + \frac{qAD_{pE}}{W_E} \cdot \frac{n_i^2}{N_{DE}} \cdot \left(e^{qV_{BE}/kT} - 1\right)$$

$$X_{n1} = \frac{X_{p1} \cdot N_{AB}}{N_{DE}} = \frac{21.9 \times 1\times10^{18}}{5\times10^{20}} \approx 0.0438\,nm$$

$$W_E = W_{EM} - X_{n1} = 0.15\,\mu m - 0.0438\,nm \approx 150\,nm$$

$$\mu_{nB} = 250\,cm^2/Vs \Rightarrow D_{nB} = \frac{kT}{q} \cdot \mu_{nB} = 0.026 \times 250 = 6.5\,cm^2/s.$$

$$\mu_{pE} = 100\,cm^2/Vs \Rightarrow D_{pE} = \frac{kT}{q} \cdot \mu_{pE} = 0.026 \times 100 = 2.6\,cm^2/s$$

$$\therefore I_E = \frac{1.6\times10^{-19}C \times 100\times10^{-8}cm^2 \times 6.5\,cm^2/s}{266.6\times10^{-7}cm} \cdot \frac{1\times10^{20}/cm^6}{1\times10^{18}/cm^3} \cdot e^{0.75/0.026}$$

$$+ \frac{1.6\times10^{-19} \times 100\times10^{-8} \times 2.6}{150\times10^{-7}} \cdot \frac{1\times10^{20}}{5\times10^{20}} \cdot e^{0.75/0.026}$$

$$\approx 1.31\times10^{-5}A + 1.87\times10^{-8}A$$

$$\approx 1.312\times10^{-5}A$$

$$I_C \approx \frac{qAD_{nB}}{W_B} \cdot \frac{n_i^2}{N_{AB}} \cdot \left(e^{qV_{BE}/kT} - 1\right) \approx 1.31\times10^{-5}A$$

$$I_B = I_E - I_C \approx 1.87\times10^{-8}A.$$

c) What are the base transport factor, α_T and the emitter injection efficiency, γ? (15 points)

$D_B = 6.5\,cm^2/s$. $\tau_B = 1\mu s$, $L_B = \sqrt{D_B \tau_B} = \sqrt{6.5 \times 1 \times 10^{-6}} \approx 2.55 \times 10^{-3}\,cm$

base transport factor $\alpha_T = \frac{\partial I_n(W_B)}{\partial I_n(0)} \approx 1 - \frac{W_B^2}{2 L_B^2}$

$$= 1 - \frac{(266.6 \times 10^{-7}\,cm)^2}{2 \times (2.55 \times 10^{-3}\,cm)^2}$$

$$\approx 0.999945$$

emitter injection efficiency $\gamma = \frac{\partial I_{pE}}{\partial I_E}$

$$\approx \frac{1}{1 + \frac{D_E W_B N_{AB}}{D_B W_E N_{DE}}}$$

$$= \frac{1}{1 + \frac{2.6 \times 266.6 \times 1 \times 10^{18}}{6.5 \times 150 \times 5 \times 10^{20}}}$$

$$\approx 0.9986$$

d) **What are the common base current gain, α_o and the common emitter current gain, β_o? (10 points)**

Common base current gain $\alpha_0 = \alpha_T \cdot \gamma \cdot M$

$$\approx \alpha_T \cdot \gamma$$

$$= 0.999945 \times 0.9986$$

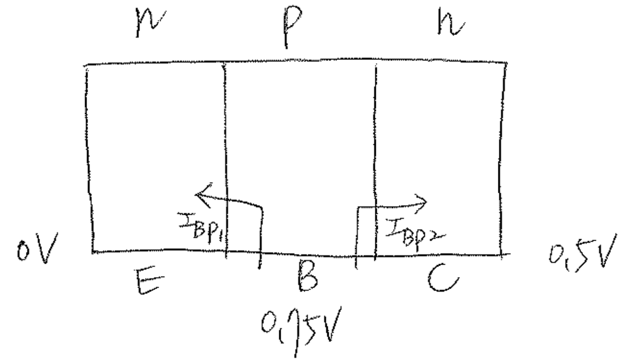
$$\approx 0.9985$$

Common emitter current gain $\beta_0 = \frac{\alpha_0}{1-\alpha_0}$

$$= \frac{0.9985}{1-0.9985}$$

$$\approx 665.7$$

(e) (Bonus question) if V_{CE} is 0.5V, that is Base-Collector junction is *forward* biased at 0.25V, what are I_C, I_E, and I_B? Discussed. (20 points)



since the BE junction is the Same as the ~~@~~ 1. (a)

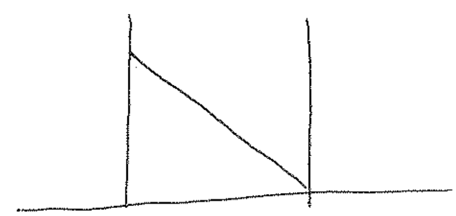
⇒ forward bias at 0.75V

and I_E is controlled by V_{BE}

∴ I_E is still the same as before (equation is the same)

At BC junction, minority carriers at base side ~~@~~ ~~also~~ is $0 \gtrsim \frac{n_i^2}{N_B} \exp(\frac{qV_{BC}}{KT})$, which is relatively small compare to the minority carries at BE junction base side

⇒ So we can still assume it's a straight line

⇒ 

So ~~and~~ $I_B = I_{BP1} + I_{BP2}$ as before

then $I_C = I_E - I_B$

2 (a) For the transistor in (1) if we want to reduce the neutral base width by 25% while maintaining the same current gain, β_o, what is the new base doping concentration, assuming the same minority carrier (i.e. electron) mobility? (15 points)

$$W_{B,neutral} = 0.3\ \mu m - \sqrt{\frac{2\varepsilon_{si}\,(\psi_{bi,BE} - 0.75)}{q\left(\frac{N_B N_E}{N_B + N_E}\right)}} \cdot \frac{N_E}{N_B + N_E} - \sqrt{\frac{2\varepsilon_{si}\,(\psi_{bi,BC} + 0.5)}{q\left(\frac{N_B . N_C}{N_B + N_C}\right)}} \cdot \frac{N_C}{N_B + N_C}$$

New $W_{B,neutral} = 0.267\ \mu m \times 0.75 = 0.200\ \mu m$

By trial and error, $N_{B,new} = \underline{1.85\ 10^{17}\ cm^{-3}}$ gives $W_{B,neutral} = 0.2\mu m$

~~W_E = const~~

W_E can be adjusted to retain the same β_o.

Note that another solution exists if the metallurgical base width can be changed instead of W_E.

two perspectives:

new $W_B = W_{B,old} \times 75\%$

- calculate N_B from W_B, adjust W_E to retain the same β_o.
- β_o remains constant → calculate N_B, adjust W_{BM} to give required W_B for the new N_B.

(b) Do the same as in (a) if the minority carrier (i.e. electron) mobility is $\propto \dfrac{1}{1+\left(\dfrac{N_A}{1\times 10^{17}\,cm^{-3}}\right)^{-0.25}}$**? (10 points)**

For $W_{B,new} = 0.75 \times W_{B,old}$, trial and error gives $N_{B,new} = 1.85\ 10^{17}\ cm^{-3}$.

Note that another solution exists if the metallurgical base width can be changed instead of W_E.

(c) List two advantages and and 2 disadvantages of this transistor (2b) compared to that in (1) (10 points)

Transistor in 2b has smaller W_B, smaller N_B:

→ Smaller base transit time, faster transistor ✓

→ Less electric field in junctions, less avalanche multiplication ✓

→ Smaller early voltage X

→ Larger base resistance X

(3) (a)The maximum voltage gain a BJT is capable of is given by $g_m r_{out}$ (EE151A). Show that $g_m r_{out}$ is to first order given by

$$g_m r_{out} = \frac{V_A}{V_T}$$

where V_A is the early voltage and V_T is the thermal voltage, kT/q. (8 points)

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_C}{\frac{kT}{q}}$$

$$r_{out} = \frac{V_A}{I_C}$$

$$\Rightarrow \quad g_m r_{out} = \frac{V_A}{\frac{kT}{q}} = \frac{V_A}{V_T} \qquad \#$$

(b) What is the maximum voltage gain of the transistor in (1) (12 points)

$$V_A = \frac{qN_B W_B^2}{\epsilon_s}$$

$$g_m r_{out} = \frac{V_A}{V_T}$$

$$V_A = \frac{qW_B N_{AB}}{C_{DBC}}, \quad W_B = 266.6\ nm, \quad N_{AB} = 1\times10^{18}/cm^3, \quad x_{p2} = 11.5\ nm$$

$$x_{n2} = \frac{x_{p2}\cdot N_{AB}}{N_{DC}} = \frac{11.5\times 1\times10^{18}}{8\times10^{16}} \approx 143.75\ nm$$

$$C_{DBC} = \frac{\epsilon_{si}}{x_{p2}+x_{n2}}$$

$$= \frac{11.7\times 8.85\times10^{-14}\ F/cm}{(11.5+143.75)\times10^{-7}\ cm}$$

$$\approx 6.67\times10^{-8}\ F/cm^2$$

$$\therefore V_A = \frac{1.6\times10^{-19}\ c \times 266.6\times10^{-7}\ cm \times 1\times10^{18}/cm^3}{6.67\times10^{-8}\ F/cm^2} \approx 63.95\ V$$

$$\therefore g_m r_{out} = \frac{V_A}{V_T} = \frac{63.95\ V}{0.026\ V} \approx 2460$$