Name: UID:

EE 121B Quiz 3 Solution

You have 1 hour to finish the quiz. There are 4 problems (8%+10%+12%+70%).

Use T=300K unless specified otherwise. Make appropriate assumptions when necessary.

Constants and Equations

$n_i = 1.5 * 10^{10} cm^{-3}$	$q = 1.6 * 10^{-19} C$	$k_B = 8.617 * 10^{-5} eV K^{-1}$ $k_B T = 0.0259 eV$ at T=300K	$\frac{\rho(x)}{\epsilon_s} = \frac{dE(x)}{dx} = \frac{-d^2\phi}{dx^2}$
$n_0 = n_i exp[\frac{E_F - E_{Fi}}{kT}]$ $p_0 = n_i exp[\frac{E_{Fi} - E_F}{kT}]$	$J_{n,drift} = q\mu_n nE$ $J_{p,drift} = q\mu_p pE$	$J_{n,diffusion} = qD_n \frac{dn}{dx}$ $J_{p,diffusion} = -qD_p \frac{dp}{dx}$	$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}$
$n_0 p_0 = n_i^2$	$\sigma = q\mu_n n + q\mu_p p$ $\rho = 1/\sigma$	$\tau_d = \frac{\epsilon}{\rho}$	$V_{bi} = \frac{kT}{q} ln(\frac{N_a N_d}{n_i^2})$
$n_p = n_{p0} exp(\frac{qV_a}{kT})$ $p_n = p_{n0} exp(\frac{qV_a}{kT})$	$L_p^2 = D_p \tau_{p0}$ $L_n^2 = D_n \tau_{n0}$	$x_n = \left\{\frac{2\epsilon_s(V_{bi}+V_R)}{q}\left[\frac{N_a}{N_d}\right][\frac{1}{N_a+N_d}]\right\}^{1/2}$ $x_p = \left\{\frac{2\epsilon_s(V_{bi}+V_R)}{q}\left[\frac{N_d}{N_a}\right][\frac{1}{N_a+N_d}]\right\}^{1/2}$	
$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + g_p - \frac{p}{\tau_{pt}}$		$J_{ideal} = (\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n})\left(exp\left(\frac{qV_a}{kT}\right) - 1\right)$	
$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + g_n - \frac{n}{\tau_{nt}}$		$D_p \frac{\partial^2(\delta p_n)}{\partial x^2} - \mu_p E \frac{\partial(\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial(\delta p_n)}{\partial t}$	
$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p}$, $\mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p}$		$D_n \frac{\partial^2(\delta n_p)}{\partial x^2} + \mu_n E \frac{\partial(\delta n_p)}{\partial x} + g' - \frac{\delta n_p}{\tau_{n0}} = \frac{\partial(\delta n_p)}{\partial t}$	
$\gamma = \frac{J_{nE}}{J_{nE} + J_{pE}}$	$\alpha_T = \frac{J_{nC}}{J_{nE}}$	$\delta = \frac{J_{nE} + J_{pE}}{J_{nE} + J_{pE} + J_R}$	$\alpha = \gamma \alpha_T \delta$
$I_C \approx \alpha I_E$ $I_C = \beta I_B$	$I_{CEO} = (\beta + 1) I_{CBO}$	$V_{CE}(sat) = kT ln[\frac{I_C(1-\alpha_R) + I_B}{\alpha_F I_B - (1-\alpha_F) I_C}(\frac{\alpha_F}{\alpha_R})]$	

Hyperbolic functions

$sinh(x) = \frac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$		$cosh(x) = \frac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$	
$tanh(x) = \frac{sinh(x)}{cosh(x)}$	$ctnh(x) = \frac{cosh(x)}{sinh(x)}$	$sech(x) = \frac{1}{cosh(x)}$	$csch(x) = \frac{1}{sinh(x)}$

Name: UID:

1. What is different between 2 back-to-back diodes and the npn BJT? Draw the npn BJT model that is based on 2 back-to-back diodes. What is this model called? How does the model take the into account the difference between 2 back-to-back diodes and a npn BJT? (8%)

Solution:

The npn BJT is two back-to-back diodes that interact with each other. For example the "forward active mode" of two back-to-back diodes will essentially be one diode "on" and one diode "off". The "off" diode does not pass significant current.

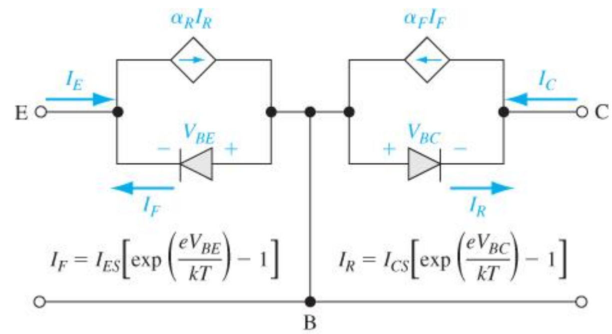
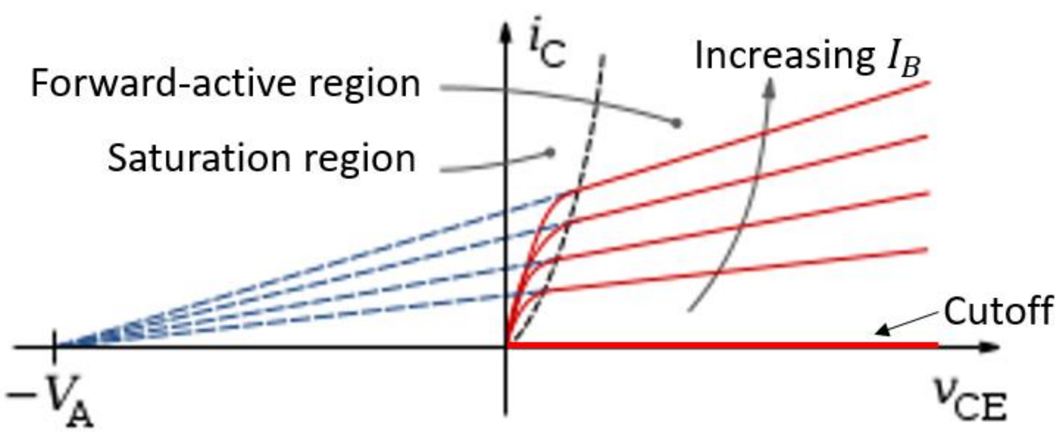


Figure 12.37 | Basic Ebers–Moll equivalent circuit.

The Ebers-Moll model uses 2 back-to-back diodes, but it adds two current sources that are individually coupled to the diode on the other side. So now the two diode parts can effectively interact with each other considering the current source in the model.

2. Draw the $I_C(V_{CE})$ characteristics of the npn BJT. Show the cases of cutoff, forward-active and saturation in your plot. Show the effect of increasing I_B. Show the Early effect and label the Early voltage in your plot. (10%)

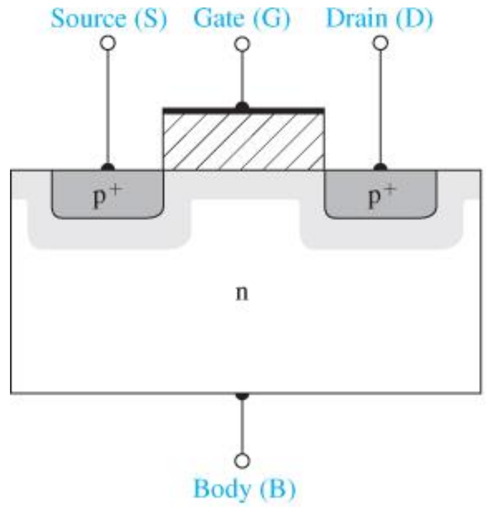
Solution:



Name: UID:

3. Draw a p channel MOSFET and show the lateral BJT it has. What is the type of this BJT? What role does this BJT play in this MOSFETs operation'? What limitation does this BJT impose on the MOSFET operation? (12%)

Solution:

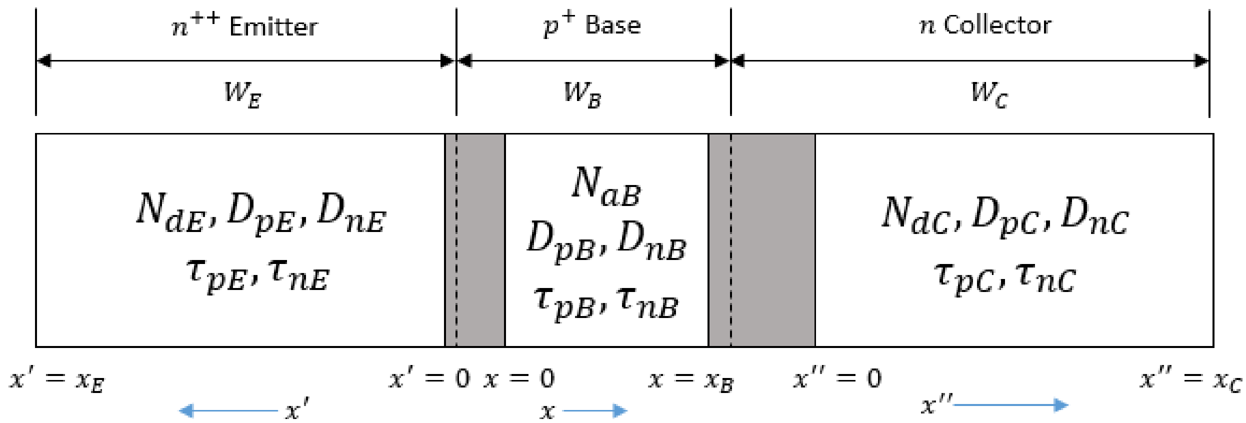


Consider the source as the emitter, substrate channel as the base and the drain as the collector.

The lateral BJT is the pnp BJT.

This BJT is responsible for the subthreshold drain current of the MOSFET. At subthreshold, the gate is (slightly) negatively biased, source is grounded and the E-B is forward biased. As the drain voltage decreases, the C-B junction is reverse biased and the current in this BJT becomes the subthreshold current of the MOSFET.

This diffusion nature of the BJT's forward-active current limits the subthreshold swing of the MOSFET to $\ln(10)\frac{kT}{q} \approx 60mV/dec$ at 300K.

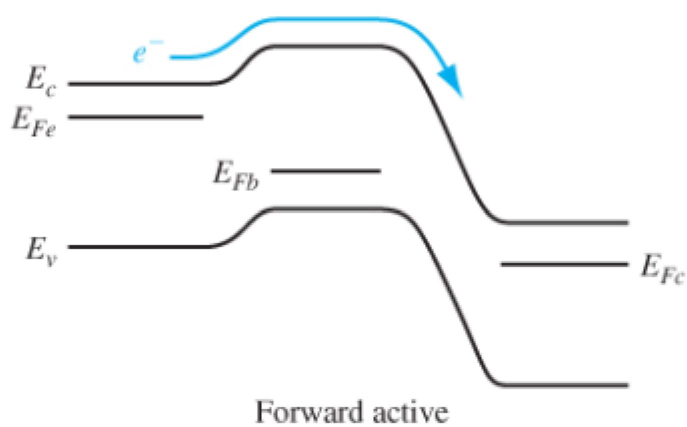


4. A long-emitter, short-base, long collector n[++]p[+]n BJT with a constant cross-sectional area of A, has the doping concentrations N_{dE}, N_{aB} and N_{dC} in each region. The diffusion coefficients and the carrier lifetimes are $D_{pE}, D_{nE}, D_{pB}, D_{nB}, D_{pC}, D_{nC}$ and $\tau_{pE}, \tau_{nE}, \tau_{pB}, \tau_{nB}, \tau_{pC}, \tau_{nC}$. It is operating in forward-active mode by V_{BE} and V_{BC}.

   Answer the following questions with the parameters and dimensions given in the figure. You can make appropriate first order assumptions and approximations. (70%)

   a. What do you know about V_{BE} and V_{BC} in this forward-active BJT? Sketch its band diagram. (8%)

      Solution: Forward-active means B-E (pn) is forward biased and B-C (pn) is reverse biased. Therefore $V_{BE} > 0$, $V_{BC} < 0$.



   b. What are the limitations on W_B for this short-base BJT? (8%)

      Solution:
      Because $x_B > 0$, so W_B should be larger than the depletion width in the base region
      $W_B > x_{p,EB} + x_{p,CB} = \left\{\frac{2\epsilon_s(V_{bi,BE} - V_{BE})}{q}\left(\frac{1}{N_{aB}}\right)\right\}^{1/2} + \left\{\frac{2\epsilon_s(V_{bi,BC} + V_{CB})}{q}\left(\frac{N_{dC}}{N_{aB}^2}\right)\right\}^{1/2}$
      Also $W_B \ll L_{nB}$.

c. To the first order, derive the excess minority carrier profile in all neutral regions. (14%)

Solution:

To apply the ambipolar transport equation, we can assume $E \approx 0$ in the neutral regions. The generation and recombination g' can be neglected and in the steady state the time differential term is zero.

In neutral base (p) region, the minority carrier distribution follows the ambipolar transport equation $D_{nB}\frac{\partial^2}{\partial x^2}\Delta n_B(x) - \frac{\Delta n_B(x)}{\tau_{B0}} = 0$, with $L_{nB}^2 = D_{nB}\tau_{B0}$.

The boundary conditions are $\Delta n_B(x=0) = n_B(x=0) - n_{B0} = n_{B0}[\exp\left(\frac{qV_{BE}}{kT}\right) - 1]$ and $\Delta n_B(x = x_B) = 0 - n_{B0} = -n_{B0}$.

Because it is short-base the profile of the excess minority carrier is approximately linear

Thus $\Delta n_B(x) \approx n_{B0}\left[exp\left(\frac{qV_{BE}}{kT}\right) - 1\right] - \frac{x}{x_B}\left[n_{B0}\left[exp\left(\frac{qV_{BE}}{kT}\right) - 1\right] - (-n_{B0})\right]$

$= \frac{n_i^2}{N_{aB}x_B}\{\left[exp\left(\frac{qV_{BE}}{kT}\right) - 1\right](x_B - x) - x\}$.

In neutral emitter (n) region, the minority carrier distribution follows the ambipolar transport equation $D_{pE}\frac{\partial^2}{\partial x^2}\Delta p_E(x') - \frac{\Delta p_E(x')}{\tau_{E0}} = 0$, with $\mathrm{L}_{\mathrm{pE}}^2 = D_{pE}\tau_{E0}$.

Boundary conditions are $\Delta p_E(x'=0) = p_{E0}[\exp\left(\frac{qV_{BE}}{kT}\right) - 1]$ and $\Delta p_E(x' \to x_E) = 0$

Because it is long-emitter, the excess minority carrier is approximately exponential

The solution is $\Delta p_E(x') = \frac{n_i^2}{N_{dE}}[\exp\left(\frac{qV_{BE}}{kT}\right) - 1]exp(-\frac{x'}{L_{pE}})$

In neutral collector (n) region, the minority carrier distribution follows the ambipolar transport equation $D_{pC}\frac{\partial^2}{\partial x^2}\Delta p_C(x'') - \frac{\Delta p_C(x'')}{\tau_{C0}} = 0$, with $L_{pC}^2 = D_{pC}\tau_{C0}$.

The boundary conditions are $\Delta p_C(x''=0) = -p_{C0}$ and $\Delta p_C(x'' \to x_C) = 0$

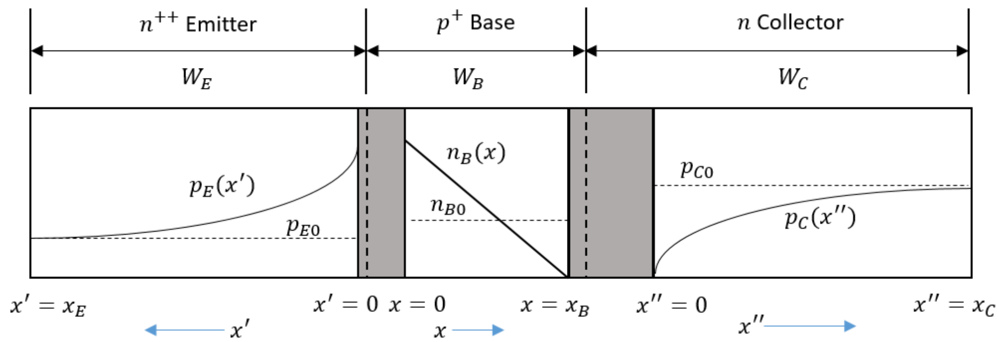
Because it is long-collector, the excess minority carrier is approximately exponential

The solution is $\Delta p_C(x'') = -\frac{n_i^2}{N_{dC}}exp(-\frac{x''}{L_{pC}})$

Name: UID:

d. Sketch the profile of the minority carriers in this device based on your solution from (c). Show the equilibrium concentrations in the sketch. (8%)

Solution：



e. Assume negligible generation and recombination current. Let the current going into the device to be positive. Derive the current in each terminal I_C, I_B, I_E. (12%)

Solution:

The current at the emitter and at the collector can be derived using the similar argument used for diode current derivation. Then we can use KCL $I_E + I_B + I_C = 0$ to get the current at the base terminal.

$$J_E = J_n(x=0) + J_p(x'=0) = qD_{nB}\frac{\partial}{\partial x}\Delta n_B(x=0) + qD_{pE}\frac{\partial}{\partial x'}\Delta p_E(x'=0)$$

$$I_E \approx -qAD_{nB}\frac{n_i^2}{N_{aB}x_B}\exp\left(\frac{qV_{BE}}{kT}\right) - qAD_{pE}\frac{n_i^2}{N_{dE}L_{pE}}\exp\left(\frac{qV_{BE}}{kT}\right)$$

Similarly,

$$J_C = J_n(x=x_B) + J_p(x''=0) = -qD_{nB}\frac{\partial}{\partial x}\Delta n_B(x=x_B) + qD_{pC}\frac{\partial}{\partial x''}\Delta p_C(x''=0)$$

$$I_C \approx qAD_{nB}\frac{n_i^2}{N_{aB}x_B}exp\left(\frac{qV_{BE}}{kT}\right) + qAD_{pC}\frac{n_i^2}{N_{dC}L_{pC}}$$

$$I_B = -I_E - I_C = qAD_{pE}\frac{n_i^2}{N_{dE}L_{pE}}exp\left(\frac{qV_{BE}}{kT}\right) - qAD_{pC}\frac{n_i^2}{N_{dC}L_{pC}}$$

f. Based on the derivation from (e), what is the <u>common emitter current gain</u> of this BJT? From a device designer's perspective, how can you improve it? (10%)

Solution:

$$\beta = \frac{I_C}{I_B} \approx \frac{qAD_{nB}\frac{n_{iB}^2}{N_{aB}x_B}exp\left(\frac{qV_{BE}}{kT}\right)}{qAD_{pE}\frac{n_{iE}^2}{N_{dE}L_{pE}}exp\left(\frac{qV_{BE}}{kT}\right)} = \frac{D_{nB}N_{dE}L_{pE}}{D_{pE}N_{aB}x_B} = \frac{D_{nB}N_{dE}\sqrt{\tau_{pE}}}{\sqrt{D_{pE}}N_{aB}x_B}$$

We can increase the difference in the doping level of the emitter and base to make larger N_{dE}/N_{aB} and decrease D_{pE}/D_{nB} at the meantime.

We can make the base shorter (within the limitation)

We can use other materials in the emitter that has larger τ_{pE}

g. Based on the previous derivations, how can you show the Early effect? You don't need to get the exact answer but you need to show the equations to use and the result to expect. (10%)

Solution:

Because $I_C \approx qAD_{nB}\frac{n_{iB}^2}{N_{aB}x_B}exp\left(\frac{qV_{BE}}{kT}\right)$, $x_B = W_B - \left\{\frac{2\epsilon_s(V_{bi,BE}-V_{BE})}{q}\left(\frac{1}{N_{aB}}\right)\right\}^{\frac{1}{2}} - \left\{\frac{2\epsilon_s(V_{bi,BC}+V_{CB})}{q}\left(\frac{N_{dC}}{N_{aB}^2}\right)\right\}^{1/2}$

As V_{CE} increases for a fixed I_B, the forward bias on the B-E junction stays constant but the reverse bias on C-B junction increases and consequently increases the depletion width of the C-B junction and decreases the neutral base width.

Therefore,

$$\frac{\partial I_C}{\partial V_{CE}} = \frac{\partial I_C}{\partial x_B}\frac{\partial x_B}{\partial V_{CE}}$$

$$= -\frac{I_C}{x_B}\frac{\partial}{\partial V_{CE}}\left(W_B - \left\{\frac{2\epsilon_s(V_{bi,BE}-V_{BE})}{q}\left(\frac{1}{N_{aB}}\right)\right\}^{\frac{1}{2}} - \left\{\frac{2\epsilon_s(V_{bi,BC}+V_{CB})}{q}\left(\frac{N_{dC}}{N_{aB}^2}\right)\right\}^{\frac{1}{2}}\right)$$

$$= -\frac{I_C}{x_B}\frac{\partial}{\partial V_{CE}}\left(-\frac{2\epsilon_s(V_{bi,BC}+V_{CB})}{q}\left(\frac{N_{dC}}{N_{aB}^2}\right)\right)^{\frac{1}{2}}$$

$$= \frac{I_C}{x_B}\frac{\partial}{\partial V_{CE}}\left(\frac{2\epsilon_s(V_{bi,BC}+V_{CE}-V_{BE})}{q}\left(\frac{N_{dC}}{N_{aB}^2}\right)\right)^{\frac{1}{2}} > 0$$

Thus, I_C increases as V_{CE} increases and the BJT has a finite output resistance.