Name: UID:

EE 121B Quiz 1

You have 1 hour to finish the quiz. There are four problems and each is 25% of the total grade. For problem 3, pick one from A ,B and C.

Use T=300K unless specified otherwise. Make appropriate assumptions when necessary.

Constants for room temperature Si and Useful equations

$n_i = 1.5 * 10^{10} cm^{-3}$ $E_g = 1.14eV$ $\epsilon_{\mathrm{Si}} = 1.04 * 10^{-12} F/cm$ $q = 1.6 * 10^{-19} C$

$h = 6.63 * 10^{-34} m^2 kg/s = 2\pi\hbar$ $k_B = 1.38 * 10^{-23} m^2 kg s^{-2} K^{-1} = 8.617 * 10^{-5} eV K^{-1}$

$$\mathrm{p} = \hbar k, \mathrm{E} = \frac{\hbar^2 k^2}{2m}, m_{\mathrm{eff}} = \hbar^2/(\frac{\partial^2 E}{\partial k^2}) \qquad \mathrm{f_F}(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

$$\mathrm{n_0} p_0 = n_i^2 = N_c N_v \exp(-\frac{E_g}{kT}) \qquad \mathrm{g_c}(E) = \frac{4\pi(2m_n^*)^{\frac{3}{2}}}{h^3}\sqrt{\mathrm{E} - \mathrm{E_C}}, \mathrm{g_v}(E) = \frac{4\pi(2m_p^*)^{\frac{3}{2}}}{h^3}\sqrt{\mathrm{E_v} - \mathrm{E}}$$

$$n_0 = N_c\, exp\left[-\frac{(E_F - E_c)}{kT}\right] = n_i \exp[\frac{E_F - E_{Fi}}{kT}] \qquad p_0 = N_v exp[-\frac{E_v - E_F}{kT}] = n_i \exp[\frac{E_{Fi} - E_F}{kT}]$$

$$\mathrm{n_0} + N_a - p_a = p_0 + N_d - n_d \qquad \mathrm{n_d} = \frac{N_d}{1+\frac{1}{2}\exp(\frac{E_d - E_F}{kT})}, \mathrm{p_a} = \frac{N_a}{1+\frac{1}{4}\exp(\frac{E_F - E_a}{kT})}$$

$$\mathrm{n_0} = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \qquad |\mathrm{v}| = \mu|\mathrm{E}|, \quad \frac{1}{\mu} = \frac{1}{\mu_{lattice}} = \frac{1}{\mu_{impurity}}$$

$$\mathrm{J_{n,drift}} = q\mu_n n E, \mathrm{J_{p,drift}} = q\mu_p p E \qquad \mathrm{J_{n,diffusion}} = qD_n \frac{dn}{dx}, \mathrm{J_{p,diffusion}} = -qD_p \frac{dp}{dx}$$

$$\sigma = q\mu_n n + q\mu_p p,\ \rho = 1/\sigma,\ \tau_{\mathrm{d}} = \frac{\epsilon}{\rho}$$
$$\mathrm{L_p^2} = D_p \tau_{p0}, \mathrm{L_n^2} = D_n \tau_{n0} \qquad \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + g_p - \frac{p}{\tau_{pt}}, \qquad \frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + g_n - \frac{n}{\tau_{nt}} \qquad D_p \frac{\partial^2(\delta p_n)}{\partial x^2} - \mu_p E \frac{\partial(\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial(\delta p_n)}{\partial t}$$

$$\mathrm{D}' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p}, \quad \mu' = \frac{\mu_n \mu_p (p-n)}{\mu_n n + \mu_p p} \qquad D_n \frac{\partial^2(\delta n_p)}{\partial x^2} + \mu_n E \frac{\partial(\delta n_p)}{\partial x} + g' - \frac{\delta n_p}{\tau_{n0}} = \frac{\partial(\delta n_p)}{\partial t}$$

$$\frac{\rho(x)}{\epsilon_s} = \frac{dE(x)}{dx} = \frac{-d^2\phi}{dx^2} \qquad V_{bi} = \frac{kT}{q}\ln(\frac{N_a N_d}{n_i^2}), \quad V_B \approx \frac{\epsilon_s E_{crit}^2}{2qN_B}$$

$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{q}\left[\frac{N_a}{N_d}\right][\frac{1}{N_a + N_d}]\right\}^{1/2}, x_p = \left\{\frac{2\epsilon_s V_{bi}}{q}\left[\frac{N_d}{N_a}\right][\frac{1}{N_a + N_d}]\right\}^{1/2} \qquad J_{ideal} = (\frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n})\left(\exp\left(\frac{qV_a}{kT}\right) - 1\right)$$

$$C' = \frac{dQ'}{dV_R} = \left\{\frac{q\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}\right\}^{1/2} \qquad n_p = n_{p0}\exp(\frac{qV_a}{kT}), \quad p_n = p_{n0}\exp(\frac{qV_a}{kT})$$

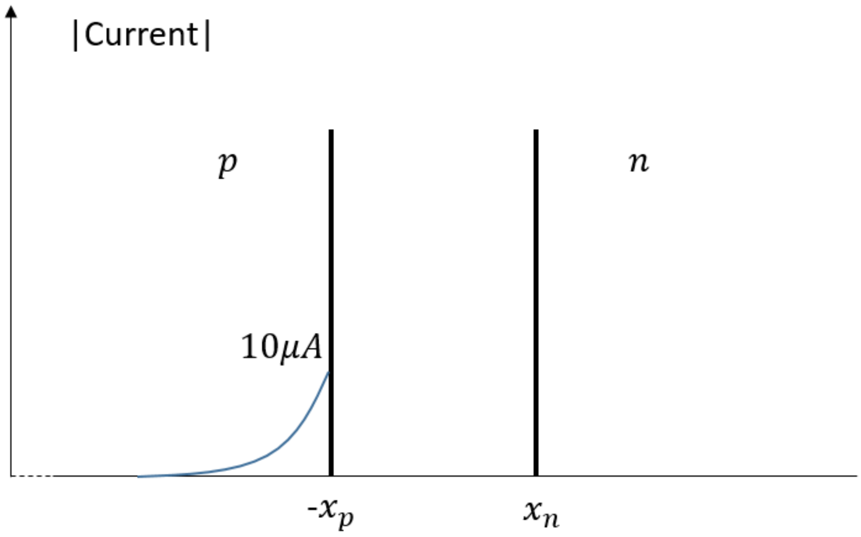
Some solutions for the ambipolar transport equation

$$\boldsymbol{\delta p(t) = \delta p(0) e^{-t/\tau_{p0}}}$$
$$\boldsymbol{\delta p(t) = g' \tau_{p0} (1 - e^{-t/\tau_{p0}})}$$

$$\delta p(x) = \delta p(0) e^{-|x|/L_p}$$
$$\delta p(x,t) = e^{-t/\tau_{p0}} * \left(4\pi D_p t\right)^{-\frac{1}{2}} * exp[-\frac{(x - \mu_p E_0 t)^2}{4D_p t}]$$

Name: UID:

1. **(a)** For intrinsic Si, use y-axis for energy and show the relative positions for the conduction band edge, valence band edge and the Fermi level. **(b)** In the figure from (a), sketch the Fermi-Dirac distribution, density of states functions for electrons and holes, carrier concentration for electrons and holes (total of 5 functions). **(c)** Now the Si is doped by a single dopant so that $E_F - E_{Fi} = 0.2eV$, What's the type of the dopant? What are the carrier concentrations now and what is the doping concentration? **(d)** Repeat (b) for the doped Si in (c).



2. **(a)** When we calculate the ideal current for a forward-biased diode (ignoring the recombination current), what are the four components of the current? In the figure, we showed one of the four components in the neutral p region. Which component is this one? **(b)** If $D_n = 25cm^2/s$, $D_p = 10cm^2/s$, $\tau_{n0} = 5 * 10^{-7}s$ and $\tau_{p0} = 5 * 10^{-6}s$ and the given current component is $10\mu A$ at $-x_p$ and $N_a = N_d$, what is the total current of the forward-biased diode? Plot the total current in the same figure. **(c)** Sketch electron current and hole current in the neutral p region, depletion region and neutral n regions based on the information from (a) and (b).

3. Choose one from problem A, B, C

A: (a) Sketch how carrier mobility changes with temperature and explain. **(b)** For a moderately doped Si ($N_d{\sim}10^{15}cm^{-3}$), sketch its conductivity ($\log(\sigma)$) versus temperature ($1/T$).

B: Si is used as dopant ($\mathrm{N_I}$) in GaAs. Suppose a fraction of the Si atoms, $\alpha\%$ replaces Ga and the other $1-\alpha\%$ of Si atoms replaces As in the lattice. How will the Fermi level change in the GaAs with respect to α?

C: **(a)** For a p-i-n junction with the linear dopant profile as shown in the figure, what's the applied reverse bias if the depletion width is $4\mu\mathrm{m}$? **(b)** Repeat (a) if the i-region is now replaced by metal ($\sigma=\infty$).

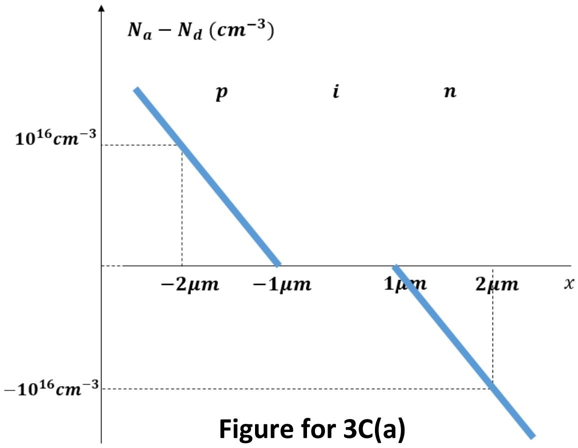


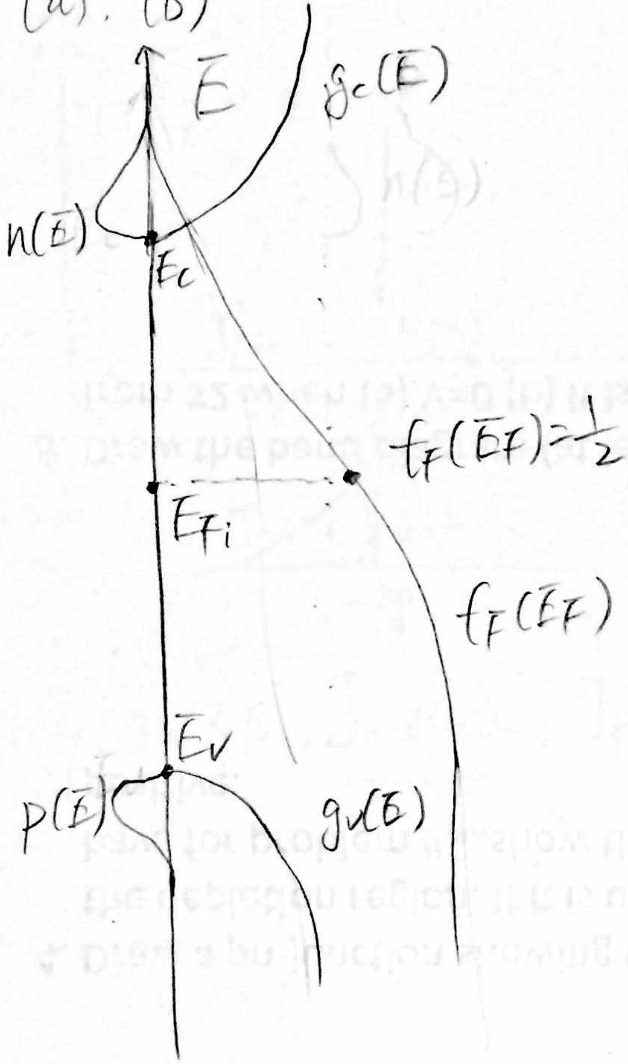
Figure for 3C(a)

Name: UID:

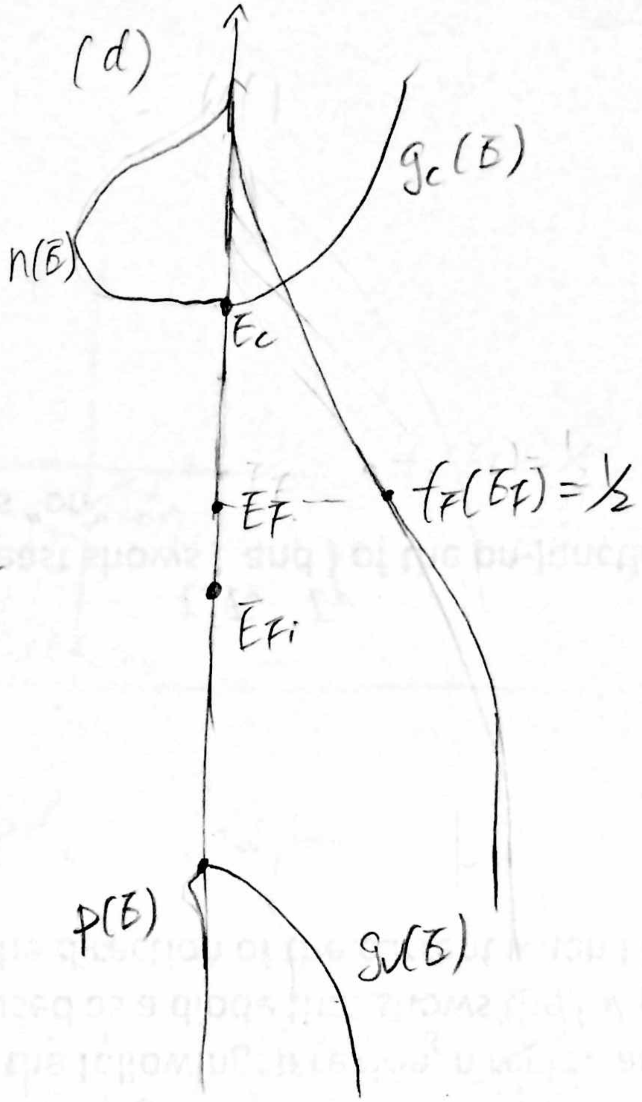
4. Assume that a finite number of electron-hole pairs is generated instantaneously at $t = 0$ and $x = 0$ in an n-type semiconductor, assume the excess carrier generation rate is zero for $t > 0$. **(a)** Sketch excess carrier concentration with respect to x for $t = t_1, t = t_2, t = t_3$ (3 curves) and $\mathrm{t}_3 > t_2 > t_1 > 0$ for no external field ($\mathrm{E}_0 = 0$). **(b)** Repeat (a) for $\mathrm{E}_0 > 0$. **(c)** Inspired by the result from (b), a very famous experiment is set up to measure the <u>minority carrier mobility</u>, <u>minority carrier diffusion coefficient</u> and <u>minority excess carrier lifetime</u>. What's the name of the experiment and what's its significance? Draw the set up for the experiment. **(d)** Describe how you can use it to measure the underlined parameters (you might need hint from the equation sheet).

Name: UID:

1. (a). (b)



(d)



(c) $E_F > E_{Fi} \Rightarrow$ n-type dopant

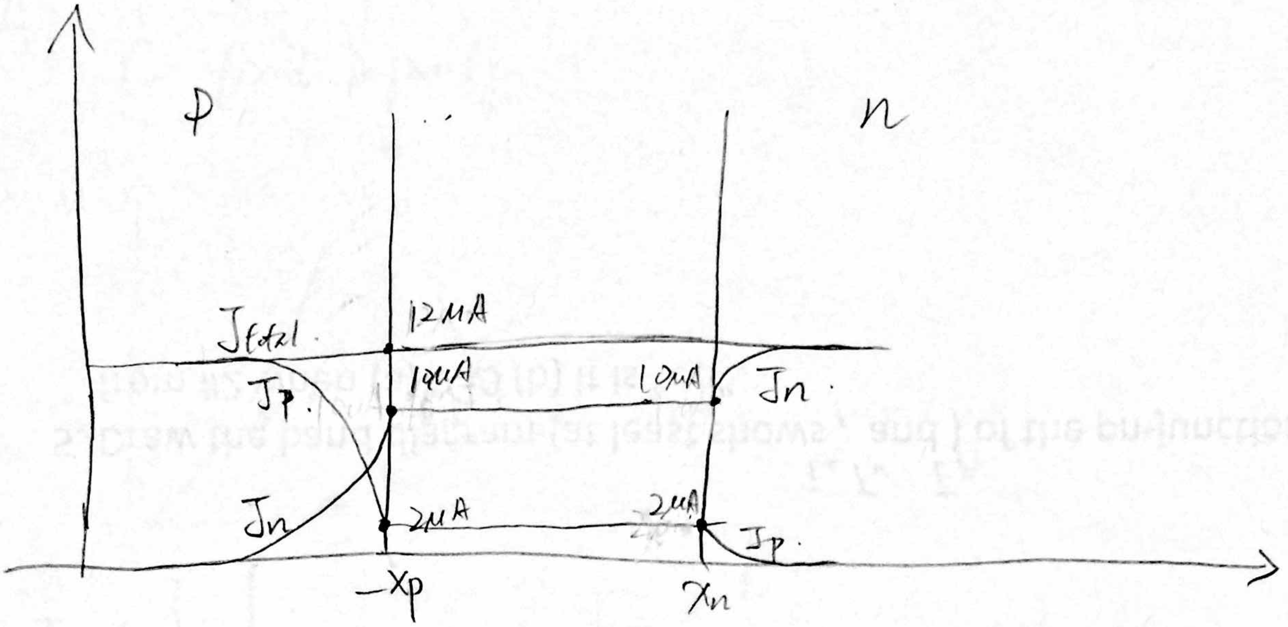
$n_0 \approx n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) = n_i \times 2.3\times10^3 = 3.4\times10^{13}/cm^3$

$p_0 = n_i^2/n_0 = 6.6\times10^6/cm^3$

$n_0 \gg n_i \Rightarrow$ carrier mostly from dopant,

assume complete ionization @ 300K $\Rightarrow N_d = n_0 = 3.4\times10^{13}/cm^3$

2,



(a) $J_{n,drift}$, $J_{n,diffusion}$, $J_{p,drift}$, $J_{p,diffusion}$

is the gime curve

(b) $$\frac{J_n(-x_p)}{J_p(x_n)} = \frac{J_n(-x_p)}{J_p(-x_p)} = \frac{D_n}{D_p}\cdot\frac{L_p}{L_n} = \frac{D_n\sqrt{D_p\tau_{p0}}}{D_p\sqrt{D_n\tau_{n0}}} = \sqrt{\frac{D_n\tau_{p0}}{D_p\tau_{n0}}} = \sqrt{\frac{25\cdot5\cdot10^{-6}}{10\cdot5\cdot10^{-7}}}$$

$$= 5.$$

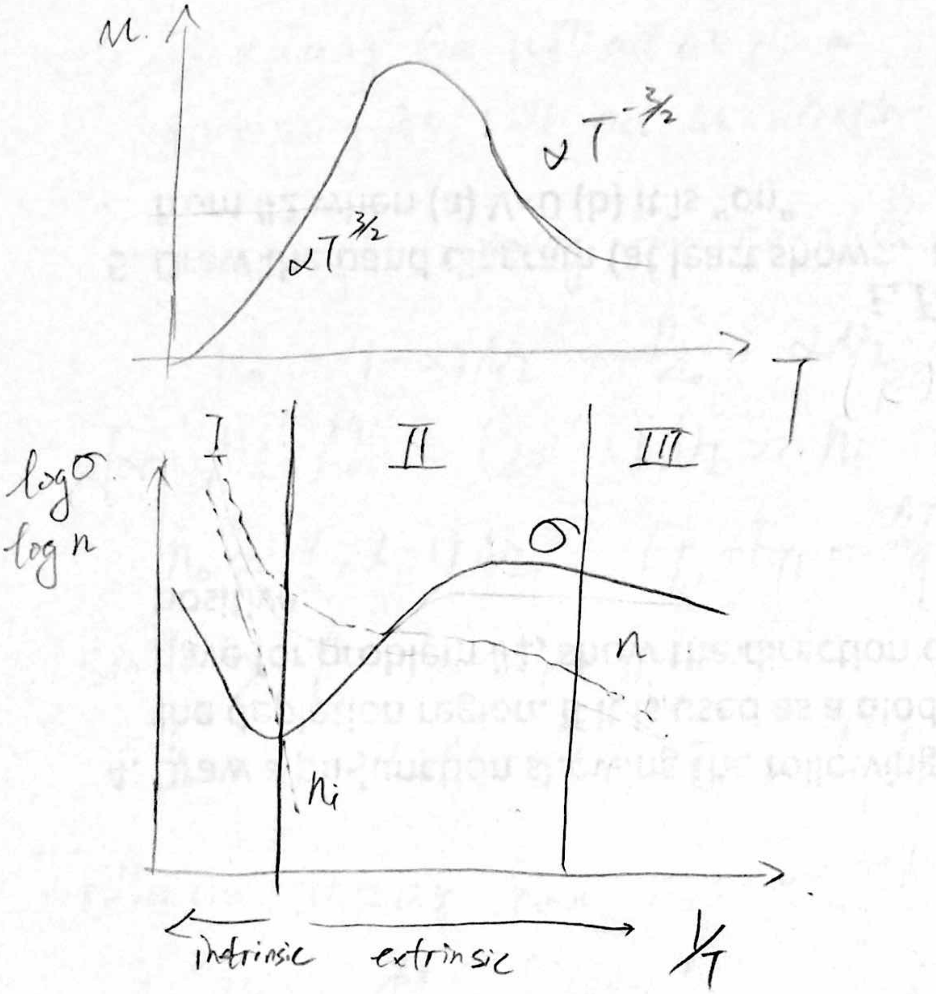
$$\Rightarrow J_n(-x_p) = 5\cdot J_p(x_n) = 5J_p(-x_p)$$

$$J_{total} = J_n(-x_p) + J_p(-x_p) = 10\mu A + \frac{10\mu A}{5} = 12\mu A$$

(c) in figure

3

A: for fixed impurity



High T (region I), $n_i \gg$ doping, $n_i \propto T^{3/2} \exp\left(\frac{-E_g}{2kT}\right)$

$\sigma \propto \mu \cdot n_i \approx T^{-3/2} \cdot T^{3/2} \exp\left(\frac{-E_g}{2kT}\right)$

$\Rightarrow \log \sigma \propto -1/T$

Medium T (region II) $n \approx N_d$ (complete ionization)

$\Rightarrow \sigma \propto \mu \Rightarrow \sigma$ goes up and down like μ

Low T (region III) carrier freezes out: $n \downarrow$ as $T \downarrow$

also $\mu \downarrow$ as $T \downarrow \Rightarrow \sigma \downarrow$ as $T \downarrow$

3.B.

GaAs is $\underset{\text{Ga}}{\underset{\uparrow}{\text{III}}} - \underset{\text{As}}{\underset{\uparrow}{\text{V}}}$ material. Si is in Group IV

$$\Rightarrow \left\{\begin{array}{l}\text{Si replacing Ga will act as donor}\\ \text{Si replacing As will act as acceptor}\end{array}\right. \Rightarrow \left\{\begin{array}{l} N_d = \alpha N_I \\ N_a = (1-\alpha) N_I\end{array}\right.$$

from charge neutrality, for complete ionization

$$n_0 + (1-\alpha) N_I = \frac{n_i^2}{n_0} + \alpha N_I$$

for $N_d - N_a = (2\alpha - 1) N_I \gg n_i$

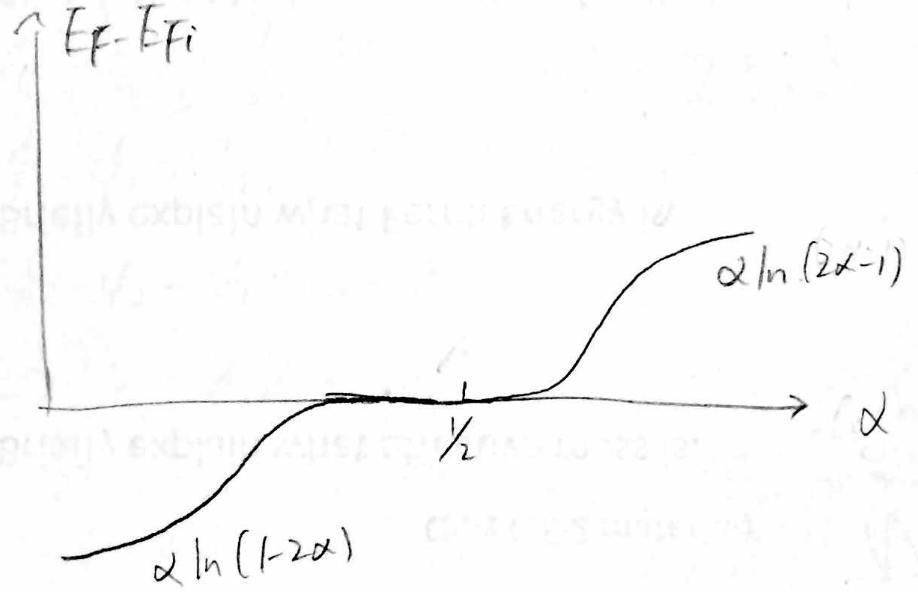
$$n_0 \approx (2\alpha-1) N_I, \quad E_F - E_{Fi} = \frac{kT}{q} \cdot \ln\left[\frac{(2\alpha-1)N_I}{n_i}\right]$$

similarly for $N_a - N_d = (1-2\alpha) N_I \gg n_i$:

$$p_0 \approx (1-2\alpha) N_I, \quad E_{Fi} - E_F = \frac{kT}{q} \ln\left[\frac{(1-2\alpha)N_I}{n_i}\right]$$

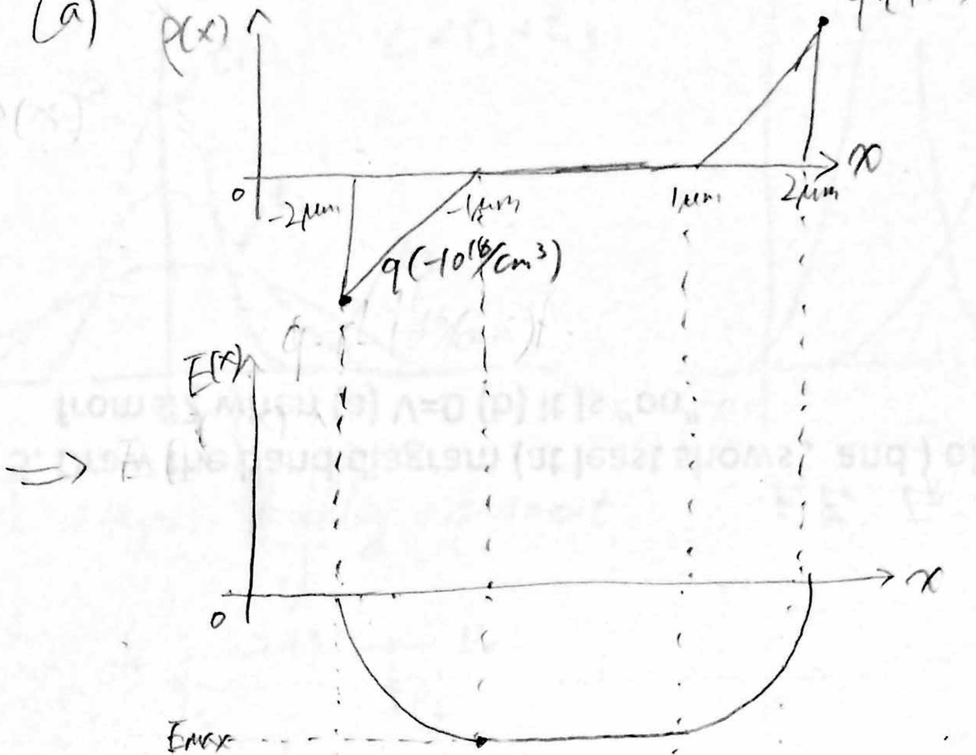
otherwise $\alpha \approx 0.5$ and $n_0 = \frac{(2\alpha-1)N_I}{2} + \sqrt{\left[\frac{(2\alpha-1)N_I}{2}\right]^2 + n_i^2}$

$$E_F - E_{Fi} = \frac{kT}{q} \ln\left\{\frac{(2\alpha-1)N_I}{2} + \sqrt{\left[\frac{(2\alpha-1)N_I}{2}\right]^2 + n_i^2}\right\}$$

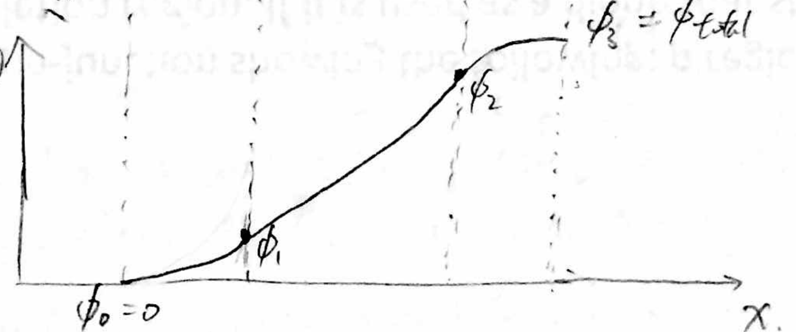


3. C. The doping is "symmetric" so $x_p = x_n = 2\mu m$

(a)



$$E_{max} = \frac{1}{\epsilon}\int_{-2\mu m}^{-1\mu m} \rho(x)\,dx = \frac{(-10^{16}/cm^3)\cdot q}{\epsilon_{Si}}\cdot 1\mu m\cdot\frac{1}{2} = -7.7\times10^{3}\,V/cm$$



$$\phi_1 - \phi_0 = \frac{q\left(\frac{10^{16}/cm^3}{1\mu m}\right)}{6\epsilon}(1\mu m)^3 = 2.57V$$

$\phi_2 - \phi_1 = |E_{max}|\cdot 2\mu m = 15.4V$

$\phi_3 - \phi_2 = \phi_1 - \phi_0 = 2.57V$

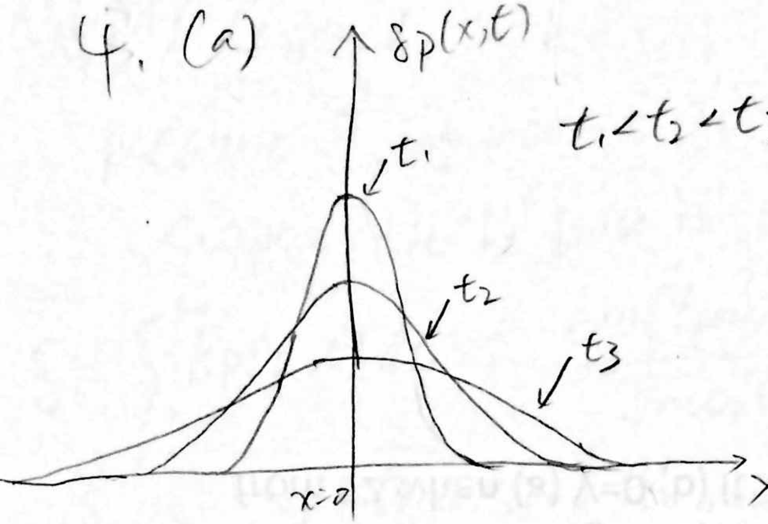
$\Rightarrow V_R \approx \phi_{total} = (\phi_3-\phi_2)+(\phi_2-\phi_1)+(\phi_1-\phi_0) = 20.5V$

(b) for p-i-n junction with same depletion width

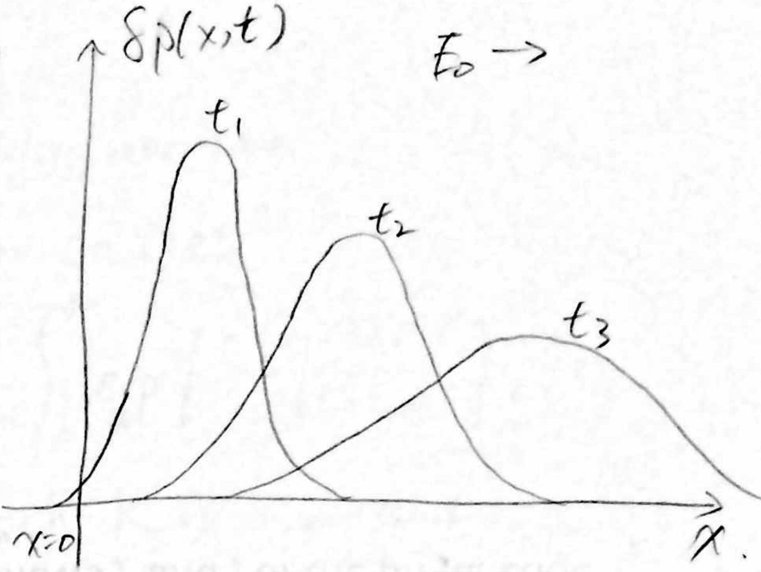
$\phi_1 - \phi_0 = 2.57V = \phi_3 - \phi_2$, $\phi_2 - \phi_1 = 0$ (in perfect conductor)

$V_R = 2\times 2.57V = 5.14V$

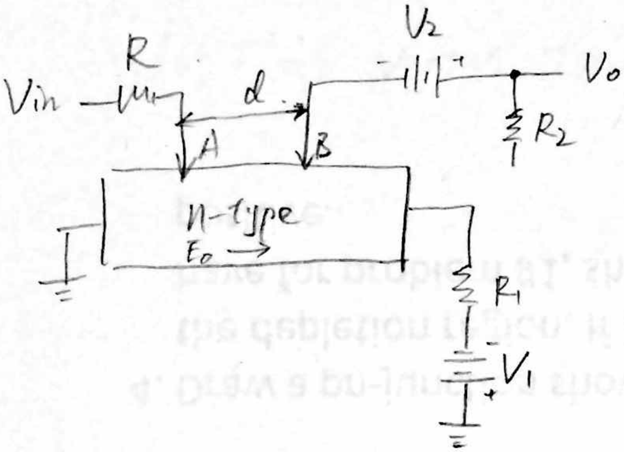
4. (a)

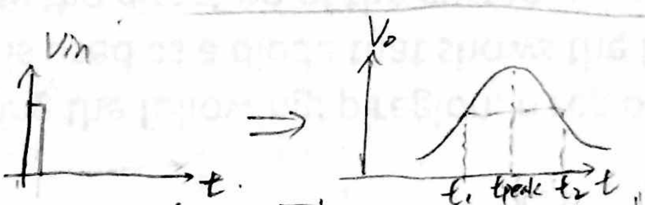


(b)



(c). Haynes-Shockley experiment.





(d). Let the V_{in} to be a square pulse. The output voltage $V_0(t)$ has a pulse shape proportional to $\delta p(x,t)$ with $x=d$.

The pulse provides an instantaneous carrier injection, the bias from V_1 provides a constant field. E_0 to the right.

$$\Rightarrow \delta p(x,t) = \frac{\exp(-t/\tau_{p0})}{\sqrt{4\pi D_p t}} \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4D_p t}\right]$$

from equation sheet

Measure μ_p: the peak of the V_0 is measured at t_{peak}.

$$\Rightarrow d-\mu_p E_0 t_{peak} = 0 \Rightarrow \mu_p = \frac{d}{E_0 t_{peak}}$$

Measure D_p: Find $t_1 \neq t_2$ with $V_0(t_1) = V_0(t_2) = V_0(t_{peak}) \cdot \frac{1}{e}$

suppose $\frac{\exp(-t/\tau_{p0})}{\sqrt{4\pi D_p t}}$ does not change much at $t_1, t_{peak}, t_2 \Rightarrow \begin{cases}(d-\mu_p E_0 t_1)^2 = 4D_p t_1 \\ (d-\mu_p E_0 t_2)^2 = 4D_p t_2\end{cases}$,

$$\Rightarrow D_p = \frac{(\mu_p E_0)^2 (t_2-t_1)^2}{16 t_{peak}}$$

4(d) continued

Measure τ_{p0}: measure the V_0 integrated over time

Suppose the V_0 pulse is narrow so that

$$S = \int_0^\infty \delta p(d,t)\, dt \approx \frac{\exp(t_{peak}/\tau_{p0})}{\sqrt{4\pi D_p t_{peak}}} \cdot \int_0^\infty \exp\left[\frac{-(x-\mu_p E_0 t)^2}{4 D_p t_{peak}}\right].$$

$S = \exp(t_{peak}/\tau_{p0}) \cdot K$, in which K is a constant

$\Rightarrow \ln S = \ln K + \frac{t_{peak}}{\tau_{p0}}$

$\Rightarrow$ plot $\ln S$ versus t_{peak}, the slope is $\frac{1}{\tau_{p0}}$.