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EE121B **Mid Term** **11.2.15**

Assume the temperature T is 300K and the semiconductor is silicon unless otherwise specified.

1. (20')

A short-based Si p-n junction (width $W_p = W_n = 1\mu m$) with cross-sectional area A = 0.001 cm^{-2} is formed with $N_a = 10^{16} cm^{-3}$ and $N_d = 10^{18} cm^{-3}$. Calculate:

(a) Built-in potential, V_{bi}. (5')

For a normal P-N junction we have

$$V_{bi} = kT \ln\left(\frac{N_a N_d}{n_i^2}\right) \approx 0.81V$$

(b) What are the minority carrier concentrations in P and N quasi-neutral region in equilibrium? (5')

In p quasi neutral region, the minority carrier concentration is given by

$$n_0 \approx \frac{n_i^2}{N_a} = 2.25 \times 10^4 cm^{-3}$$

while in n quasi neutral region

$$p_0 \approx \frac{n_i^2}{N_d} = 2.25 \times 10^2 cm^{-3}$$

(c) When a forward bias of 0.4V is applied, what is the current density for the electrons at the edge of the depletion in the P region? Assume μ_n= 600 cm^2/V-S, μ_p = 200 cm^2/V·S, and τ_n=τ_p=25 μs. (5')

The width of the depletion in P and N region is given by

$$x_n = \sqrt{\frac{2\epsilon_{Si}(V_{bi} - 0.4)}{q(N_a + N_d)}\frac{N_a}{N_d}} \approx 2.3nm$$

$$x_p = \sqrt{\frac{2\epsilon_{Si}(V_{bi} - 0.4)}{q(N_a + N_d)}\frac{N_d}{N_a}} \approx 230nm$$

The neutral region width is can be calculated by

$$d_n = W_n - x_n$$

$$d_p = W_p - x_p$$

$$J_n(x_p) = \frac{qD_n n_i^2}{N_a d_p}\left(\exp\frac{qV}{kT} - 1\right) \approx 3.7 \times 10^{-3} A/cm^2$$

The current flows out of p region.

$$\delta n = \frac{n_i^2}{N_a}\left(\exp\frac{qV}{kT} - 1\right) \approx 1.1 \times 10^{11} cm^{-3}$$

(d) Ignore the recombination/generation current, when the P-N diode is reverse-biased, what is the saturation current? (5')

As reverse bias >> kT, the saturation current can be approximated

$$\mathrm{x}_p \approx \sqrt{\frac{2\epsilon_{Si}(V_{bi} + V_R)}{q(N_a + N_d)}\frac{N_d}{N_a}} \approx 322nm, x_n \approx 3.2nm$$

$$\mathrm{I}_0 \approx qA\left(\frac{D_n n_i^2}{N_a(W_n - x_n)} + \frac{D_n n_i^2}{N_a(W_p - x_p)}\right) \approx 8.2 \times 10^{-13} A$$

2. (20’)

A long-based Si p-n junction with cross-sectional area A = 0.001 cm^{-2} is formed with N_a = 10^{17}cm^{-3} and N_d= 10^{17}cm^{-3}. Assume μ_n= 600 cm^2/V-S, μ_p = 200 cm^2/V·S, and τ_n=τ_p=25 μs.

(a) At forward bias of 0.5V, calculate the diffusion charge stored in the n and p quasi-neutral region (5’)

The diffusion charge stored in the quasi neutral region is given by

$$Q_{n,diff} = qA\int n\cdot \mathrm{d}x \approx qA\frac{n_i^2}{N_a}\exp\frac{qV}{kT}\int_0^\infty \exp\left(-\frac{x}{L_n}\right)\mathrm{d}x \approx qA\frac{n_i^2}{N_a}\exp\frac{qV}{kT}L_n \approx 1.7\times 10^{-12}\ \mathrm{C}$$

$$Q_{p,diff} = qA\int p\cdot \mathrm{d}x \approx qA\frac{n_i^2}{N_d}\exp\frac{qV}{kT}\int_0^\infty \exp\left(-\frac{x}{L_p}\right)\mathrm{d}x \approx qA\frac{n_i^2}{N_a}\exp\frac{qV}{kT}L_p \approx 9.9\times 10^{-13}\ \mathrm{C}$$

$(\delta\mathrm{n} = \delta\mathrm{p} \approx \frac{n_i^2}{N_d}\exp\frac{qV}{kT} \approx 5.44\times 10^{11}cm^{-3}$

$\mathrm{L_n} = \sqrt{D_n\tau_n} \approx 1.9\times 10^{-2}cm$

$\mathrm{L}_p = \sqrt{D_p\tau_p} \approx 1.1\times 10^{-2}cm)$

(b) Find the depletion width and depletion capacitance of this diode when it is forward biased at 0.5V (5’)

The built-in potential is given by

$$V_{bi} = kT\ln\left(\frac{N_aN_d}{n_i^2}\right) \approx 0.81V$$

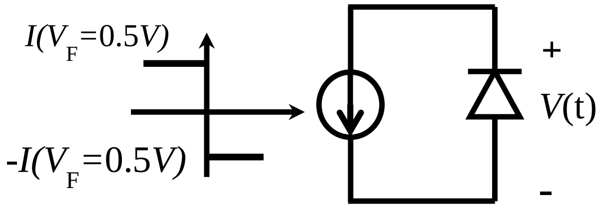
And the depletion width is

$$\mathrm{W_d} = \sqrt{\frac{2\epsilon_{Si}(V_{bi}-V_F)}{qN_aN_d/(N_a+N_d)}} \approx 90.1\mathrm{nm}$$

Therefore, the depletion capacitance is

$$C_{dep} = \frac{A\varepsilon_{\mathrm{Si}}}{W_D} \approx 1.16\times 10^{-10}F$$

(c) Now at t=0, the bias current is reversed and find $\left.\frac{dV(t)}{dt}\right|_{t=0}$ (10')



After time t=0, the current start to discharge the capacitance, as well as the recombination (which has the same current under the forward bias), i.e.,

$$\left.\frac{dV(t)}{dt}\right|_{t=0} = \frac{2I}{C_{dep}+C_{diff}} \approx \frac{2qAn_i^2}{C_{dep}+\frac{\left(Q_n+Q_p\right)}{\frac{kT}{q}}}\left(\frac{D_p}{N_dL_p}+\frac{D_n}{N_aL_n}\right)\left[\exp\left(\frac{qV_F}{kT}\right)-1\right]$$

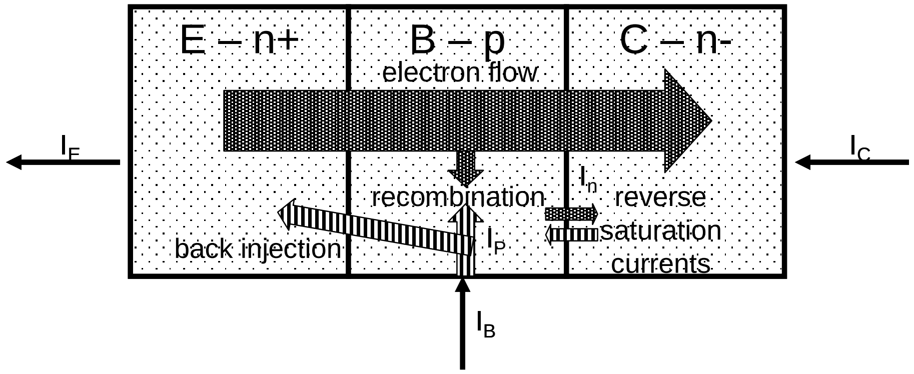
$$\approx 10^3 V/s$$

$$I(V_F = 0.5V) = qAn_i^2\left(\frac{D_p}{N_dL_p}+\frac{D_n}{N_aL_n}\right)\left[\exp\left(\frac{qV_F}{kT}\right)-1\right] \approx 1.1\times10^{-7}A$$

3. (20')

Consider the npn BJT below.

(a) Sketch the current components of the NPN bipolar transistor between the emitter, base and collector. (5')



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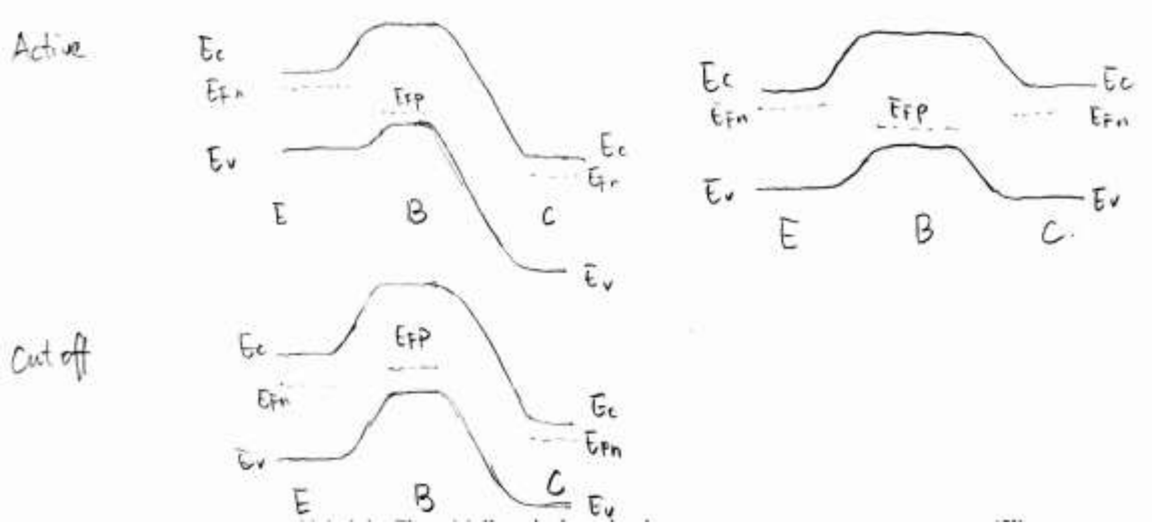
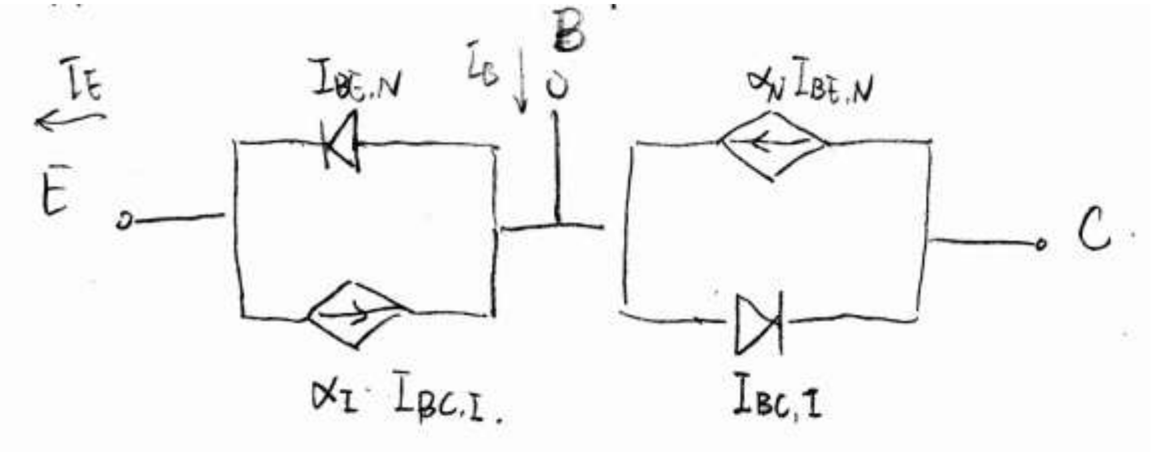
(b) Draw and label the band diagrams in the in active, cutoff and saturation mode (10’)



(c) Draw and label the Ebers-Moll equivalent circuit (5’)



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4. (20’)

A Si NPN BJT has emitter, base, and collector doping levels of 10^{19} cm^{-3}, 10^{18} cm^{-3}, and 10^{17} cm^{-3}, respectively. Assume both the quasi-neutral base and emitter width is 500nm, and the collector width is large. Approximately consider the widths of quasi-neutral region do not change with depletion width. Assume electron and hole mobility of 100 and 50 cm^2/V-s, respectively, in all regions and that the carrier lifetimes are 1 μs everywhere.

(a) When it is biased in the normal active mode, with an emitter-base voltage of 0.6V, if we have negligible base recombination, calculate the emitter current density, emitter injection efficiency, and base transport factor. (5’)

The emitter current density can be calculated as

$$J_E = qn_i^2\left(\frac{D_n}{W_B N_{AB}} + \frac{D_p}{W_E N_{DE}}\right)\cdot\left(\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right) \approx 2.26\times 10^{-2} A\cdot cm^{-2}$$

Emitter injection efficiency

$$\gamma_{\mathrm{BE}} \approx \frac{1}{1+\left(\frac{D_n W_E N_{DE}}{D_p W_B N_{AB}}\right)^{-1}} \approx 0.95$$

As we ignore the recombination in the base region, base transport factor

$$\mathrm{B} \approx 1$$

(b) Find the emitter(collector) injection efficiency when the collector-base junction is forward biased (V_F=0.4V) and emitter-base junction is reverse biased. (5’)

Since the collector has a long neutral region, collector injection efficiency is given by

$$\gamma_{\mathrm{BC}} \approx \frac{1}{1+\left(\frac{D_n L_{pC} N_{DC}}{D_p W_B N_{AB}}\right)^{-1}} \approx 0.814$$

And $\mathrm{L_{pc}} = \sqrt{D_p\tau_p} = 1.1\times 10^{-3} cm$

$$J_C \ or\ J_{C,I} = qn_i^2\left(\frac{D_n}{W_B N_{AB}} + \frac{D_p}{L_{pC} N_{DC}}\right)\cdot\left(\exp\left(\frac{qV_{BC}}{kT}\right) - 1\right) \approx 1.2\times 10^{-5} A\cdot cm^{-2}$$

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(c) When it is biased in the saturation mode (V_E=0V, V_B=0.6V, V_c=0.2V) use Ebers-Moll model to find the emitter, base and collector current density. (10')

Let's define the direction of the current the same way as the one in active mode.

Using Ebers-Moll model and previous calculated J_E and J_C, we have

$$J_E = J_{E,N} - \alpha_{BC} J_{C,I}$$
$$J_C = -J_{C,I} + \alpha_{BE} J_{C,I}$$

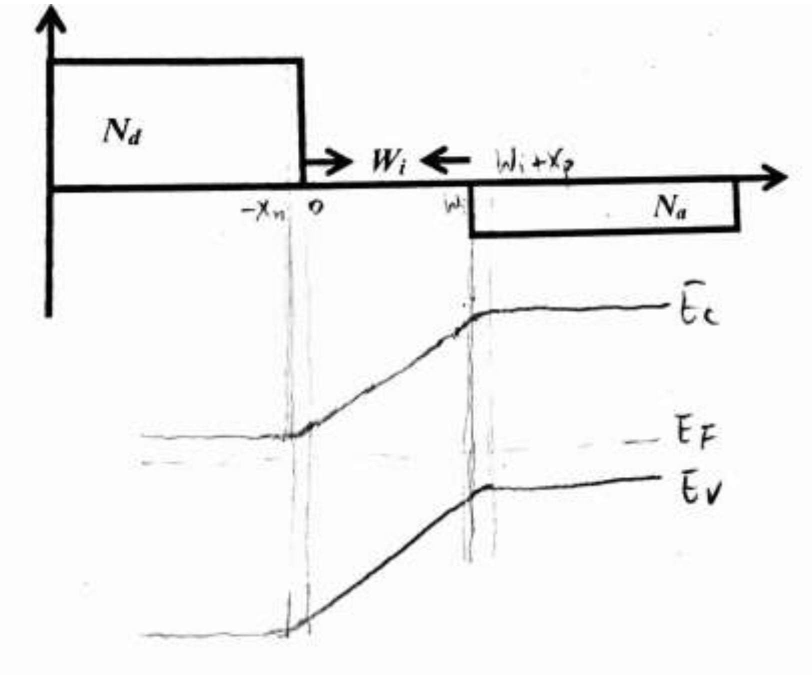
As B ≈ 1 for both cases, we have

$$J_E = J_{E,N} - \gamma_{BC} J_{C,I} \approx 2.26 \times 10^{-2}\, A/cm^2$$
$$J_C = -J_{C,I} + \gamma_{BE} J_{C,I} \approx 2.15 \times 10^{-2} A/cm^2$$
$$J_B = J_E - J_C \approx 1.1\text{m}A/cm^2$$

5. (20')

The PIN silicon diode consists of a P-region with doping of N_a=10^{17}cm^{-3}, an intrinsic region and an N-region with doping of N_d=10^{18}cm^{-3}, as plotted in the figure. Assume the width of the intrinsic region W_i=1μm.

(a) Sketch the band diagram of this PIN diode (5')



(b) At equilibrium, calculate the total depletion width under depletion approximation and the depletion capacitance (5')

Built-in potential $V_{bi} = kT \cdot \ln\left(\frac{N_a N_d}{n_i^2}\right) \approx$ 0.87V

(Under depletion approximation, we can have

$$\frac{1}{2}\frac{qN_a}{\varepsilon_{Si}}x_p^2 + \frac{1}{2}\frac{qN_d}{\varepsilon_{Si}}x_n^2 + \frac{qN_d x_n}{\varepsilon_{Si}}w_i = V_{bi}$$

Using charge neutrality, we have

$$qN_d x_n = qN_a x_p = Q$$

So

$$\frac{1}{2}\frac{Q^2}{qN_a\varepsilon_{Si}} + \frac{1}{2}\frac{Q^2}{qN_d\varepsilon_{Si}} + \frac{Q}{\varepsilon_{Si}}w_i = V_{bi}$$

Solving for Q

$$Q = q\varepsilon_{Si}\left(\frac{N_a N_d}{N_a + N_d}\right)\left[-\frac{w_i}{\varepsilon_{Si}} + \sqrt{\frac{w_i^2}{\varepsilon_{Si}^2} + \frac{2V_{bi}}{q\varepsilon_{Si}\frac{N_a N_d}{N_a+N_d}}}\right] \approx 9.1 \times 10^{-5} C/m^2)$$

On the other hand, we can do the calculation with approximation when Q is small and *w_i* is large,

$$\frac{qN_d x_n}{\varepsilon_{Si}}w_i \gg \frac{1}{2}\frac{Q^2}{qN_d\varepsilon_{Si}}, and\ \frac{1}{2}\frac{Q^2}{qN_d\varepsilon_{Si}}$$

We have $\mathrm{Q} \approx \frac{V_{bi}}{w_i}\varepsilon_{Si} \approx 9.1 \times 10^{-5} C/m^2$

Thus, total depletion width is

$$x_n + x_p + w_i = \frac{Q}{qN_d} + \frac{Q}{qN_a} + w_i \approx 1.006\,\mu m$$

Therefore,

$$\mathrm{C_{dep}} = \frac{\varepsilon_{Si}}{x_n + x_p + w_i} = 1.04 \times 10^{-8}\,F/m^2$$

(c) When the PIN diode is reversely biased at V_R=0.5V, what is the total capacitance across the diode? (5')

Using the same approximation,

$$\mathrm{Q} \approx \frac{V_{bi} + V_R}{w_i}\varepsilon_{Si} \approx 1.43 \times 10^{-4}\,C/m^2$$

$$x_n + x_p + w_i = \frac{Q}{qN_d} + \frac{Q}{qN_a} + w_i \approx 1.01\,\mu m$$

$$\mathrm{C_{dep}} = \frac{\varepsilon_{Si}}{x_n + x_p + w_i} \approx 1.04 \times 10^{-8}\,F/m^2$$

(d) Where and how would we want to use such a PIN diode? (5')

In such a diode, we can expect the depletion width is much wider as normal diode, and the depletion cap remains roughly constant. It is very useful as a light sensor/photodetector or as RF switch.

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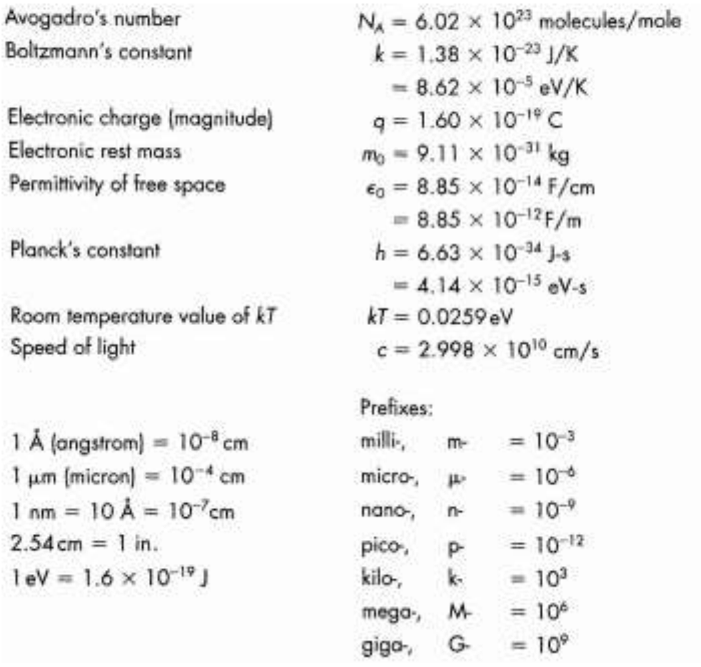
Table of Constants



Avogadro's number	$N_A = 6.02 \times 10^{23}$ molecules/mole
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K
	$= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$q = 1.60 \times 10^{-19}$ C
Electronic rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm
	$= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.63 \times 10^{-34}$ J-s
	$= 4.14 \times 10^{-15}$ eV-s
Room temperature value of kT	$kT = 0.0259$ eV
Speed of light	$c = 2.998 \times 10^{10}$ cm/s

1 Å (angstrom) = 10^{-8} cm
1 μm (micron) = 10^{-4} cm
1 nm = 10 Å = 10^{-7}cm
2.54cm = 1 in.
1 eV = 1.6×10^{-19} J

Prefixes:

milli-,	m-	$= 10^{-3}$
micro-,	μ-	$= 10^{-6}$
nano-,	n-	$= 10^{-9}$
pico-,	p-	$= 10^{-12}$
kilo-,	k-	$= 10^{3}$
mega-,	M-	$= 10^{6}$
giga-,	G-	$= 10^{9}$

General Properties of Silicon	
Atomic Density	5 x 10^{22}cm^{-3}
Atomic Weight	28.09
Density (ρ)	2.328 g cm^{-3}
Energy Bandgap (E_G)	1.12 eV
Intrinsic Carrier Concentration (n_i) at 300K	1.5 x 10^{10}cm^{-3}
Lattice Constant	0.54 nm
Melting Point	1415 ℃
Thermal Conductivity	1.5 Wcm^{-1}K^{-1}
Thermal Expansion Coefficient	2.6 x 10^{-6}K^{-1}
Effective Density of States in the Conduction Band (N_C)	3 x 10^{19}cm^{-3}
Effective Density of States in the Conduction Band (N_V)	1 x 10^{19}cm^{-3}
Relative Permittivity (ε_r)	11.8
Electron Affinity	4.05 eV