EE121B – Midterm

UCLA Department of Electrical & Computer Engineering
EE121B – Principles of Semiconductor Device Design
Winter 2020
Midterm, Jan. 30, 2020 (100 minutes)

Name [redacted]_______________ **Student number** [redacted]

This is a closed book exam – you are allowed 1 pages (A4 size) of notes (front + back). You are allowed to use a calculator. You are NOT allowed to use other electronic devices such as laptops and cell phones. Check to make sure your test booklet has all of its pages – both when you receive it and when you turn it in. Remember – there are several questions, with varying levels of difficulty, be careful not to spend too much time on any one question to the exclusion of all others.

Exam grading: When grading, we focus on evaluating your level of understanding, based on what you have written out for each problem. For that reason, you should make your work clear, and provide any necessary explanation. In many cases, a correct numerical answer with no explanation will not receive full credit, and a clearly explained solution with an incorrect numerical answer will receive close to full credit. CIRCLE YOUR FINAL ANSWER.

If an answer to a question depends on a result from a previous section that you are unsure of, be sure to write out as much of the solution as you can use symbols before plugging in any numbers, that way at you will still receive the majority of credit for the problem, even if your previous answer was numerically incorrect.

Please be neat – we cannot grade what we cannot decipher.

	Max Points	Your points
Problem 1	10	6
Problem 2	30	27
Problem 3	30	30
Problem 4	30	27
Problem 5	10	10
Total	110	100

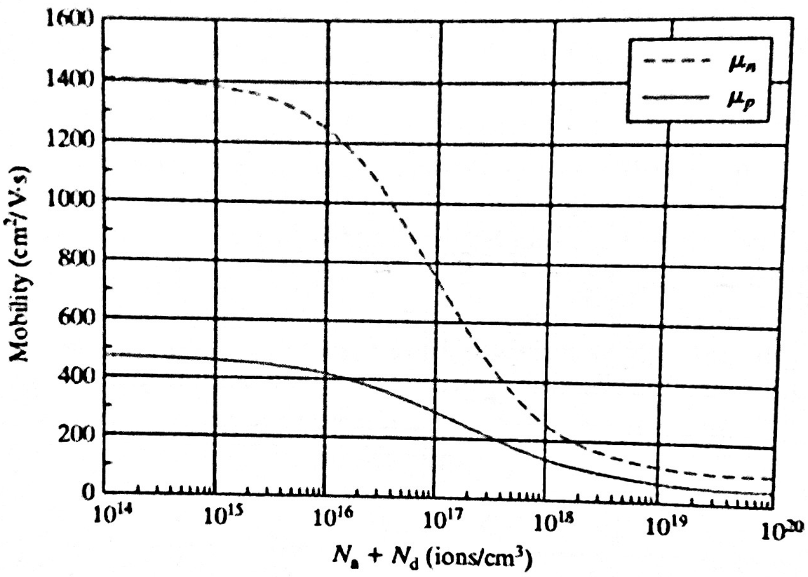
Properties of silicon (Si at 300K).

Description	Symbol	Value
Band gap energy	E_G	1.12 eV
Intrinsic carrier concentration	n_i	10^{10} cm^{-3}
Electron affinity	χ	4.05 eV
Permittivity	ε_{si}	10^{-12} F/cm
Effective density of states in conduction band	N_c	3.2×10^{19} cm^{-3}
Effective density of states in valence band	N_v	1.8×10^{19} cm^{-3}

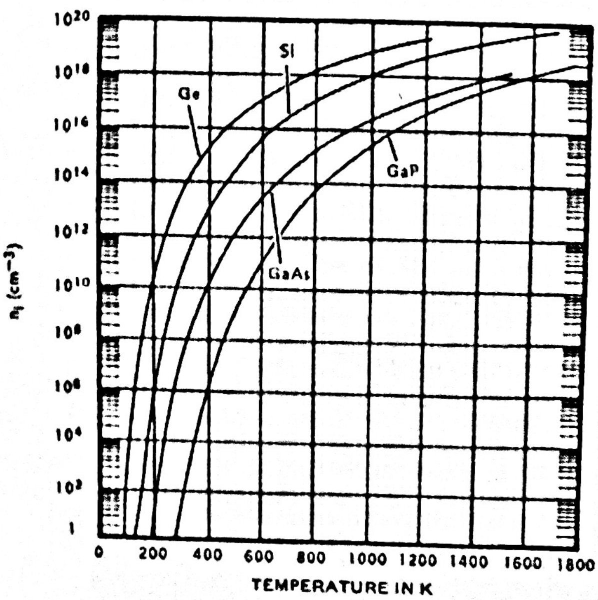
Physical constants

Description	Symbol	Value
Electronic charge	q	1.6×10^{-19} C
Thermal voltage	kT/q	0.026 V

Carrier mobility in silicon



Intrinsic carrier concentration vs. temperature

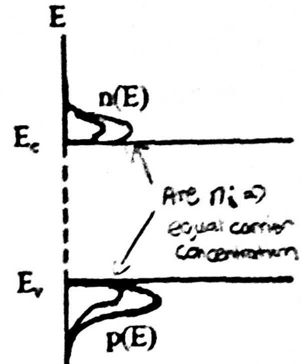


Question 1) [10pts] To the right is a plot of the electron and hole distributions within the conduction and valence bands, respectively, for a lightly doped ($|N_D - N_A| < 10^{17}\,cm^{-3}$) silicon sample maintained at T = 300 K.

a) [5 pts] Is this material *n-type* or *p-type* (Circle one.) Briefly justify your answer.

since the carrier concentration in holes is greater than election carrier concentration

-2



b) [5 pts] Show qualitatively (by adding 2 curves to the plot) how $n(E)$ and $p(E)$ would change if the temperature were to be increased significantly (*e.g.* to 1000 K).

$n(E) = n_i e^{(E_F - E_i)/kT}$ AS $T \to \infty$, $n(E) = n_i$

$p(E) = n_i e^{(E_i - E_F)/kT}$ $p(E) = n_i$

$\frac{n}{n_i} = \frac{N_c e^{-(E_c - E_F)/kT}}{N_c e^{-(E_c - E_i)/kT}} = e^{(E_F - E_c + E_c - E_i)/kT}$ ← scratch work

-2

$np = n_i^2$

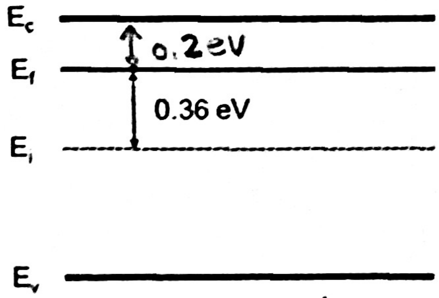
$n = n_i e^{(E_F - E_i)/kT}$

$p = n_i e^{(E_i - E_F)/kT}$

Question 2) [30 pts]

The energy band diagram for a non-compensated and uniformly-doped Si sample maintained at T = 300 K is shown below.

a) [2 pts] Is this sample *n-type* or *p-type*? (Circle one.)



b) [8 pts] Calculate the mobile charge carrier concentration, n and p. (Remember kTln(10) = 0.06 eV at 300K)

$n_i = 10^{10}\,cm^{-3}$

$E_F - E_i = 0.36\,eV$

$n = N_d$ completely ionized.

$N_c = 3.2 \times 10^{19}\,cm^{-3}$

$n = N_c \left(\frac{1}{1 + e^{(E_c - E_F)/kT}}\right)$

$\boxed{n = 1.5 \times 10^{16}\,cm^{-3}} \gg 10^{10}\,cm^{-3}$ n_i

, therefore we can use approximate that $np = n_i^2$

$\boxed{p = n_i^2/n \cong 6.7 \times 10^{-3}\,cm^{-3}}$

X

since $0.2eV < 3kT$,

$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$

$E_c - E_F = \frac{E_g}{2} - (E_F - E_i)$

$E_c - E_F = 0.2eV > 3kT = 0.078$

No Boltzmann approximation

-1

-2

c) [8 pts] Roughly estimate the resistivity of this sample.

$\mu_n \cong 1200\ cm^2/V\text{-}s$

$\mu_p \cong 410\ cm^2/V\text{-}s$

$N_A + N_D \cong 1.5\times10^{16}\ cm^{-3}$

let $n = 10^{16}\ cm^{-3}$

$p = 10^{4}\ cm^{-3}$

from part b)

X

$$\rho = \frac{1}{q\mu_n n + q\mu_p p_o}, \text{ since } n \gg p$$

$$\rho \cong \frac{1}{q\mu_n n} \cong \frac{1}{(1.6\times10^{-19}\ C)(1200\ cm^2/V\text{-}s)(1.5\times10^{16}\ cm^{-3})} \cong \boxed{0.35\ \Omega\cdot cm}$$

d) [4 pts] If this sample were to be subjected to an electric field of strength 400 V/cm, what would be the electron drift velocity?

since $E < 1000\ V/cm$,

let $\mu_n = 1200\ cm^2/V\text{-}s$ $\quad V_d = \mu_n E$

$V_d = (1200\ cm^2/V\text{-}s)(400\ V/cm)$

$\boxed{V_d = 4.8\times10^{5}\ cm/s}$

e) [8 pts] **Qualitatively**, how would your answer to **(c)** and **(d)** change if this sample were to be additionally doped with Boron atoms (Group III) to a concentration of $10^{16}\ cm^{-3}$. Briefly, justify your answer.

$N_A = 10^{16}\ cm^{-3}$, where we let $N_D = 10^{16}\ cm^{-3}$ from part a)

$\cancel{N_D} + p = \cancel{N_A} + n \Rightarrow \underbrace{n = p = n_i^2}_{\text{mass action law}}$

Since the dopant atom cancel each other out, the semiconductor concentration becomes intrinsic. However, due to the added doping, $N_A + N_D = 2\times10^{16}\ cm^{-3}$, the mobility for both hole and electron will change s.t $\mu_n \cong 1000\ cm^2/V\cdot s$ and $\mu_p \cong 400\ cm^2/V\cdot s$. For part c) we cannot ignore the hole mobility and resistivity decreases as $n_i^2 > N_d = 10^{16}\ cm^{-3}$. For part d) the ~~ne~~ electron mobility decreases from impurity scattering and will have a smaller electron drift velocity than part d).

Question 3) [30 pts]

Please refer to the metal-semiconductor (NiSi-Si) device below, and assume: $\Phi_{Bn} = 0.65$ eV, $\Phi_{Bp} = 0.47$ eV. For each of the following cases (a, b, c):

- Calculate the width of the depletion region (W)
- Sketch and annotate the energy-band diagrams indicating the Schottky barrier height (Φ_B), the width of the depletion region (W), and V_{bi}-V_A (where V_{bi} is the built-in voltage and V_A is the applied voltage across the metal-semiconductor device).

a) [10 pts] A contact between NiSi and uniformly doped n-type silicon with $N_D = 10^{17}$ cm^{-3}, $V_A = -1$ V

$\Phi_{Bn} = 0.65$eV

$N_c = 3.2\times10^{19}$ cm^{-3}

$V_{bi} = \dfrac{\Phi_{Bn} - (E_c - E_F)}{q}$

$V_{bi} \cong 0.5$V, where $V_A = 0$

$n = N_d = N_c e^{-(E_c - E_F)/kT} \Rightarrow E_c - E_F = -kT\ln\left(\frac{n}{N_c}\right)$

completely ionized

$E_c - E_F = kT\ln\left(\frac{N_c}{n}\right)$

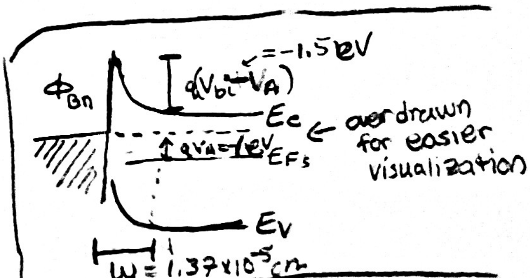
$= kT\ln\left(\frac{3.2\times10^{19} cm^{-3}}{10^{17} cm^{-3}}\right)$

$\cong 0.15$eV

$W = \sqrt{\dfrac{2\varepsilon_s(V_{bi} - V_A)}{qN_D}}$

$W = \sqrt{\dfrac{2(10^{-12} F/cm)(0.5 - (-1))}{1.6\times10^{-19}C\,(10^{17} cm^{-3})}}$

$\boxed{W \cong 1.37\times10^{-5} cm}$



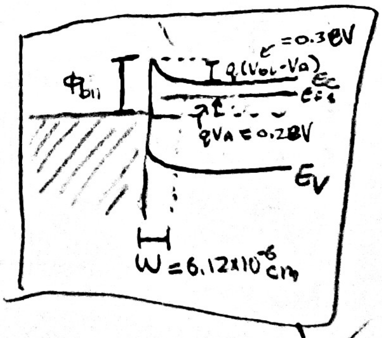
b) [10 pts] A contact between NiSi and uniformly doped n-type silicon with $N_D = 10^{17}$ cm^{-3}, $V_A = 0.2$ V

from part a) where $E_c - E_F \cong 0.15$eV & $V_{bi} \cong 0.5$V

$W = \sqrt{\dfrac{2\varepsilon_s(V_{bi} - V_A)}{qN_D}}$

$= \sqrt{\dfrac{2(10^{-12} F/cm)(0.5 - 0.2V)}{(1.6\times10^{-19}C)(10^{17} cm^{-3})}}$

$\boxed{W \cong 6.12\times10^{-6} cm}$



c) [10 pts] A contact between NiSi and uniformly doped p-type silicon with $N_A = 10^{17}$ cm^{-3}, $V_A = 0$ V

$N_V = 1.8\times10^{19}\ \text{cm}^{-3}$

$\phi_{Bp} = 0.47\ \text{eV}$

$p = N_A = N_V e^{-(E_F-E_V)/kT} \Rightarrow (E_F-E_V) = -kT\ln\left(\frac{N_A}{N_V}\right)$

$= kT\ln\left(\frac{N_V}{N_A}\right)$

$E_F - E_V \cong 0.14\ \text{eV}$

$\phi_{Bp} = qV_{bi} + (E_F - E_V)$

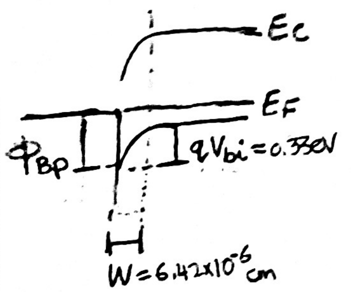
$V_{bi} = \frac{\phi_{Bp} - (E_F - E_V)}{q}$

$V_{bi} = 0.33\ \text{V}$ were $V_A = 0$

$$W = \sqrt{\frac{2\varepsilon_s(V_{bi} + \overset{0}{\cancel{V_A}})}{qN_A}}$$

$$W = \sqrt{\frac{2(10^{-12}\ \text{F/cm})(0.33\ \text{V})}{(1.6\times10^{-19})(10^{17}\ \text{cm}^{-3})}}$$

$\boxed{W = 6.42\times10^{-6}\ \text{cm}}$



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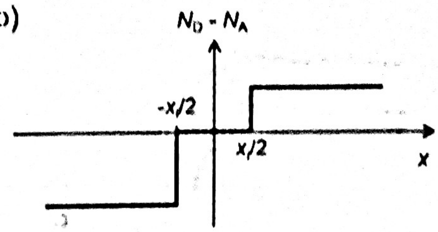
Question 4) [30 pts]

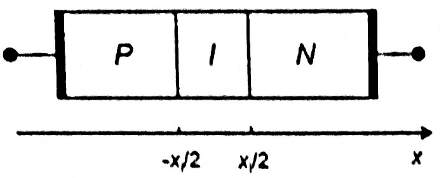
Consider a *PIN* junction (where *I* stands for intrinsic Si, Fig. a) with the doping profile shown in Fig. b.

Hint: the profile of the net charge density is shown in Fig. c.

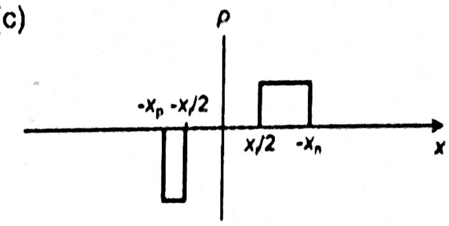
(b)



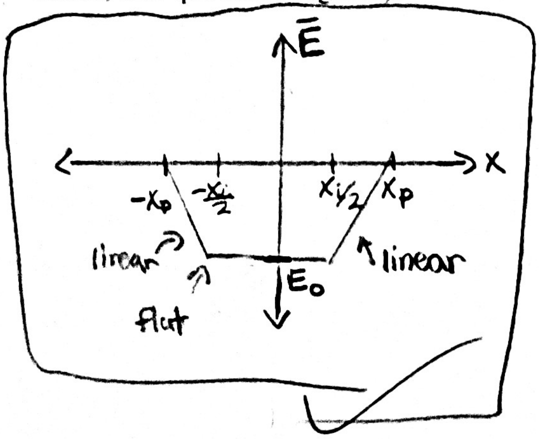
(a)

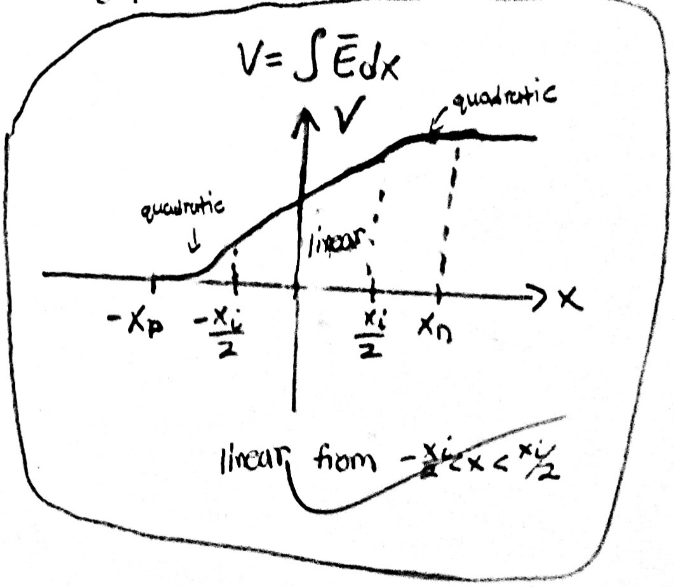


(c)

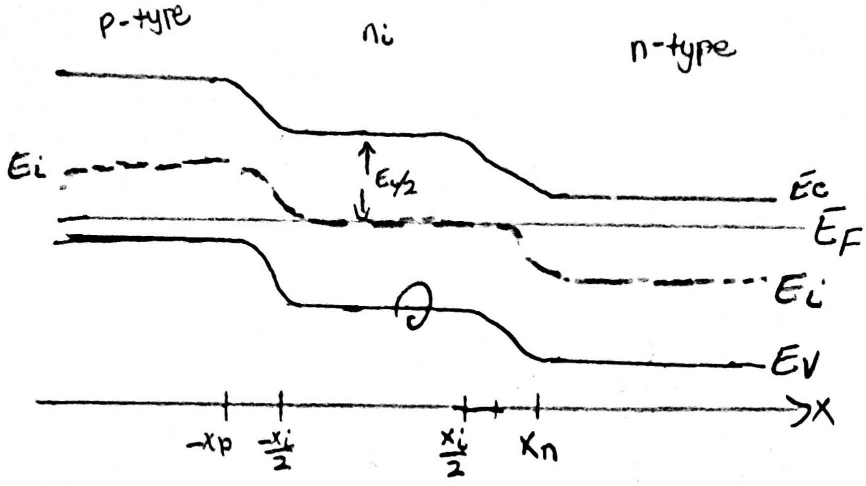


a) [10 pts] Make rough sketches of the electric field and voltage profiles across the device (annotate the flat, linear, and quadratic regions).





b) [10 pts] Draw the equilibrium energy band diagram for the device and identify the location of E_i with respect to E_F at the two ends of the junction.



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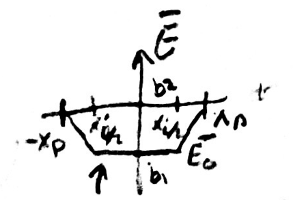
c) [10 pts] Derive an expression for the built-in voltage (V_{bi}) that exists across the junction under equilibrium conditions.



we can determine V by area under the curve

$V = \int E\,dx$

absolute value

$$V_{bi} = \frac{qN_A}{2E_s}\left(\frac{x_i}{2} - x_p\right) + \frac{qn_i}{E_s}x_i + \frac{qN_D}{2E_s}\left(x_n - \frac{x_i}{2}\right)$$

$$\boxed{V_{bi} = \frac{q}{E_s}\left(\frac{N_A}{2}(x_i/2 - x_p) + n_i x_i + \frac{N_D}{2}\left(x_n - \frac{x_i}{2}\right)\right)}$$

Question 5) [10pts]

A uniformly-doped n-type silicon sample has $N_D = 10^{18}$ cm^{-3}. Use $\mu_n = 1200$ cm^2/V·s, $\mu_p = 400$ cm^2/V·s. The carrier lifetimes are $\tau_n = \tau_p = 10^{-7}$ s. ($T = 300$ K)

Let a fixed excess hole concentration $\Delta p_n = \delta p_n$ (x = 0) be injected and maintained at one edge of the sample, at x = 0. Then the excess hole concentration profile throughout the sample will be given by:

$$\delta p(x) = \Delta p_n \exp\left(\frac{-x}{L_p}\right)$$

Find the Δp_n level required to maintain a hole diffusion current density $J_{p,diff} = 10^{-5}$ A/cm^2 = 10 μA/cm^2 at the edge x = 0 at room temperature.

$$p = \frac{n_i^2}{N_D} = 10^2\,cm^{-3}$$

$$n \approx N_D$$

$$\frac{d\delta p(x)}{dx} = -\frac{\Delta p_n}{L_p}\exp\left(\frac{-x}{L_p}\right)$$

$$J_{p,diff} = -qD_p\frac{d(p_0+\delta p)}{\delta x}$$

$$= -qD_p\frac{d\delta p(x)}{dx}$$

$$L_p = \sqrt{D_p\tau_p}$$

$$J_p(x) = -qD_p\left(-\frac{\Delta p_n}{L_p}e^{-x/L_p}\right)$$

$$D_p = \frac{kT}{q}\mu_p$$

$$= (26mV)(400\,cm^2/Vs)$$

$$J_p(x) = q\sqrt{\frac{D_p}{\tau_p}}\Delta p_n e^{-x/L_p}$$

$$D_p = 10.4\,cm^2/s$$

$$J_p(x=0) = q\sqrt{\frac{D_p}{\tau_p}}\Delta p_n \cancel{e^{0}} = 10^{-5}\,A/cm^2$$

$$\Delta p_n = \frac{10^{-5}\,A/cm^2}{q}\sqrt{\frac{\tau_p}{D_p}}$$

$$= \frac{10^{-5}\,A/cm^2}{1.6\times10^{-19}\,C}\sqrt{\frac{10^{-7}\,s}{10.4\,cm^2/s}}$$

$$\boxed{\Delta p_n = 6.1\times10^{9}\,cm^{-3}}$$