# EE 121B Principles of Semiconductor Device Design
## Winter 2014 Midterm Exam
## February 13th, 2014, 4:05-5:45pm

**Name** : ________SOLUTIONS________

**Student ID#** : ______________________________

**Seat No.** : ______________________________

FORMAT:
- TOTAL: 100 POINTS
- TIME ALLOTTED: 100 MINUTES
- CLOSED BOOK
- ONE PAGE OF LETTER SIZE SHEET OF NOTES AND A SCIENTIFIC POCKET CALCULATOR ALLOWED

INSTRUCTIONS:
- USE THE FOLLOWING PHYSICAL CONSTANTS AND GENERAL ASSUMPTIONS IF NECESSARY
- SHOW ALL WORK AS CLEARLY AS POSSIBLE TO MAXIMIZE OPPORTUNITY FOR PARTIAL CERDIT
- USE COMMON SENSE TO INTERPRET THE QUESTIONS, OR ASK IF YOU ARE NOT SURE
- INSERT YOUR ONE PAGE NOTES INTO THE EXAM BOOKLET WHEN YOU TURN IN YOUR EXAM

**Physical constants:**

**I. Electronic charge = $1.60 \times 10^{-19}$ C**
**II. Vacuum Permittivity = $8.85 \times 10^{-14}$ F/cm**
**III. Boltzmann constant = $8.62 \times 10^{-5}$ eV/K**
**IV. Planck constant = $6.63 \times 10^{-34}$ J-s**
**V. Electron mass in vacuum = $9.10 \times 10^{-31}$ kg**

**General assumptions in all problems unless specifically stated otherwise:**

**I. Temperature = 300 K**
**Semiconductor = Silicon**
- **a. Intrinsic carrier concentration ≈ $1 \times 10^{10}$ cm$^{-3}$**
- **b. Permittivity = 11.8**
- **c. Bandgap energy = 1.12 eV**

**Questions:**

1) Figure 1 illustrates the ideal (dashed lines) and practical (solid lines) PN junction *I*-*V* characteristics for both reverse and forward biases.
   (a) Briefly explain the deviations from ideality in regions I to V **{10 points}**

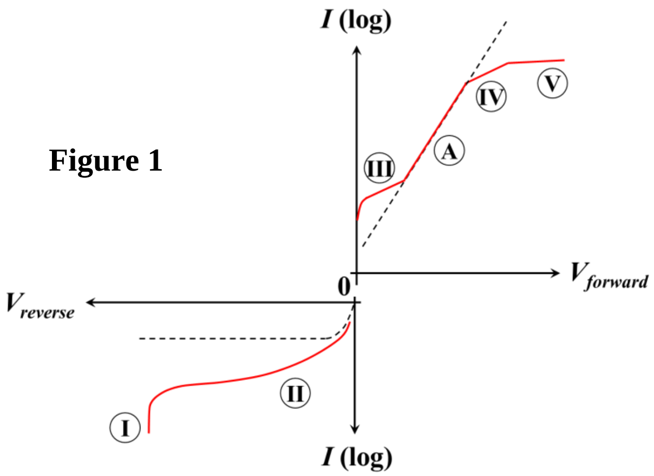


**Figure 1**

**I**. Breakdown – avalanche multiplication of carriers at large reverse bias within transition region

**II**. Generation – EHP creation due to thermal energy absorption

**III**. Recombination – EHP recombines within the transition region due to trap level existence → extra current besides $I_{diff}$ & $I_{drift}$

**IV**. High Leven Injection – injected minority carriers become comparable to the majority carriers in concentration → less effective diffusion and $I_{diff}$ is less than ideal

**V**. Ohmic Loss – at large current, I-R drop in the neutral region is significant → Effective voltage drop in the junction is less and thus lower I

(b) In region A,

(i) Suggest, with a short explanation, a method to lengthen this region **{4 points}**

Increase the doping of the lower doping side of the PN junction

∵ To delay the on-set of **IV**, which is the high level injection

(ii) Calculate the inverse of its slope in the unit of volt **{4 points}**

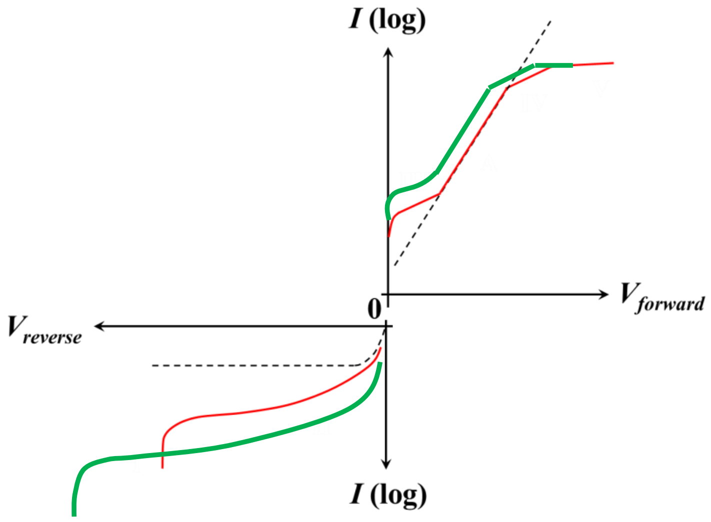
$$I = I_0\left(e^{qV/kT} - 1\right)$$

$$\left(\frac{d\, log_{10} I}{dV}\right)^{-1} = \left(\frac{1}{ln\, 10}\frac{d\, ln\, I}{dV}\right)^{-1} = ln\, 10 \times \frac{kT}{q} = 0.060\, V/dec$$

at RT

(c) Assume $N_a = 10^{19}$ cm$^{-3}$ and $N_d = 10^{17}$ cm$^{-3}$, sketch and justify the new *I*-*V* characteristics when

(i) $N_d$ is decreased to $10^{15}$ cm$^{-3}$ **{8 points}**



$N_a$>>$N_d$ ⇒*p*+-*n* (one side) junction

$N_d$ is the lower doping side

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{qV/kT} - 1\right)$$

Rev bias $I \approx -qA\left(\frac{D_p}{L_p}\frac{n_i^2}{N_d}\right)$

Fwd bias $I \approx qA\left(\frac{D_p}{L_p}\frac{n_i^2}{N_d}\right)\left(e^{qV/kT} - 1\right)$

Now, $N_d$ is ↓

I. higher $V_{BD}$⇒smaller E-field in $N_d$ junction

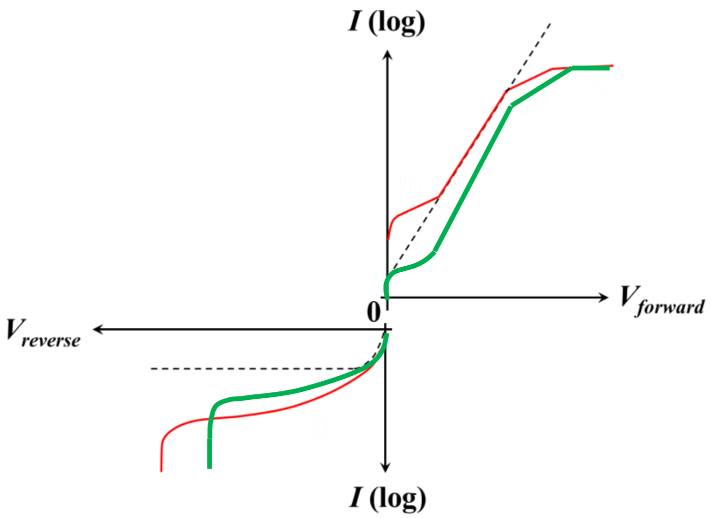
⇒more external bias is needed for BD

II, III. More generation & recombination (‘W’ is longer) & $I_0$ is larger

A. smaller ideal region with the same slope

IV, V. the on-set of high level injection & ohmic offsets are moved forward

(ii) Temperature is decreased **{8 points}**



With the same one-sided PN junction

I. lower VB ⇒less scattering/collision at lower T such that carriers are less likely to lose energy. Therefore we need a smaller bias to provide enough energy to BD

II, III less thermal generation & recombination

A. Slope of the ideal region is $\frac{1}{ln\,10}\frac{q}{kT}$ and a lower T would increase the slope

IV, V. Increased slope with the same argument as for A

2) This question is about PN junction capacitance.

(a) Name the type(s) of capacitance seen in a generic PN junction, and briefly explain its(their) origin(s). **{6 points}**

1. Junction capacitance – distributed charge dipole inside transition region

2. Diffusion capacitance – the lagging behind of voltage as current changes, due to charge storage effects

(b) The small-signal capacitance values at different DC bias measured from a $p^+$-$n$ junction are listed in Table 1. Determine the type(s) of the capacitance and explain your answer. **{4 points}**

Junction capacitance

– under reverse bias conditions, junction capacitance is dominant

**Table 1**

| DC Voltage (V) | Small-Signal Capacitance (pF) |
|---|---|
| 0 | 1.155 |
| -0.2 | 1.048 |
| -0.4 | 0.967 |
| -0.6 | 0.902 |
| -0.8 | 0.848 |
| ≤-1.0 | 0.803 |

(c) The $p^+$-$n$ junction cross sectional area is $10^{-5}$ cm$^2$. Estimate the doping concentration of the $n$-doped region using the data in Table 1. **{8 points}**

$$\frac{1}{C_j^2} = \frac{2(V_0 - V)}{q\varepsilon A^2 N_d} = k \times (V_0 - V)$$

$k = \frac{2}{q\varepsilon A^2 N_d}$ is the slope of the $1/C_j^2$-V plot.

Pick any two data from the table, and we can calculate $k$.

$k = 0.8 \times 10^{24} F^{-2} \cdot V^{-1}$

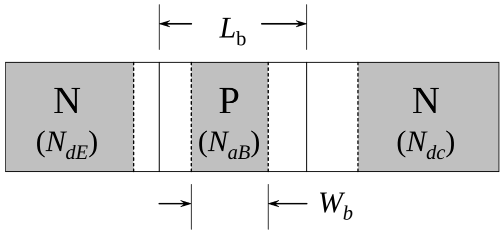
$$N_d = \frac{2}{q\varepsilon A^2 k} = \frac{2}{1.6 \times 10^{-19} \times 11.8 \times 8.85 \times 10^{-14} \times 10^{-10} \times 0.8 \times 10^{24}}$$
$$= 1.5 \times 10^{17} cm^{-3}$$

(d) Estimate the length of the $n$-doped region. (Hint: The small-signal capacitance stopped changing at DC voltage less than -1.0 V. Why?) **{6 points}**

When capacitance stopped changing, the $n$-doped region was fully depleted. If the length of $n$-doped region is $x_{n0}$, $x_{n0} \cong W$ because the concentration of $n$-doped region is much lower than $p$-doped region.

$$C = \frac{\varepsilon A}{W}$$

$$W = \frac{\varepsilon A}{C} = \frac{11.8 \times 8.85 \times 10^{-14} \times 10^{-5}}{0.803 \times 10^{-12}} = 1.3 \times 10^{-5} cm$$



| | Emitter | Base | Collector |
|---|---|---|---|
| Metallurgical Base Width, $L_b$ (μm) | | 10 | |
| Doping (cm$^{-3}$) | $5\times10^{18}$ | $10^{18}$ | $5\times10^{15}$ |
| Minority Carrier Lifetime (s) | $10^{-9}$ | $10^{-8}$ | $3\times10^{-6}$ |
| Minority Carrier Diffusion Length (μm) | 0.6 | 2 | 74 |

**Figure 2**

3) (a) When a 0.7 V forward bias and 10.0V reverse bias are respectively applied across the emitter-base and collector-base junctions to the BJT shown in Fig. 2, <u>calculate</u> **{18 points}**

(i) The emitter injection efficiency,

(ii) The base transport factor,

(iii)The common base current gain, and

(iv)The common emitter current gain

$$D_{pE} = \frac{L_{pE}}{\tau_{pE}} = \frac{(0.6\times10^{-4})^2}{10^{-9}} = 3.6\ cm^2/s$$

$$D_{nB} = \frac{L_{nE}}{\tau_{nE}} = \frac{(2\times10^{-4})^2}{10^{-8}} = 4\ cm^2/s$$

$$V_{0,EB} = \frac{kT}{q} ln\frac{N_aN_d}{n_i^2} = 0.026\, ln\frac{5\times10^{18}\times10^{18}}{10^{20}} = 1\ V$$

$$V_{0,BC} = 0.026\, ln\frac{10^{18}\times5\times10^{15}}{10^{20}} = 0.82\ V$$

$$W_{EB} = \sqrt{\frac{2\times11.8\times8.85\times10^{-14}\times(1-0.7)}{1.6\times10^{-19}}\times\left(\frac{1}{5\times10^{18}}+\frac{1}{10^{18}}\right)} = 2.17\times10^{-6}\ cm$$

$$W_{BC} = \sqrt{\frac{2\times11.8\times8.85\times10^{-14}\times(0.82+10)}{1.6\times10^{-19}}\times\left(\frac{1}{10^{18}}+\frac{1}{5\times10^{15}}\right)} = 1.68\times10^{-4}\ cm$$

$$x_{p,EB} = W_{EB}\times\frac{5\times10^{18}}{5\times10^{18}+10^{18}} = 1.81\times10^{-6}\ cm$$

$$x_{p,BC} = W_{BC}\times\frac{5\times10^{15}}{10^{18}+5\times10^{15}} = 8.36\times10^{-7}\ cm$$

$$W_b = L_b - x_{p,EB} - x_{p,BC} = 9.98\times10^{-4}\ cm \qquad \Rightarrow \text{long base}$$

(i) $\gamma = \left[1+\frac{D_{pE}L_{nB}N_{aB}}{D_{nB}L_{pE}N_{dE}}\right]^{-1} = 0.625$

(ii) $B = sech\left(\frac{W_b}{L_{nB}}\right) = 0.0136$

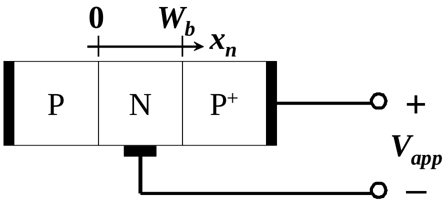
(iii) $\alpha = B\gamma = 0.0085$

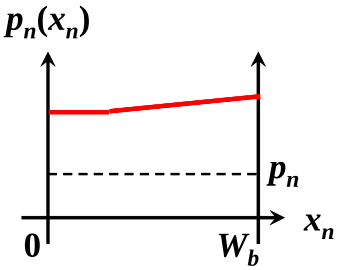
(iv) $\beta = \frac{\alpha}{1-\alpha} = 0.00857$

(b) Fill in the table below on whether the quantities at the leftmost column will increase (↑), decrease (↓), or remain unchanged (0) when the parameters in the top row are changed. Assume the terminal voltages are kept the same throughout. State your assumption(s) and explain your choice if necessary. **{12 points}**

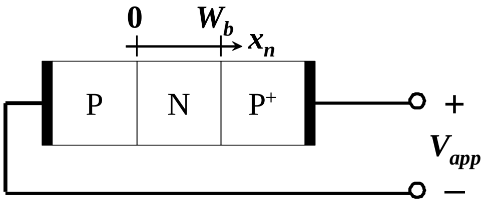
| | Emitter Doping ↓ | Base Doping ↑ | Collector Doping ↓ | Emitter Minority Carrier Lifetime ↑ |
|---|---|---|---|---|
| Common Emitter Gain | ↓<br>($I_{En}$ ↓, $\gamma$↓) | ↓<br>($\gamma$↓,) | ↓<br>(Depletion width at BC junction ↓, $W_b$ ↑) | ↑<br>(Assumption : mobility is not changed<br>$L_{Ep}$ ↑, $I_{Ep}$↓) |
| $I_{En}/I_{Ep}$ | ↓<br>( $I_{En}$ ↓, $I_{Ep}$ ↑) | ↓<br>( $I_{En}$ ↓, $I_{Ep}$ ↑) | 0 | ↑<br>($I_{Ep}$ ↓) |

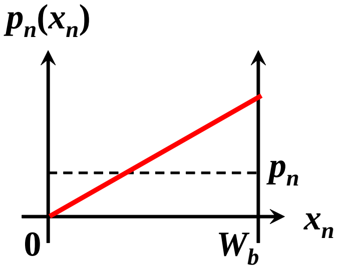
4) Sketch and briefly justify the hole distribution inside the neutral base region of a PNP BJT for the following three connection schemes when $V_{app} >> +kT/q$ **{12 points}**



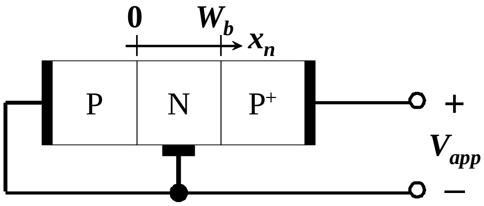


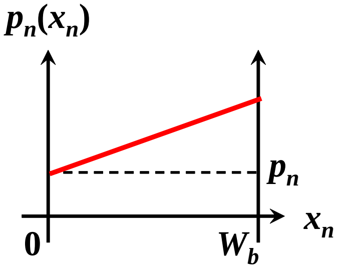
$I_C = 0$
There are a certain amount of excess carriers near p-n junction, but excess carrier gradient is 0





p+-n : forward
n-p : reverse





$$\Delta P_C = p_n\left(e^{q\times 0/kT} - 1\right) = 0$$

No excess carrier at p-n junction