EE 115B

Midterm Exam, Group I

Winter 2007

Your Name:

Solutions

Name of Person to Your Left: ______________________

Name of Person to Your Right: ______________________

Open Book, Open Notes.

Where applicable, place answers inside designated boxes.

Use all approximations specified in each problem.

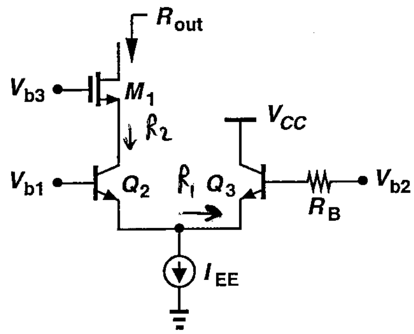
Make sure your name is checkmarked on the class list when you turn in your exam.

1. 12
2. 8
3. 8
4. 12

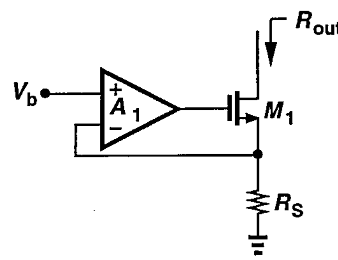
Total: 40

12

1. Assuming $\lambda > 0$ and $V_A < \infty$, calculate the output impedance of each circuit. In part (b), the op amp has a gain of A_1 but is otherwise ideal.



(a)



(b)

$$R_1 = \frac{r_\pi + R_B}{\beta + 1} \quad (2)$$

$$R_2 = \left[1 + g_{m_2}(R_1 \,\|\, r_{\pi 2})\right] r_{o2} + R_1 \,\|\, r_{\pi 2} \quad (2)$$

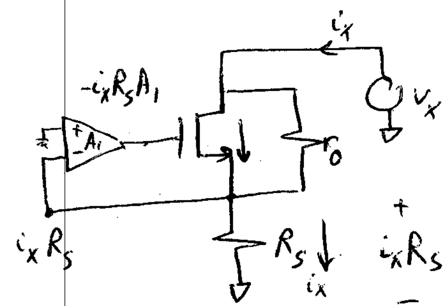
$$R_{out} = \left[1 + g_{m_1} R_2\right] r_{o1} + R_2 \quad (2)$$

Group I:

$$\boxed{R_{out} = \left[1 + g_{m_1} R_2\right] r_{o1} + R_2 \quad \text{with } R_2 = \left[1 + g_{m_2}\left(\left(\frac{r_\pi + R_B}{\beta+1}\right) \,\|\, r_{\pi_2}\right)\right] r_{o2} + \left(\frac{r_\pi + R_B}{\beta+1}\right) \,\|\, r_{\pi_2}}$$

Group II:

$$\boxed{R_{out} = \left[1 + g_{m_1} R_2\right] r_{o_1} + R_2 \quad \text{with } R_2 = \left[1 + g_{m_2}\left(\left(\frac{r_\pi + 2R_B}{\beta+1}\right) \,\|\, r_{\pi_2}\right)\right] r_{o2} + \left(\frac{r_\pi + 2R_B}{\beta+1}\right) \,\|\, r_{\pi_2}}$$



$$i_x = g_m v_{gs} + \frac{v_x - i_x R_s}{r_o} \quad (2)$$

$$= -g_m i_x R_s (1 + A_1) + \frac{v_x - i_x R_s}{r_o} \quad (2)$$

$$\frac{v_x}{i_x} = \left[1 + g_m R_s (1 + A_1) + \frac{R_s}{r_o}\right] r_o \quad (2)$$

Group I:

$$\boxed{R_{out} = R_s + r_o + g_m r_o R_s A_1}$$

Group II:

$$\boxed{R_{out} = 2R_s + r_o + 2 g_m r_o R_s A_1}$$

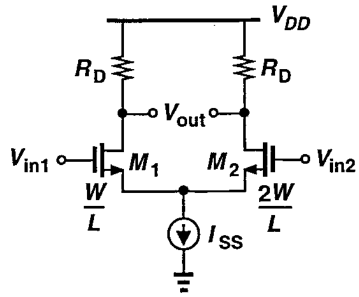
(a): R_{out} =

(b): R_{out} =

2. Due to a manufacturing error, one of the transistors in the differential pair shown below has twice the width of the other. The circuit is otherwise symmetric and $\lambda = 0$. 8

(a) Assuming $V_{in1} = V_{in2}$, calculate the bias current of each transistor and the differential output voltage.

(b) Determine the differential input voltage required to force the differential output voltage to zero. (This is called the input-referred offset voltage of the circuit and denoted by V_{OS}.)



a) $I_{SS} = I_{D_1} + I_{D_2}$ 1/2

$I_{D_1} = \frac{1}{2} I_{D_2}$ 1/2

$\therefore$ for Group I: $\boxed{I_{D_1} = \frac{I_{SS}}{3}}$, $\boxed{I_{D_2} = \frac{2I_{SS}}{3}}$ 1

for Group II: $\boxed{I_{D_1} = \frac{4I_{SS}}{3}}$, $\boxed{I_{D_2} = \frac{2I_{SS}}{3}}$ 1

$V_{out_{diff}} = (I_{D_2} - I_{D_1}) R_D$ 1 $\Longrightarrow$ Group I: $\boxed{V_{out\,diff} = \frac{I_{SS} R_D}{3}}$ 1

Group II: $\boxed{V_{out\,diff} = \frac{2I_{SS} R_D}{3}}$

b) We want $I_{D_1} = I_{D_2} = \frac{I_{SS}}{2}$ 1

$\therefore V_{in_1} - V_{th} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$, $V_{in_2} - V_{th} = \sqrt{\frac{I_{SS}}{2\mu_n C_{ox} \frac{W}{L}}}$ 1

Group I: $\boxed{V_{os} = V_{in_1} - V_{in_2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \left(1 - \frac{1}{\sqrt{2}}\right)}$ 1

Group II: $\boxed{V_{os} = V_{in_2} - V_{in_1} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \left(1 - \frac{1}{\sqrt{2}}\right)}$

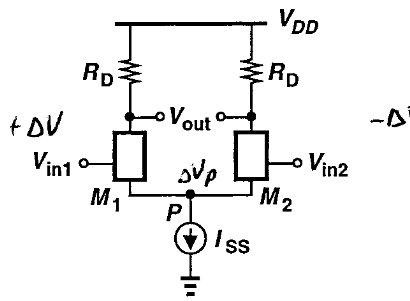
I_{D1} =		I_{D2} =:		V_{OS} =:	

8

3. Suppose a new type of MOS transistor has been invented that exhibits the following I-V characteristic:

$$I_D = \gamma (V_{GS} - V_{TH})^3, \tag{1}$$

where γ is a proportionality factor. Shown below is a symmetric differential pair employing such transistors.
(a) For small differential inputs, can we consider node P to be at ac ground? Explain your reasoning. 2
(b) Calculate the overdrive voltage of each transistor if $V_{in1} = V_{in2}$. 3
(c) At what value of $V_{in1} - V_{in2}$ does one transistor turn off? (We denote this value by $V_{in,max}$.) 3



Assume we have ΔV, ΔV_P as in figure

a) $\Delta I_1 = g_m(\Delta V - \Delta V_P)$

$\Delta I_2 = g_m(-\Delta V - \Delta V_P)$

$\Delta I_1 = -\Delta I_2 \Rightarrow g_m(\Delta V - \Delta V_P) = g_m(\Delta V + \Delta V_P) \Rightarrow 2g_m \Delta V_P = 0 \Rightarrow \Delta V_P = 0$ ac ground.

Even that BJT is exponential not quadratic or cubic, it was a virtual ground. Whatever the device is, provided that the circuit is symmetric and we have equal and opposite in sign inputs, P is going to be an ac ground.

b) $V_{in1} = V_{in2} \Rightarrow I_{D1} = I_{D2} = \frac{I_{SS}}{2} = \gamma (V_{GS} - V_{th})^3 \Rightarrow \boxed{V_{ov} = \sqrt[3]{\frac{I_{SS}}{2\gamma}}}$ for Group I

$\boxed{V_{ov} = \sqrt[3]{\frac{2I_{SS}}{\gamma}}}$ for Group II

c) M_1 passes all the current $\Rightarrow I_{SS}$

M_2 is at edge of conduction $V_{in2} - V_P = V_{th}$

$\therefore V_{ov_1} = \sqrt[3]{\frac{I_{SS}}{\gamma}} = 2^{1/3} V_{ov_{eq}} \Rightarrow V_{in\,max} = V_{in1} - V_{in2} = 2^{1/3} V_{ov\,eq}$

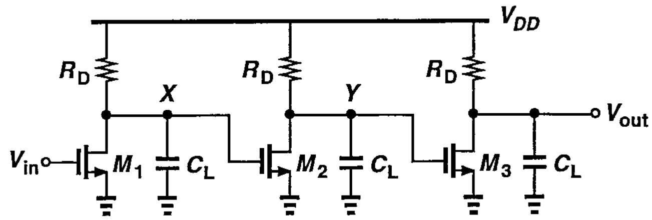
for Group I: $\boxed{V_{in\,max} = \sqrt[3]{\frac{I_{SS}}{\gamma}}}$

for Group II: $\boxed{V_{in\,max} = \sqrt[3]{\frac{4I_{SS}}{\gamma}}}$

Overdrive = [ ] $V_{in,max}$: [ ]

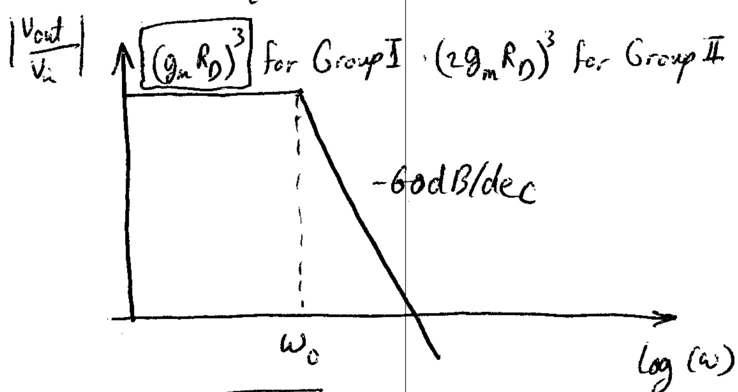
(12)

4. Shown below is a cascade of three identical CS stages. Neglecting channel-length modulation and other capacitances, construct the Bode plot of $|V_{out}/V_{in}|$. Calculate the -3-dB bandwidth of the circuit (i.e., the frquency at which $|V_{out}/V_{in}|$ reaches $1/\sqrt{2}$ times its low-frequency value).



$$\frac{V_{out}(s)}{V_y(s)} = \frac{V_y(s)}{V_x(s)} = \frac{V_x(s)}{V_{in}(s)} = \frac{-g_m R_D}{1+sR_DC_L} = \frac{-g_m R_D}{1+\frac{s}{\omega_o}} \quad \text{with } \omega_o = \frac{1}{R_D C_L}$$

$$\therefore \frac{V_{out}(s)}{V_{in}(s)} = \frac{-(g_m R_D)^3}{\left(1+\frac{s}{\omega_o}\right)^3}$$



$\boxed{\omega_o = \frac{1}{R_D C_L}}$ for Group I

$\boxed{\omega_o = \frac{1}{2R_D C_L}}$ for Group II

$$\left|\frac{V_{out}}{V_{in}}(\omega_{3dB})\right| = \frac{1}{\sqrt{2}}\left|\frac{V_{out}}{V_{in}}(0)\right|$$

$$\frac{(g_m R_D)^3}{\left(\sqrt{1+\left(\frac{\omega_{3dB}}{\omega_o}\right)^2}\right)^3} = \frac{(g_m R_D)^3}{\sqrt{2}}$$

$$1+\left(\frac{\omega_{3dB}}{\omega_o}\right)^2 = 2^{1/3}$$

$$\omega_{3dB} = \omega_o\sqrt{2^{1/3}-1}$$

Group I: $\boxed{\omega_{3dB} = \frac{\sqrt{2^{1/3}-1}}{R_D C_L}}$

Group II: $\boxed{\omega_{3dB} = \frac{\sqrt{2^{1/3}-1}}{2R_D C_L}}$

-3-dB Bandwidth=