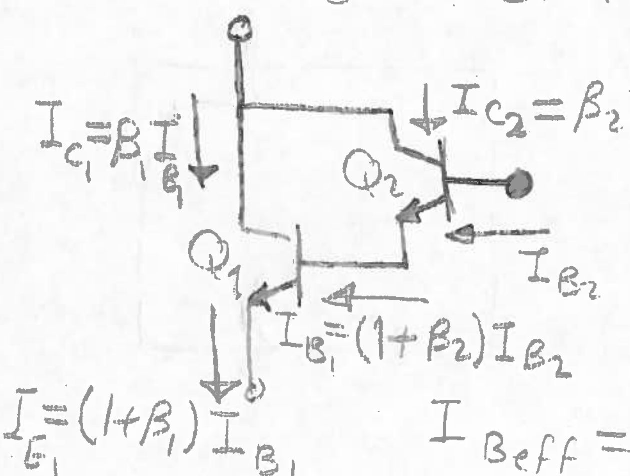
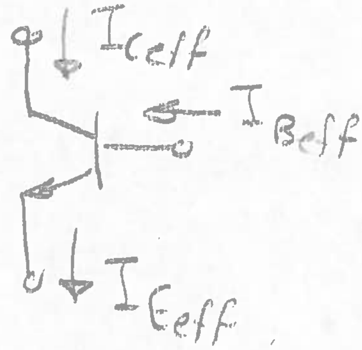
EE 115B

Mid-term Solution:

1- A) A Darlington pair can be represented using a single BJT with an effective $\beta = \beta_{eff}$



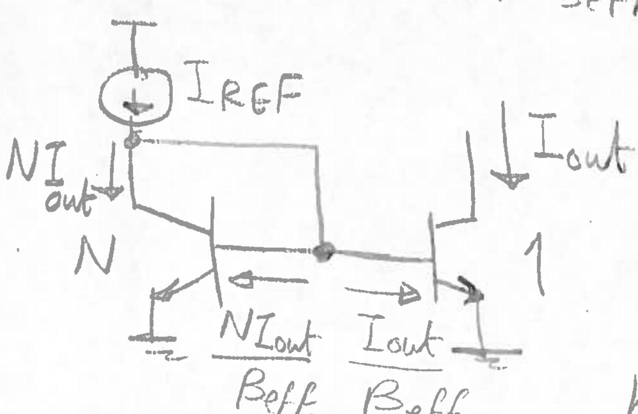
$\equiv$



$$I_{Beff} = I_{B_2}, \quad I_{ceff} = \beta_1 I_{B_1} + \beta_2 I_{B_2} = \underbrace{[\beta_1(1+\beta_2) + \beta_2]}_{\beta_{eff}} I_{B_2}$$

$$I_{Eeff} = (1+\beta_1) I_{B_1} = \underbrace{(1+\beta_1)(1+\beta_2)}_{1+\beta_{eff}} I_{B_2}$$

$$\therefore \left.\begin{aligned} I_{ceff} &= \beta_{eff} I_{Beff} \\ I_{Eeff} &= (1+\beta_{eff}) I_{Beff} \end{aligned}\right\} \Rightarrow \beta_{eff} = \beta_1 + \beta_2 + \beta_1\beta_2$$



$$I_{REF} = N I_{out} + \frac{N I_{out}}{\beta_{eff_1}} + \frac{I_{out}}{\beta_{eff_2}}$$

$$\therefore I_{out} = \frac{I_{REF}}{N + \frac{N}{\beta_{eff_1}} + \frac{1}{\beta_{eff_2}}}$$

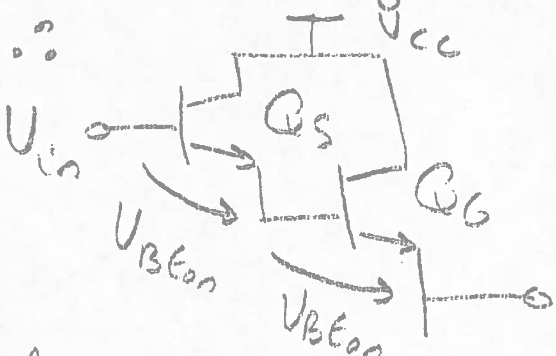
but $\beta_{eff_1} = \beta_1 + \beta_2 + \beta_1\beta_2$

$\beta_{eff_2} = \beta_3 + \beta_4 + \beta_3\beta_4$

$$\therefore I_{out} = \frac{I_{REF}}{N + \frac{N}{\beta_1+\beta_2+\beta_1\beta_2} + \frac{1}{\beta_3+\beta_4+\beta_2\beta_4}}$$

B) For Q_5 & Q_6 to be on $(V_{BE})_5 = (V_{BE})_6 = V_{BEon}$

$\therefore$



$V_o = V_{in} - 2V_{BEon}$

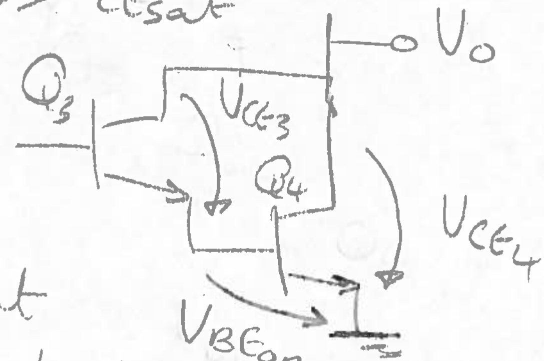
Also for Q_3 & Q_4 to be in active

$(V_{CE})_4 \geq V_{CEsat} \implies V_{C4} \geq V_{CEsat}$

Also, $(V_{CE})_3 \geq V_{CEsat}$

But $V_{E3} = V_{BEon}$

$\therefore$ $V_{C3} \geq V_{BEon} + V_{CEsat}$



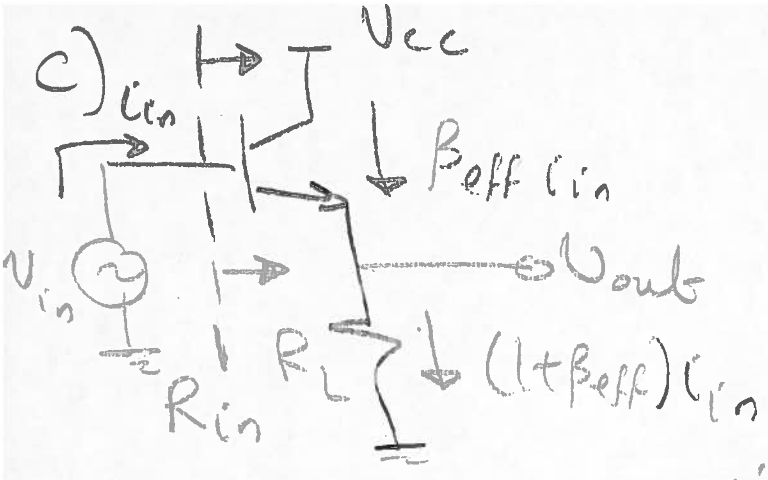
Since both devices need to be active we take the larger of the two constraints

$\therefore$ $V_o \geq V_{BEon} + V_{CEsat}$

$\therefore$ $V_{o\,min} = V_{BEon} + V_{CEsat}$

$V_{in\,min} = 3V_{BEon} + V_{CEsat}$

c)



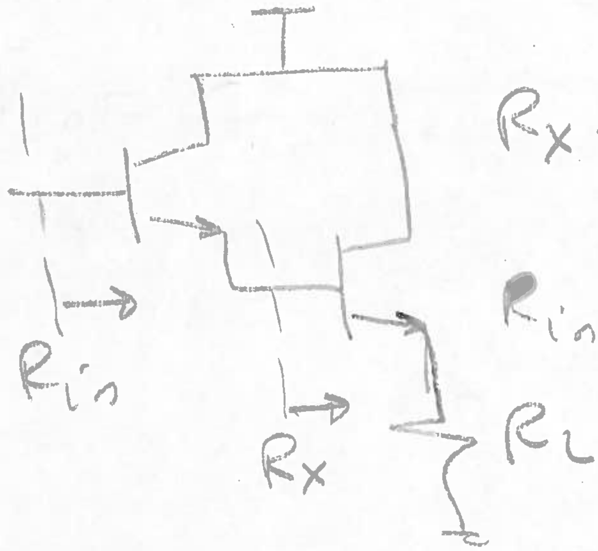
$$i_{in} = \frac{v_{in}}{R_{in}}$$

$$v_{out} = (1+\beta_{eff}) i_{in} R_L$$

$$\therefore v_{out} = \frac{(1+\beta_{eff}) R_L v_{in}}{R_{in}}$$

$$\therefore A_v = \frac{(1+\beta_{eff}) R_L}{R_{in}}$$

But



$$R_X = \underbrace{r_{\pi_6} + (1+\beta_1) R_L}_{\text{reflection rule}}$$

$$R_{in} = \underbrace{r_{\pi_5} + (1+\beta_2) R_X}_{\text{reflection rule}}$$

$$R_{in} = r_{\pi_5} + (1+\beta_2)\left[r_{\pi_6} + (1+\beta_1) R_L\right]$$

Also, $\beta_{eff} = \beta_1 + \beta_2 + \beta_1\beta_2$

$$\therefore A_v = \frac{(1+\beta_1+\beta_2+\beta_1\beta_2) R_L}{r_{\pi_5} + (1+\beta_2)\left[r_{\pi_6} + (1+\beta_1) R_L\right]}$$

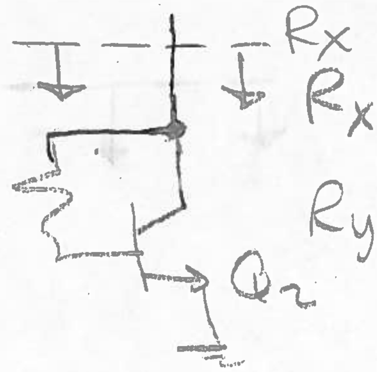
You may also use: $r_{\pi_5} = \frac{V_T}{I_{B_5}}$ & $r_{\pi_6} = \frac{V_T}{I_{B_6}}$

But $I_{B_6} = (1+\beta_2) I_{B_5} \Rightarrow r_{\pi_6} = \frac{r_{\pi_5}}{1+\beta_2}$

$$\therefore \boxed{A_v = \frac{(1+\beta_1)(1+\beta_2) R_L}{2 r_{\pi_5} + (1+\beta_1)(1+\beta_2) R_L}}$$

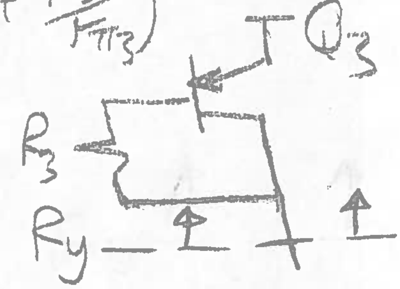
$$r_{\pi_5} = \frac{\beta_2 V_T}{I_{C_5}} = \frac{(1+\beta_2)(1+\beta_1) V_T}{I_{out}}$$

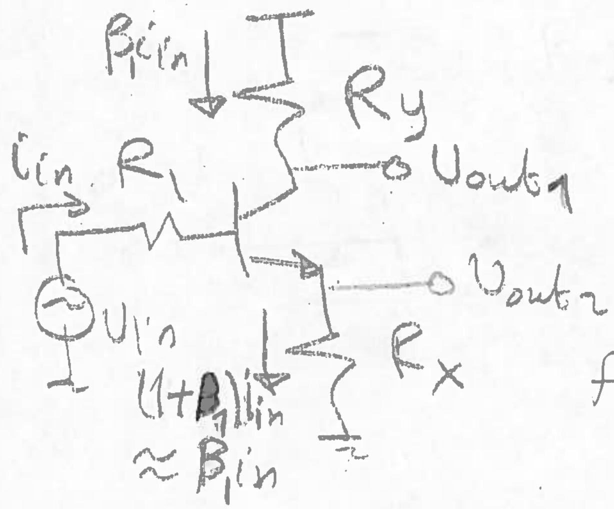
2- A)



$$R_x = (R_2 + r_{\pi_2}) \,//\, \frac{1}{g_{m_2}}\left(1 + \frac{R_2}{r_{\pi_2}}\right)$$

$$R_y = (R_3 + r_{\pi_3}) \,//\, \frac{1}{g_{m_3}}\left(1 + \frac{R_3}{r_{\pi_3}}\right)$$





$v_{out_1} = -\beta_1 i_{in} R_y$

$v_{out_2} = (1+\beta_1) i_{in} R_x$, $\beta_1 >> 1$

$\approx \beta_1 i_{in} R_x$

for $v_{out_1} = -v_{out_2}$

$$R_x = R_y$$

$$R_x = (R_2 + r_{\pi_2}) \,//\, \frac{1}{g_{m_2} r_{\pi_2}}(R_2 + r_{\pi_2}) = \frac{R_2 + r_{\pi_2}}{1 + \beta_2}$$

But because $\beta_2 >> 1 \Rightarrow R_x = \frac{R_2 + r_{\pi_2}}{\beta_2}$

$$= \frac{R_2}{\beta_2} + 1/g_{m_2}$$

Similarily, $R_y = \frac{R_3}{\beta_3} + \frac{1}{g_{m_3}}$

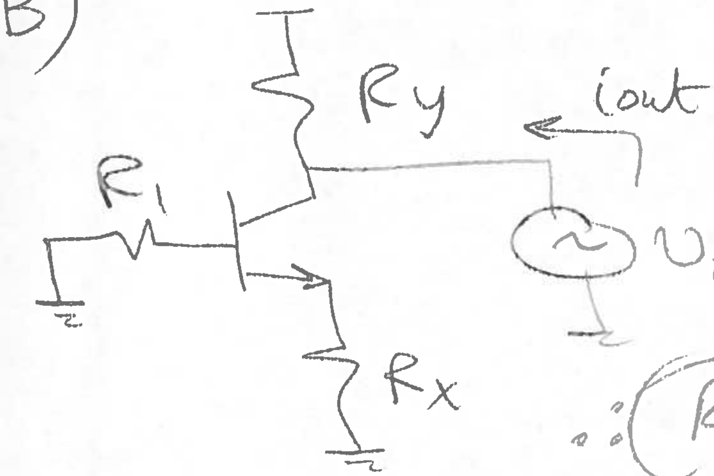
$\frac{1}{g_{m_2}} = \frac{V_T}{I_{c_2}}$ & $\frac{1}{g_{m_3}} = \frac{V_T}{I_{c_3}}$

But $I_{c_2} = I_{c_3}$ because $\beta_1, \beta_2, \beta_3 >> 1$

$\therefore$ For $R_x = R_y$ we need:

$$\frac{R_2}{\beta_2} = \frac{R_3}{\beta_3}$$

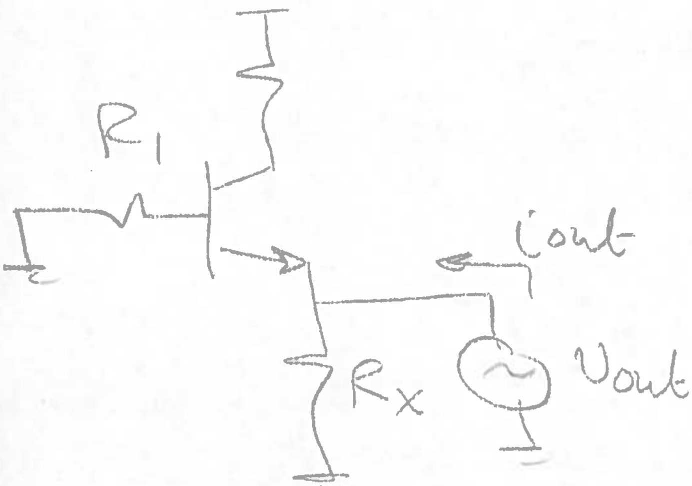
B)



Because $r_{o_1} = \infty$

$$\therefore \frac{v_{out}}{i_{out}} = R_y$$

$$\therefore R_{out_1} = (R_3 + r_{\pi_3}) \,//\, \frac{1}{g_{m_3}}\left(1+\frac{R_3}{r_{\pi_3}}\right)$$

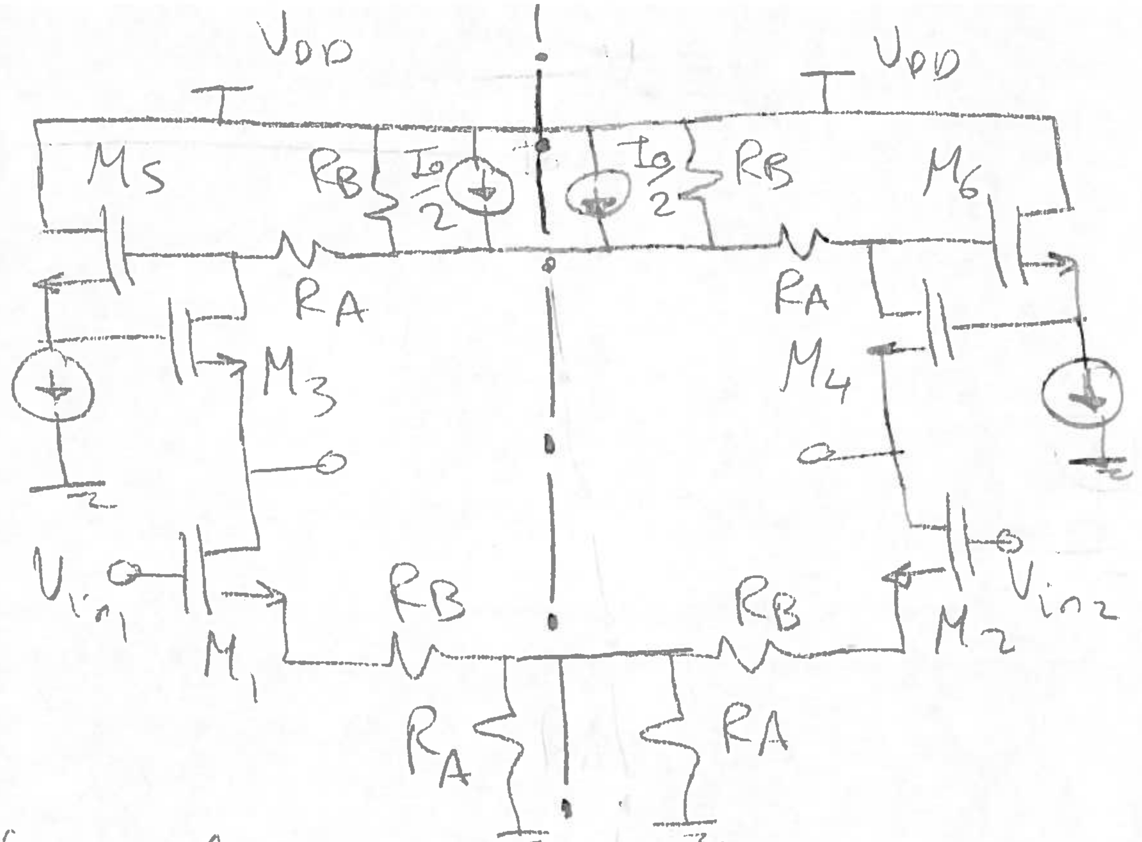


Because $r_{o_1} = \infty$

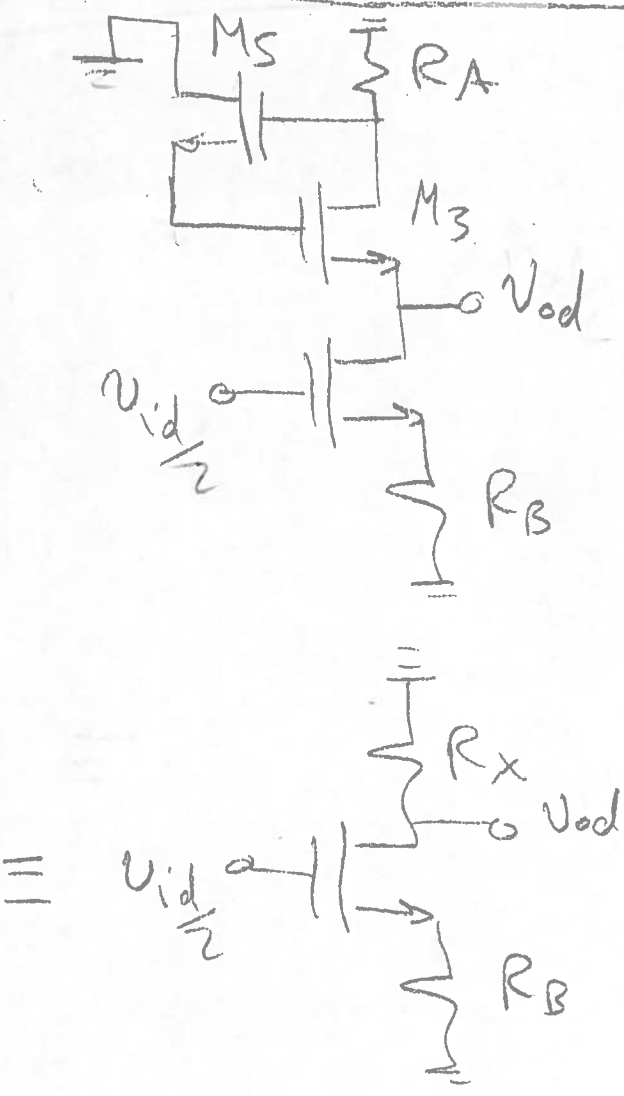
$$R_{out} = \underbrace{\frac{R_1 + r_{\pi_1}}{(1+\beta_1)}}_{\text{reflection rule}} \,//\, R_x$$

$$R_{out_2} = \frac{R_1 + r_{\pi_1}}{(1+\beta_1)} \,//\, \left[(R_2 + r_{\pi_2}) \,//\, \frac{1}{g_{m2}}\left(1+\frac{R_2}{r_{\pi_2}}\right)\right]$$

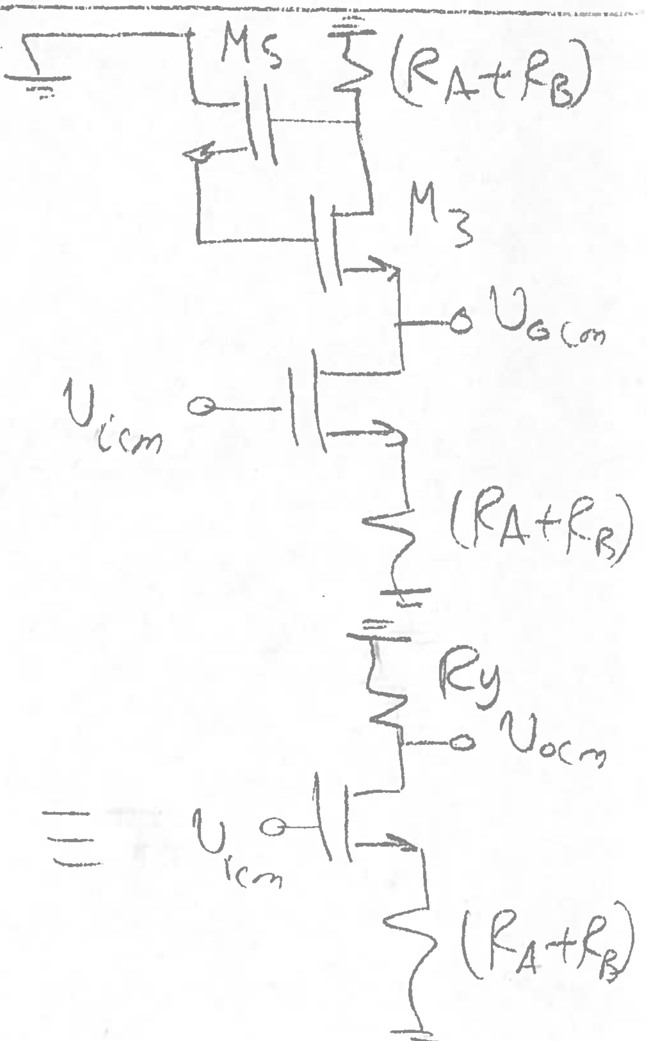
3-



Differential Half-Circuit



Common Mode Half-Circuit



$$A_{vd} = -G_m \cdot R_{out}$$

ignoring $r_{o1} \Rightarrow G_m = \dfrac{g_{m1}}{2(1+g_{m1}R_B)}$

$R_{out} = R_x$

$$\therefore A_{vd} = -\frac{g_{m1}R_x}{2(1+g_{m1}R_B)}$$

Also $A_{cm} = -G_m R_{out}$

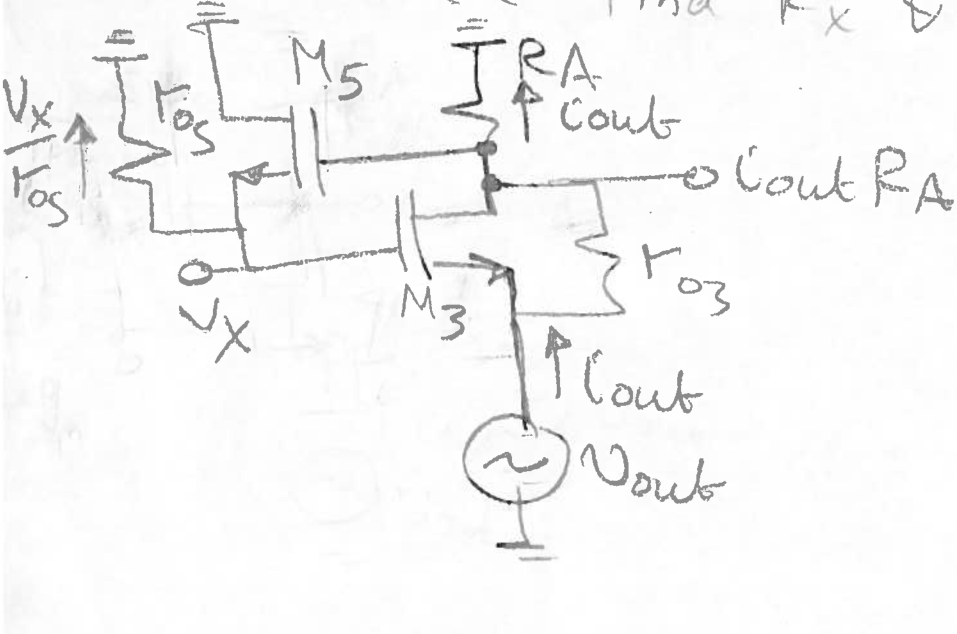
ignoring $r_{o1} \Rightarrow G_m = \dfrac{g_{m1}}{1+g_{m1}(R_A+R_B)}$

$R_{out} = R_y$

$$\therefore A_{cm} = -\frac{g_{m1}R_y}{1+g_{m1}(R_A+R_B)}$$

$$\therefore CMRR = \frac{A_d}{A_{cm}} = \frac{R_x}{2R_y}\left(\frac{1+g_{m1}(R_A+R_B)}{1+g_{m1}R_B}\right)$$

Now, we need to find R_x & R_y:



Apply KCL @ source of M_5

$$\frac{v_x}{r_{o5}} = g_{m5}(i_{out} R_A - v_x)$$

$$\therefore\ v_x\left(\frac{1}{r_{o5}} + g_{m5}\right) = g_{m5}\, i_{out} R_A$$

$$v_x = \frac{g_{m5} r_{o5}}{1+g_{m5} r_{o5}}\, i_{out} R_A$$

Apply KCL @ source of M_3

$$i_{out} = \frac{v_{out} - i_{out} R_A}{r_{o3}} - g_{m3}(v_x - v_{out})$$

$$= \frac{v_{out}}{r_{o3}} - \frac{i_{out} R_A}{r_{o3}} - \frac{g_{m3} g_{m5} r_{o5}}{1+g_{m5} r_{o5}}\, i_{out} R_A + g_{m3} v_{out}$$

$$i_{out}\left[1 + \frac{R_A}{r_{o3}} + \frac{g_{m3} g_{m5} r_{o5}}{1+g_{m5} r_{o5}} R_A\right] = v_{out}\left(\frac{1}{r_{o3}} + g_{m3}\right)$$

$\therefore$

$$R_x = \frac{r_{o3} + R_A + \dfrac{g_{m3} g_{m5} r_{o3} r_{o5}}{1+g_{m5} r_{o5}} R_A}{1+g_{m3} r_{o3}}$$

but $g_{m3} r_{o3} \gg 1$ & $g_{m5} r_{o5} \gg 1$

$$\therefore R_x \simeq \frac{1}{g_{m3}} + \frac{R_A}{g_{m3} r_{o3}} + R_A \simeq R_A + \frac{1}{g_{m3}}$$

Similarily $R_y \simeq (R_A + R_B) + \dfrac{1}{g_{m3}}$

$$\therefore CMRR = \frac{1}{2}\, \frac{1+g_{m3} R_A}{1+g_{m3}(R_A+R_B)} \cdot \frac{1+g_{m1}(R_A+R_B)}{1+g_{m1} R_B}$$