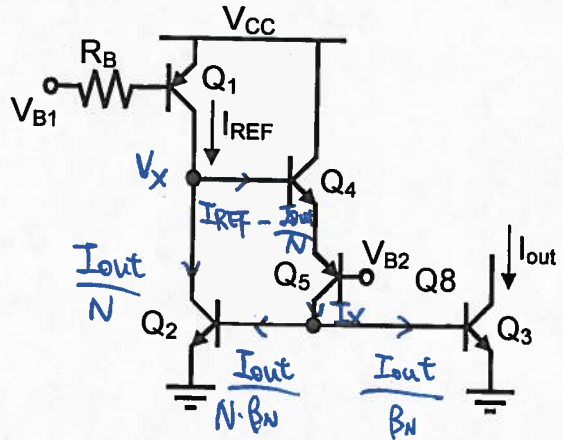


1- A) Find an exact expression for I_{out} in terms of I_{REF} , β_N , and β_P . Q_3 is matched but N times larger than Q_2 . B) Find the minimum and maximum value of V_{B2} for all the transistors to stay in the active region. Assume a piece-wise linear model for the transistors with a turn-on voltage of V_{BEon} and saturation voltage of V_{CEsat} .

(10+15=25 points)



(A)

$$I_x = \frac{(\beta_N + 1)\beta_P}{\beta_P + 1} \left[I_{REF} - \frac{I_{out}}{N} \right] = I_{out} \left(\frac{1}{\beta_N} + \frac{1}{N - \beta_N} \right)$$

$$\Rightarrow I_{out} = \frac{N \cdot I_{REF}}{1 + \frac{(N+1)(\beta_P+1)}{(\beta_N+1)\beta_P}}$$

(B)

Make Q_1 active:

$$|V_{CE, Q1}| > V_{CEsat}$$

$$\Rightarrow V_{CC} - V_x > V_{CEsat} \Rightarrow V_{CC} - (V_{B2} + 2V_{BEon}) > V_{CEsat}$$

$$\Rightarrow V_{B2} < V_{CC} - V_{CEsat} - 2V_{BEon}$$

Make Q_5 active:

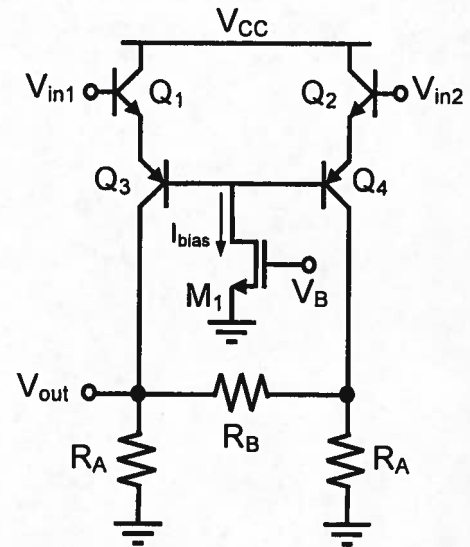
$$|V_{CE, Q5}| > V_{CEsat} \Rightarrow (V_{B2} + V_{BEon}) - V_{BEon} > V_{CEsat}$$

$$\Rightarrow V_{B2} > V_{CEsat}$$

2- Assuming that $V_T=25\text{mV}$, $|V_{BEon}|=0.7\text{V}$, $|V_{CEsat}|=0.2\text{V}$, $V_A=\infty$, $\beta=100$, $V_{TH}=0.3\text{V}$, $\lambda=0.1\text{V}^{-1}$, $V_{CC}=10\text{V}$, $V_B=0.5\text{V}$, $R_A=5\text{K}\Omega$, $R_B=10\text{K}\Omega$, and $I_{bias}=20\mu\text{A}$: A) Find the bounds for the input common mode level for all the transistors to stay in the active region. B) If the input common mode level is set at V_{CC} find maximum differential swing at the input for all the transistors to stay in the active region. C) Find $A_{Vd}=V_{out}/(V_{in1}-V_{in2})$. D) Find CMRR.

Note: Anything can be ignored compared to something 100 times larger. Also $\beta/(\beta+1)\approx 1$

(10+10+5+10=35 points)



(A) $I_{bias} = 20\mu\text{A} \Rightarrow I_{B3} = I_{B4} = 10\mu\text{A} \Rightarrow I_{C1} = I_{C2} = I_{C3} = I_{C4} = 1\text{mA}$

$\Rightarrow V_{out,DC} = 5\text{V}$

Make M_1 sat.:

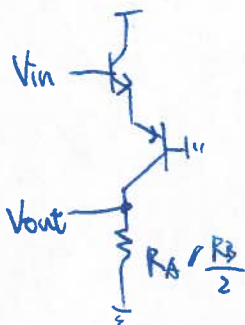
$$V_{CM} \gg V_{BE} + (V_{ES} - V_{TH}) = 2 \times 0.7 + 0.2 = 1.6$$

Make Q_1 act.:

$$V_{EC,Q1} = V_{CC} - (V_{CM} - V_{BEon}) > V_{CESat}$$

$$\Rightarrow V_{CM} < V_{CC} + V_{BEon} - V_{CESat} = 10.5$$

(B) Find differential gain:



$$G_m = - \frac{1}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}} = -20\text{mS}$$

$$R_{out} = R_A \parallel \frac{R_B}{2} = 2.5\text{k} \Rightarrow AV = 50 = \frac{V_{out}}{V_{in}}$$

$$\text{However } A_{Vd} = \frac{V_{out}}{V_{in1} - V_{in2}} = \frac{1}{2} \times AV = 25$$

The output can swing lower without affecting the transistors but if it swings higher, Q_3 may go into saturation.

That means

$$V_{EC, Q_3} \geq V_{CESat} \Rightarrow \left(V_{CM} + \frac{V_{id}}{2} - V_{BE(on)} \right) - \left(V_{out, DC} + 25 \times V_{id} \right) \geq 0.2$$

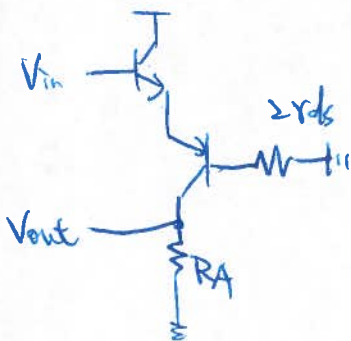
\parallel
 V_{E, Q_3}

$$\Rightarrow 10 + \frac{V_{id}}{2} - 0.7 - 5 - 25 V_{id} \geq 0.2$$

$$\Rightarrow V_{id} \leq 0.167 \text{ V}$$

(c) $A_{vd} = 25$

(D) Find common-mode gain:



$$G_{cm} = - \frac{1}{\frac{1}{g_{m1}} + \frac{r_{e3} + 2r_{ds}}{\beta}}$$

$$= -100 \mu S$$

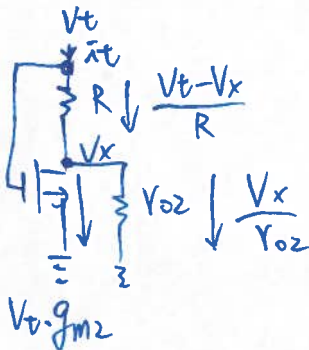
$$R_{out} = 5k$$

$$\Rightarrow A_{cm} = 0.5 \Rightarrow CMRR = \frac{25}{0.5} = 50$$

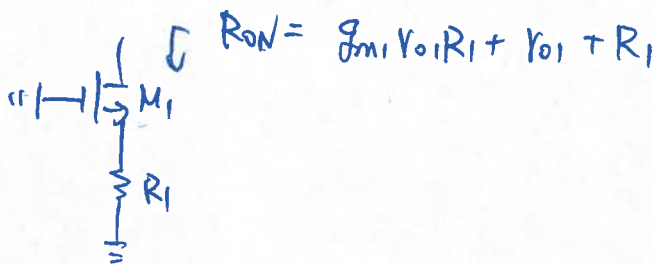
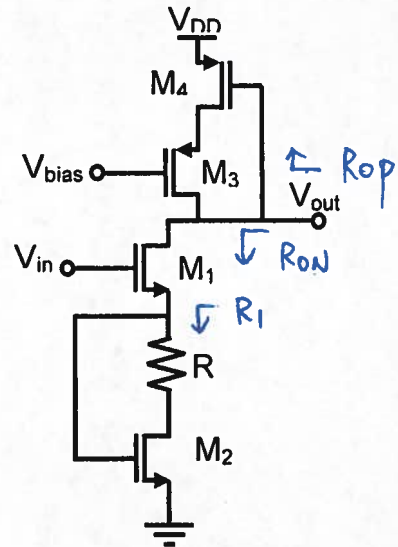
3- In this amplifier, without making any simplifying assumption: A) Find the output impedance. B) Find the voltage gain.

(20+20=40 points)

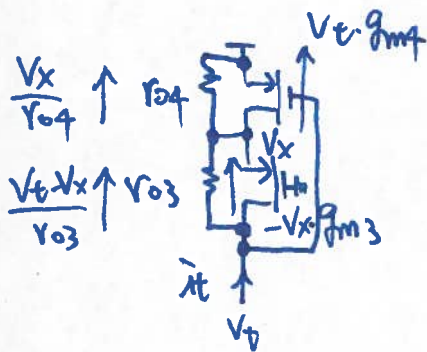
(A)



$$\Rightarrow \frac{V_t}{i_t} = \frac{R + R_{o2}}{1 + g_{m2} R_{o2}} = R_1$$



$$R_{ON} = g_{m1} R_{o1} R_1 + R_{o1} + R_1$$

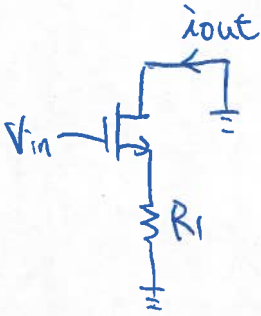


$$\Rightarrow \frac{V_t}{i_t} = \frac{R_{o4} + R_{o3} + g_{m3} R_{o3} R_{o4}}{1 + g_{m4} R_{o4} + g_{m3} g_{m4} R_{o3} R_{o4}} = R_{opp}$$

$$\Rightarrow R_{out} = R_{opp} \parallel R_{ON}$$

(B)

$$G_m = \frac{g_{m1}}{1 + g_{m1}R_1 + \frac{R_1}{r_{o1}}}$$



$$A_v = -G_m \cdot R_{out}$$