EE 115A

Midterm Exam

Fall 2015
Group I

Your Name:

____

Name of Person to Your Left: ______________________________

Name of Person to Your Right: ______________________________

Time Limit: 1 Hour and 50 Minutes

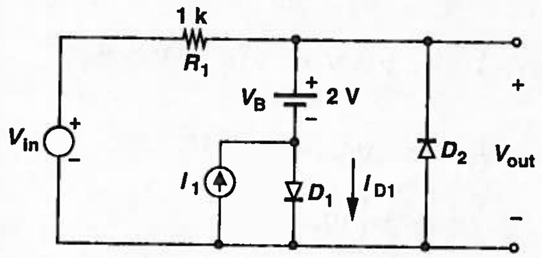
1. 10
2. 15
3. 10
4. 15
Total: 50

Total marks → 10

1. Assuming a constant-voltage diode model with $V_{D,on} = 0.8$ V,

(a) plot V_{out} and I_{D1} as a function of V_{in} if $I_1 = 0$. Show the details of your calculations.

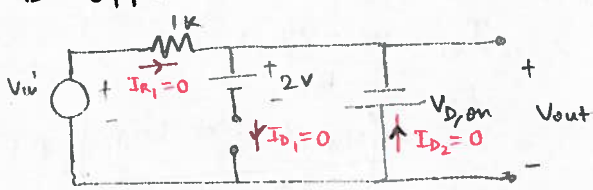
(b) plot V_{out} and I_{D1} as a function of V_{in} if I_1 is constant and equal to 1 mA. Show the details of your calculations.



(a)

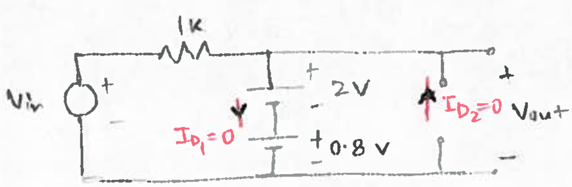
Solution: Step 1: Find Breakpoint

Assume, D2 just turns off while D1 is off.



$\therefore \; V_{in}\Big|_{\text{Breakpoint 1}} = -V_{D,on} = -0.8\text{ V}$ $\frac{1}{2}$

Now, we have to find another Breakpoint when D1 turns on.



$\therefore \; V_{in}\Big|_{\text{Breakpoint 2}} = 2 + V_{D,on} = 2.8\text{ V}$ $\frac{1}{2}$

Hence

$V_{in} \le -0.8$ V D2 ON, D1 OFF

$-0.8 < V_{in} < 2.8$ V D2 OFF, D1 OFF

$V_{in} \geq 2.8$ V D2 OFF, D1 ON

Case 1: $V_{in} \le -0.8$ V

$\boxed{V_{out} = -0.8\text{ V}}$ 1

$\boxed{I_{D1} = 0}$

Case 2: $-0.8\text{ V} < V_{in} < 2.8\text{ V}$

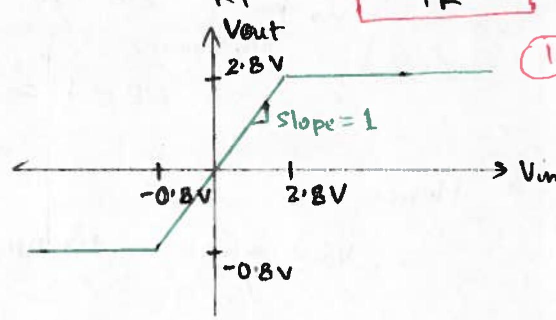
$\boxed{V_{out} = V_{in}}$ 1

$\boxed{I_{D1} = 0}$

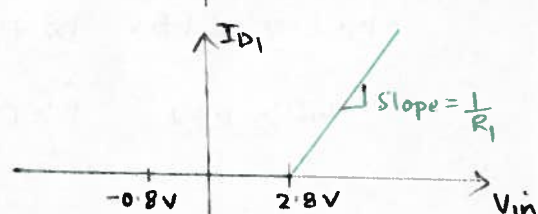
Case 3: $V_{in} \geq 2.8$ V

$\boxed{V_{out} = 2.8\text{ V}}$ 1

$I_{D1} = \dfrac{V_{in} - 2.8}{R_1} = \boxed{\dfrac{V_{in} - 2.8}{1K}}$



1

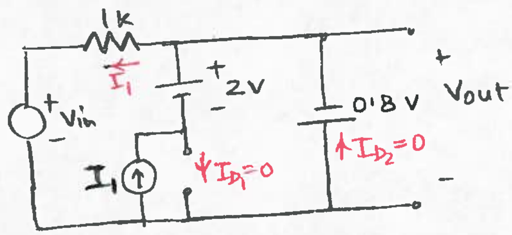


(b)

Solution :

Step 1 : Find Breakpoint

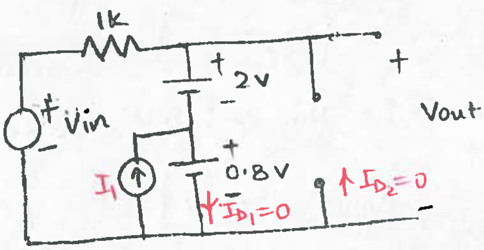
Assume, D2 just turns off while D1 is off.



$\therefore V_{in}\big|_{\text{Breakpoint 1}} = -0.8 - I_1 \times 1k$

$= -0.8 - 1 = -1.8\text{ V}$ $\frac{1}{2}$

Now, we have to find another Breakpoint when D1 turns on.



$\therefore V_{in}\big|_{\text{Breakpoint 2}} = 2.8 - I_1 \times 1k$

$= 2.8 - 1 = 1.8\text{ V}$ $\frac{1}{2}$

Hence,

$V_{in} \le -1.8\text{ V}$ D2 ON, D1 OFF

$-1.8 < V_{in} < 1.8\text{ V}$ D2 OFF, D1 OFF

$V_{in} \ge 1.8\text{ V}$ D2 OFF, D1 ON

Case 1 : $V_{in} \le -1.8\text{ V}$

$\boxed{V_{out} = -0.8\text{ V}}$ 1

$\boxed{I_{D_1} = 0}$

Case 2 : $-1.8\text{ V} < V_{in} < 1.8\text{ V}$

$V_{out} = V_{in} + I_1 \times 1k$

$= \boxed{V_{in} + 1}$ 1

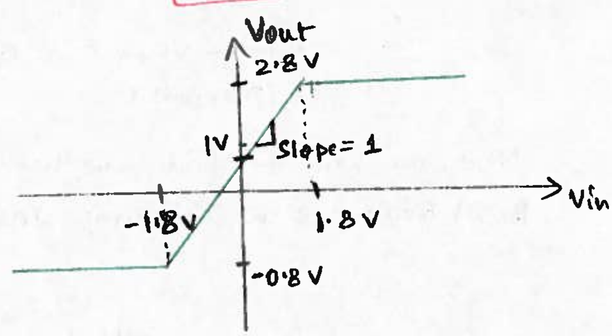
$I_{D_1} = \boxed{0}$

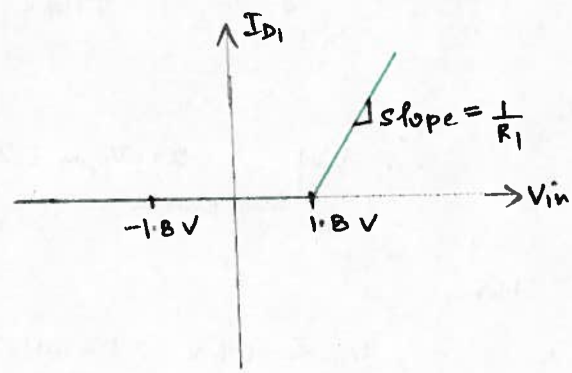
Case 3 : $V_{in} \ge 1.8\text{ V}$

$\boxed{V_{out} = 2.8\text{ V}}$ 1

$I_{D_1} = \dfrac{V_{in} - 2.8}{R_1} + I_1$

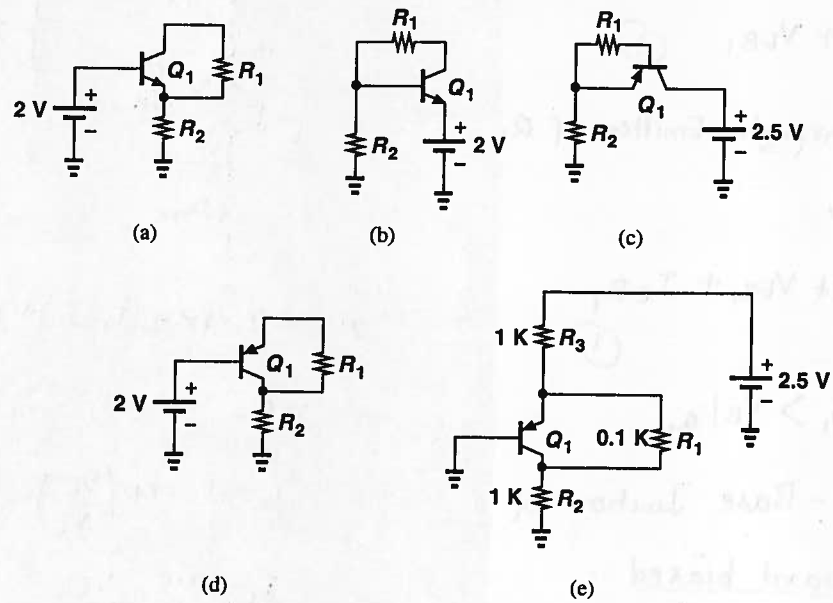
$= \boxed{\dfrac{V_{in} - 2.8}{1k} + 1\text{mA}}$ 1





Total marks → 15

2. Determine the region of operation of Q_1 in each of the circuits shown below. Assume $I_S = 2 \times 10^{-16}$ A, $\beta = 100$, $V_A = \infty$. You need only show whether the transistor is in the forward active region, saturated, at the edge of saturation, or off ($I_C = 0$). Indicate which region and and explain why.



(a)

For Q_1 to be ON, it needs a positive V_{BE_1}

$\therefore \; V_E|_{Q_1} = 2 - V_{BE_1}$ (1)

Now if +ve current I_c goes through collector of Q_1.

$$V_c|_{Q_1} = V_E|_{Q_1} - I_c R_1$$

$$= 2 - V_{BE_1} - I_c R_1$$

As $V_c|_{Q_1} < V_B|_{Q_1}$ (1)

$\Rightarrow$ Collector–Base Junction of Q_1 is forward biased

$\therefore$ $\boxed{Q_1 \text{ lies in saturation region}}$

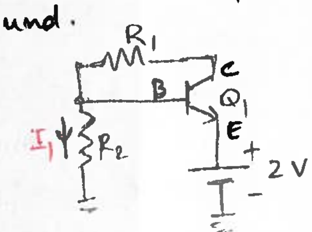
(1)

(b)

For Q_1 to be ON, it needs positive V_{BE_1}

$\Rightarrow \; V_B = 2 + V_{BE_1}$ (1)

Thus, positive current $I_1 = \dfrac{2 + V_{BE_1}}{R_2}$ flows from Base of Q_1 to ground.



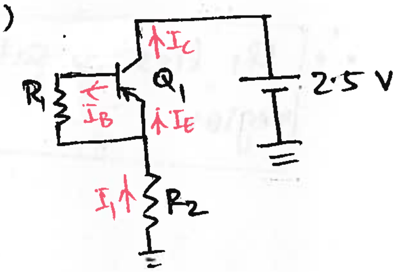
If I_B is neglected then $I_c = -I_1$

Collector current can't be negative while Q_1 is ON.

Hence, $\boxed{Q_1 \text{ is OFF}}$

(2)

(c)



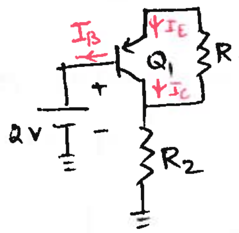
For Q_1 to be ON, it needs to follow current direction Convention labelled in the diagram above.

Now, current flow from ground to 2.5 V. Also, V_{BE} cannot be positive for pnp transistors (1)

$\therefore$ $\boxed{Q \text{ is OFF}}$

(2)

(d) For Q_1 to be ON,

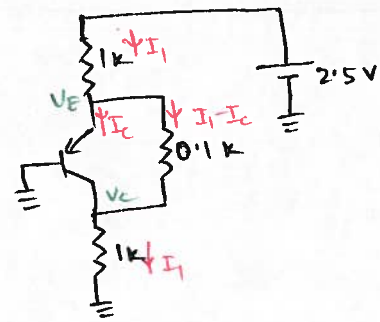


the Above mentioned convention of currents needs to be followed.

This implies I_B needs to charge 2V Battery which is not possible. In this circuit 2V is the highest possible voltage source. Thus, voltage at Emitter will be lower than voltage at Base. 1

$\therefore$ $\boxed{Q_1 \text{ is OFF}}$ 2

(e)



$$2.5 = I_1 \times 1k + (I_1 - I_C)0.1k + I_1 \times 1k$$

$$2.5 = 2.1k\, I_1 - I_C \times 0.1k$$

$$I_C = I_S \exp\left(\frac{V_E}{V_T}\right)$$

$$I_1 = \frac{2.5 - V_E}{1k}$$

$$2.5 = 2.1k\left(\frac{2.5 - V_E}{1k}\right) - I_S \exp\left(\frac{V_E}{V_T}\right) \times 0.1k$$

$$2.5 = 2.1 \times 2.5 - 2.1 V_E - I_S \exp\left(\frac{V_E}{V_T}\right) \times 0.1$$

$$I_S \exp\left(\frac{V_E}{V_T}\right) \times 0.1k = 1.1 \times 2.5 - 2.1 V_E$$

$$\boxed{V_E = V_T \ln\left(\frac{1.1 \times 2.5 - 2.1 V_E}{I_S \times 0.1k}\right)}$$

Putting $V_E = 0.8$ V and performing iterations. 1

$$V_E = 0.8218 \text{ V}$$

Now putting $V_E = 0.8218$ V in 1, we get $\boxed{V_E = 0.8207 \text{ V}}$. 1

This is the final converged value.

$$I_1 = \frac{2.5 - 0.8207}{1k} = 1.679 \text{ mA}$$

$$V_C|_{Q_1} = I_1 \times 1k = 1.679 \text{ V}$$

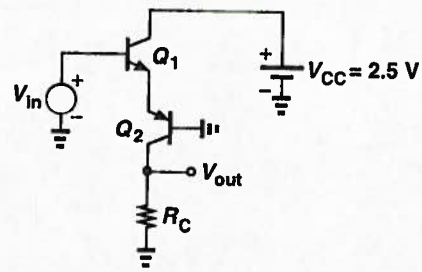
Now $\boxed{V_C|_{Q_1} > V_B|_{Q_1} \Rightarrow Q_1 \text{ lies in saturation region}}$

Total marks → 10

3. Consider the circuit shown below, where $I_{S1} = 3I_{S2} = 2 \times 10^{-16}$ A, $\beta_{npn} = 2\beta_{pnp} = 100$, and $V_A = \infty$. You can assume the collector and emitter currents are equal.

(a) We wish to forward-bias the collector-base junction of Q_2 by no more than 150 mV. Determine the maximum allowable value of V_{in}.

(b) Suppose $V_{in} = 1.5$ V. What is the maximum value of R_C that forward-biases the collector-base junction of Q_2 by no more than 150 mV?



(a)

Solution :

Given :

$$V_{CB2} < 150\,mV$$

$$I_{C2} R_C < 150\,mV$$

$$I_{C2} < \frac{150\,mV}{R_C}$$

$$V_{EB2} = V_{E2} = V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right)$$

$$< V_T \ln\left(\frac{150\,mV}{R_C I_{S2}}\right) \quad 1$$

$$\frac{\beta_2}{1+\beta_2} I_{E2} R_C < 150\,mV$$

$$\frac{1+\beta_1}{\beta_1} \frac{\beta_2}{1+\beta_2} I_{C1} R_C < 150\,mV$$

$$I_{C1} < \frac{150\,mV}{\left(\frac{1+\beta_1}{\beta_1}\right)\left(\frac{\beta_2}{1+\beta_2}\right) R_C} \quad 1$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right)$$

$$< V_T \ln\left(\frac{150\,mV}{\frac{1+\beta_1}{\beta_1} \frac{\beta_2}{1+\beta_2} R_C I_{S1}}\right) \quad 1$$

$$V_{in} = V_{BE1} + V_{EB2} \quad 1$$

$$V_{in} < V_T \ln\left(\frac{150\,mV}{I_{S2} R_C} \frac{150\,mV}{\frac{1+\beta_1}{\beta_1} \frac{\beta_2}{1+\beta_2} R_C I_{S1}}\right)$$

$$= V_T \ln\left[3 \times \left(\frac{V_{out}}{I_{S1} R_C}\right)^2 \times \frac{1}{\frac{1+\beta_1}{\beta_1}} \times \frac{1}{\frac{\beta_2}{1+\beta_2}}\right]$$

$$V_{in} < 1.81\,V + 0.026 \ln\left(\frac{1}{R_C^2}\right) V$$

$$\boxed{V_{in} < 1.81\,V - 0.052 \ln(R_C)\,V}$$

1

(b)

Given $I_{C2} R_C < 150\,mV$

$$V_{in} = V_{BE1} + V_{EB2}$$

$$= V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) + V_T \ln\left(\frac{I_{C2}}{I_{S1}} \frac{1+\beta_2}{\beta_2} \frac{\beta_1}{1+\beta_1}\right) \quad 1$$

$$V_{in} = V_T \ln\left(\frac{I_{C2}^2}{I_{S2} I_{S1}} \frac{1+\beta_2}{\beta_2} \frac{\beta_1}{1+\beta_1}\right)$$

$$I_{C2}^2 = I_{S1} I_{S2} \exp\left(\frac{V_{in}}{V_T}\right) \frac{\beta_2}{1+\beta_2} \frac{1+\beta_1}{\beta_1}$$

$$I_{C2} = \sqrt{I_{S1} I_{S2}} \left(\exp\left(\frac{V_{in}}{V_T}\right)\right)^{\frac{1}{2}} \left(\frac{\beta_2}{1+\beta_2} \frac{1+\beta_1}{\beta_1}\right)^{\frac{1}{2}}$$

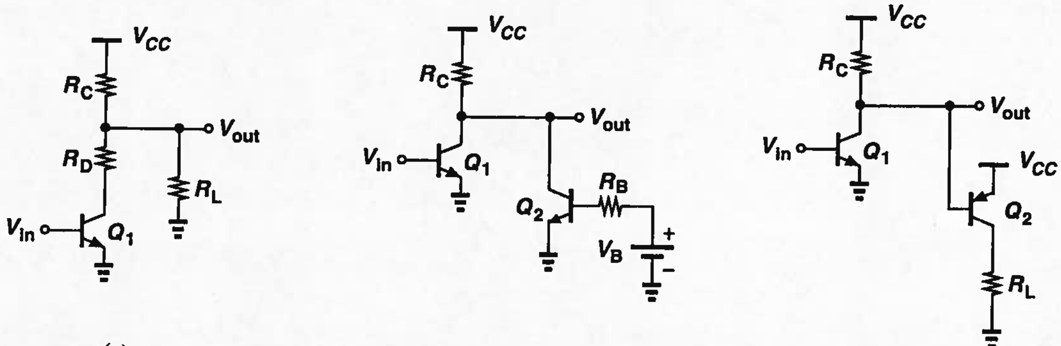
$$R_c < \frac{150\,mV}{\sqrt{I_{S_1} I_{S_2}}\left(\exp\left(\frac{V_{in}}{V_T}\right)\right)^{\frac{1}{2}}\left(\frac{\beta_2}{1+\beta_2}\,\frac{1+\beta_1}{\beta_1}\right)^{\frac{1}{2}}}$$

$$\boxed{R_c < 387.28\,\Omega}$$

1

Total marks → 15

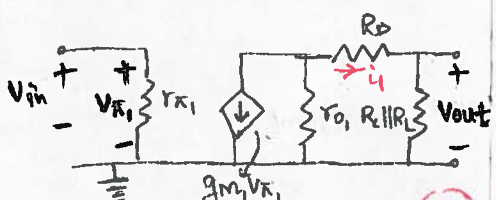
4. Compute the voltage gain of the circuits shown below, assuming that all transistors are biased in the forward active region and $V_A < \infty$ and $\beta \gg 1$.



(a) (b) (c)

Solution (a)

From small-signal model



2

Using current division

$$i_1 = \frac{-g_{m_1} v_{\pi_1} r_{o_1}}{r_{o_1} + R_D + R_c \| R_L}$$ 1

$$V_{out} = i_1 R_c \| R_L$$

$$= \frac{-g_m v_{\pi_1} r_{o_1} R_c \| R_L}{r_{o_1} + R_D + R_c \| R_L}$$

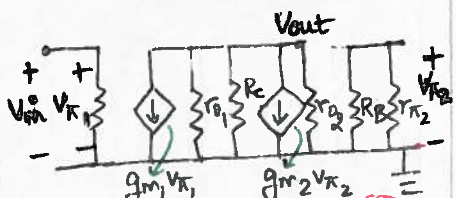
$$= \frac{-g_{m_1} v_{in} r_{o_1} R_c \| R_L}{r_{o_1} + R_D + R_c \| R_L}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{-g_{m_1} r_{o_1} R_c \| R_L}{r_{o_1} + R_D + R_c \| R_L}}$$

2

(b)

From small-signal model



2

Now $v_{\pi_2} = 0$ 1

$\therefore g_{m_2} v_{\pi_2} = 0$

$$v_{out} = -g_{m_1} v_{\pi_1} R_c \| r_{o_1} \| r_{o_2}$$

$$= -g_{m_1} v_{in} R_c \| r_{o_1} \| r_{o_2}$$

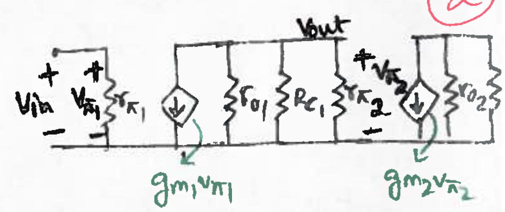
$$\boxed{\frac{v_{out}}{v_{in}} = -g_{m_1} R_c \| r_{o_1} \| r_{o_2}}$$

2

(c)

Small-signal model.

2



$$v_{out} = -g_{m_1} v_{\pi_1} r_{o_1} \| R_{c_1} \| r_{\pi_2}$$

$$= -g_{m_1} v_{in} r_{o_1} \| R_{c_1} \| r_{\pi_2}$$

$$\boxed{\frac{v_{out}}{v_{in}} = -g_{m_1} r_{o_1} \| R_{c_1} \| r_{\pi_2}}$$

3