EE 115A

Midterm Exam

Fall 2009
Group II

Your Name:

Solutions

Name of Person to Your Left: ______________________

Name of Person to Your Right: ______________________

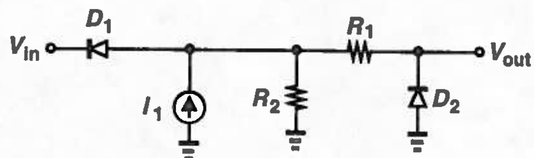
Time Limit: 2 Hours

1.
2.
3.
4.
5.
Total:

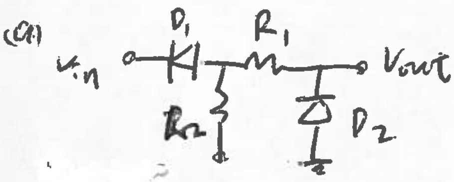
1. Assuming a constant-voltage model,

4 (a) plot V_{out} as a function of V_{in} if $I_1 = 0$. Show the details of your calculations.

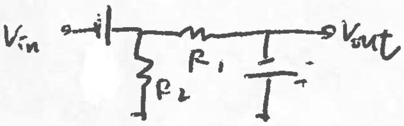
6 (b) plot V_{out} as a function of V_{in} if I_1 is a constant, positive, ideal current source. Show the details of your calculations. Assume $I_1 R_2 > V_{D,on}$.



(a)



D1 on D2 on

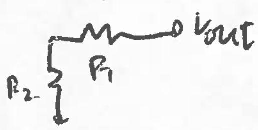


$V_{out} = -V_{D,on}$

$V_{in} + V_{D,on} < V_{out}$

$V_{in} < -2V_{D,on}$

both off

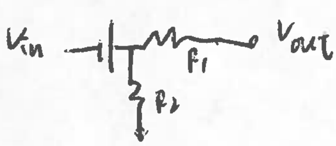


$V_{out} = 0$

$V_{out} - V_{in} < V_{D,on}$

$V_{in} > -V_{D,on}$

D1 on, D2 off

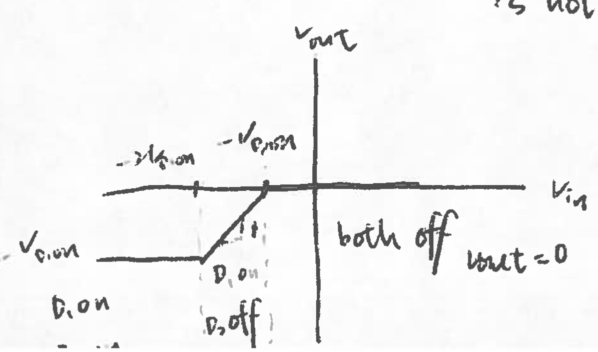


$V_{out} = V_{in} + V_{D,on}$

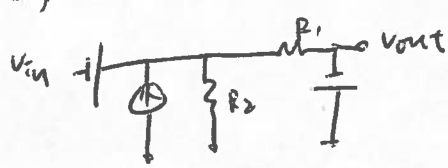
$V_{out} > -V_{D,on}$

$V_{in} > -2V_{D,on}$

D1 off, D2 on: impossible (current flow is not right)



(b) both on

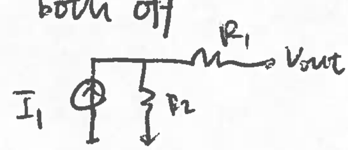


$V_{out} = -V_{D,on}$

$V_{out} - V_{in} > V_{D,on}$

$V_{in} < -2V_{D,on}$

both off



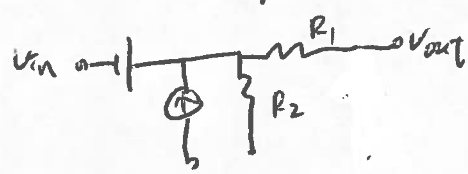
$V_{out} = I_1 R_2$

$V_{out} - V_{in} < V_{D,on}$

$V_{in} > I_1 R_2 - V_{D,on}$ (>0)

$V_{out} > -V_{D,on}$

D1 on, D2 off

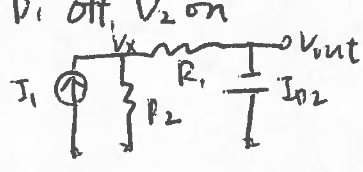


$V_{out} = V_{in} + V_{D,on}$

$V_{out} > -V_{D,on}$

$V_{in} > -2V_{D,on}$

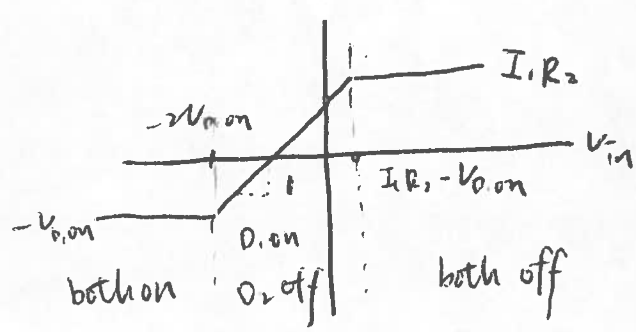
D1 off, D2 on



not possible

∵ D2 on

conflict $\{$ $\Rightarrow V_x < V_{out} = -V_{D,on}$; but $V_x = (I_1 + I_{D2})R_2$

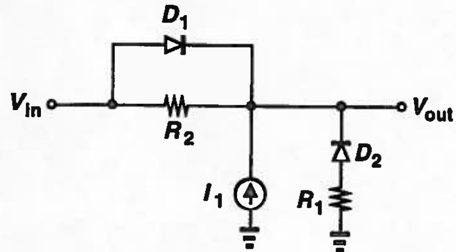


(for group 1 replace R_2 with R_1)

2. Assuming a constant-voltage model,

4 (a) plot V_{out} as a function of V_{in} if $I_1 = 0$. Show the details of your calculations.

6 (b) plot V_{out} as a function of V_{in} if I_1 is a constant, positive, ideal current source. Show the details of your calculations. Assume $I_1 R_1 > V_{D,on}$.



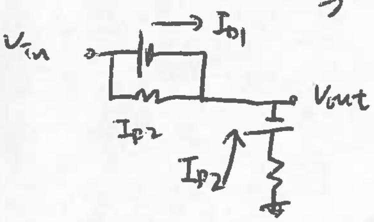
(a) both off

$V_{out} = V_{in}$

$V_{out} > -V_{D,on}$

$\Rightarrow V_{in} > -V_{D,on}$

both on



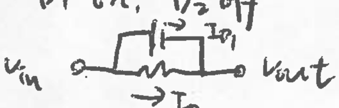
not possible

$\because I_{D1} + I_{D2} + I_{R2} = 0$

$\Rightarrow I_{D1} = I_{D2} = I_{R2} = 0$

$\Rightarrow$ both diodes off

D_1 on, D_2 off



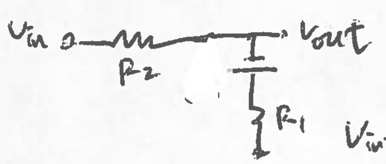
not possible

$\because I_{D1} + I_{R2} = 0$

$\Rightarrow I_{D1} = I_{R2} = 0$

$\Rightarrow D_1$ off

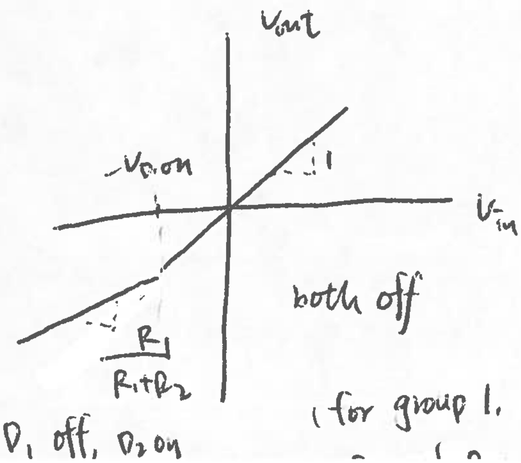
D_1 off, D_2 on



$V_{out} = \dfrac{R_1 V_{in} - R_2 V_{D,on}}{R_1 + R_2}$

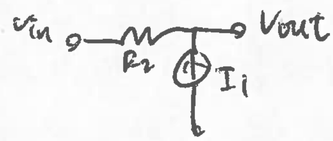
$V_{in} - V_{out} < V_{D,on}$

$V_{in} < -V_{D,on}$



(for group 1, exchange ...)

(b) both off

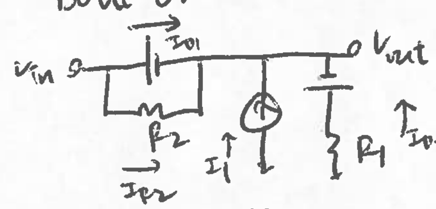


$V_{out} = V_{in} + I_1 R_2$

$V_{out} > -V_{D,on}$

$V_{in} > -V_{D,on} - I_1 R_2$

both on



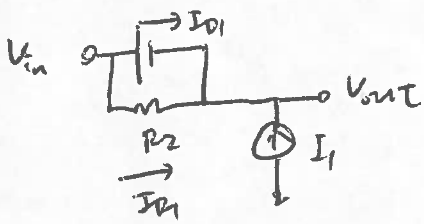
not possible

$\because I_1 + I_{D2} + I_{D1} + I_{R2} = 0$

$\Rightarrow I_1 = I_{D2} = I_{D1} = I_{R2} = 0$

D_1, D_2 off

D_1 on, D_2 off



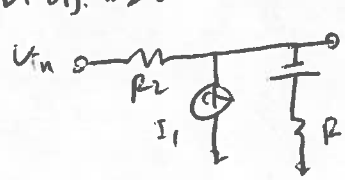
not possible

$\because I_{D1} + I_{R1} + I_1 = 0$

$\Rightarrow I_{D1} = I_{R1} = I_1 = 0$

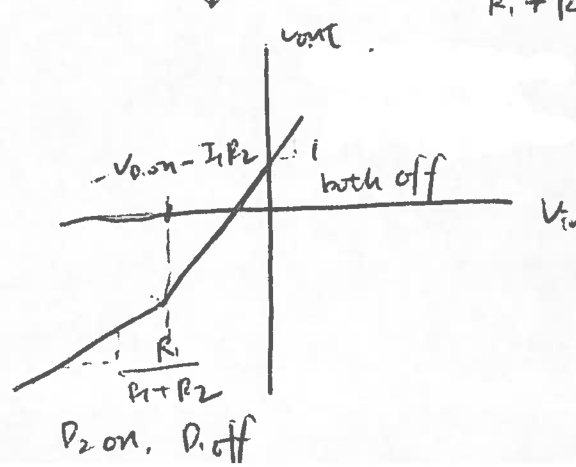
$\Rightarrow D_1$ off

D_1 off, D_2 on

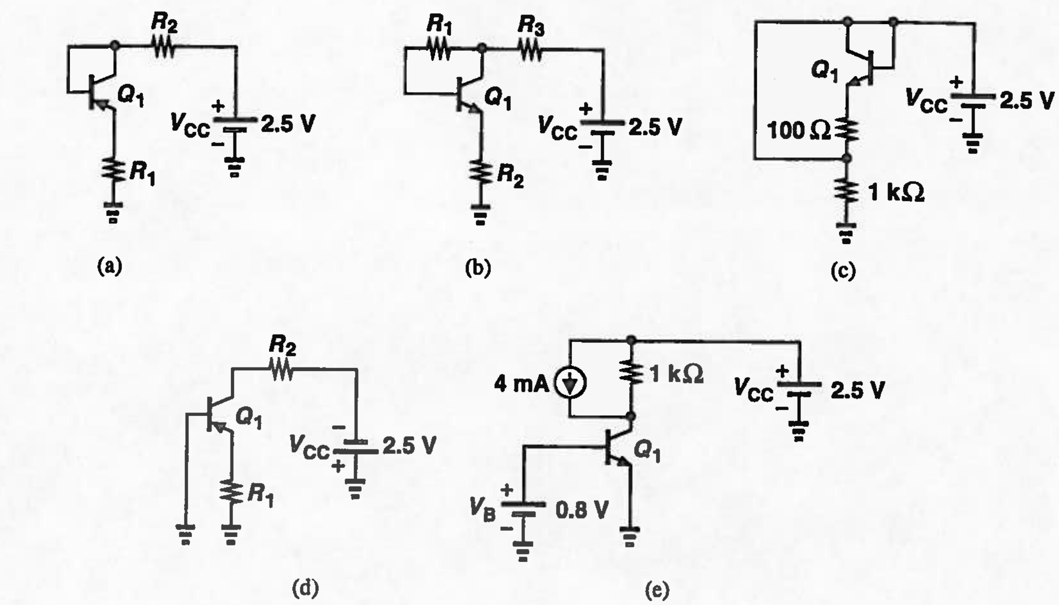


$I_1 = \dfrac{V_{out} - V_{in}}{R_2} + \dfrac{V_{out} + V_{D,on}}{R_1}$

$V_{out} = \dfrac{R_1 V_{in} - R_2 V_{D,on} + I_1 R_1}{R_1 + R_2}$



3. Determine the region of operation of Q_1 in each of the circuits shown below. Assume $I_S = 2 \times 10^{-16}$ A, $\beta = 100$, $V_A = \infty$. You need only show whether the transistor is in active region, saturated, at the edge of saturation, or off. Indicate which region and why.



a) off ∵ current flow cannot from ground to 2.5V

b) active ∵ $V_C > V_B$

c) off ∵ $V_E > V_B$ if turn on

d) off ∵ $V_E < V_B$ if on

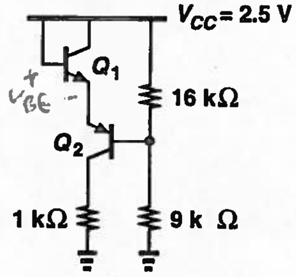
e) $I_C = 2\times10^{-16} \exp\frac{0.8}{0.026} = 4.61$ mA

$V_B = 2.5 - 1k \cdot 0.61$ mA $= 1.89$

active region

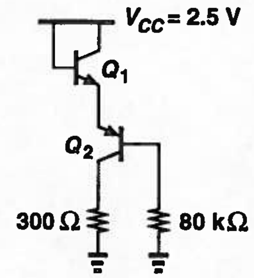
4. (a) Consider the circuits shown below, where $I_S = 5 \times 10^{-17}$ A, $\beta_{npn} = 2\beta_{pnp} = 100$, and $V_A = \infty$. Calculate the operating point of Q_1 and Q_2.

5



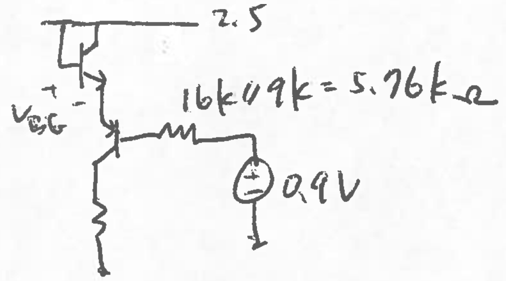
(a)

5



(b)

(a)



① $V_{BE} = 26m \ln \frac{I_c}{5\times10^{-17}}$

② $2.5 - 2V_{BE} - \frac{5.76k \cdot I_c}{50} = 0.9$

apply iteractions;

$V_{BE} = 0.775\ V$

$I_c = 0.434\ mA$

∴ Q_1 : $V_{BE} = 0.775V$

$I_c = 0.434\ mA$

$I_B = 4.34\ uA$

Q_2 : $V_{EB} = 0.775V$

$I_c = 0.434\ mA$

$I_B = 8.68\ uA$

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for Group 1

① $V_{BE} = 26 \ln \frac{I_c}{5\times10^{-17}}$

② $2.5 - 2V_{BE} - \frac{11.52k \cdot I_c}{50} = 0.9$

Q_1 : $V_{BE} = 0.765\ V$

$I_c = 0.304\ mA$, $I_B = 3.04\ uA$

Q_2: $V_{EB} = 0.765V$

$I_c = 0.304\ mA$ $I_B = 6.08\ uA$

① $2.5 - 2V_{BE} = 80k \cdot \frac{I_c}{50}$

② $V_{BE} = 26mV \cdot \ln \frac{I_c}{5\times10^{-17}}$

apply iteractions

$V_{BE} = 782.4mV$

$I_c = 0.585\ mA$

∴ for Q_1: $V_{BE} = 782.4mV$

$I_c = 0.585\ mA$

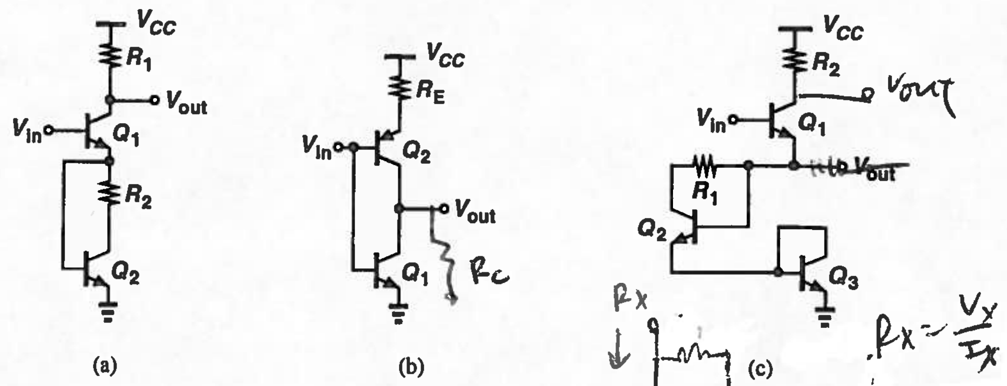
$I_B = 5.85\ uA$

for Q_2: $V_{EB} = 782.4mV$

$I_c = 0.585\ mA$

$I_B = 11.7\ uA$

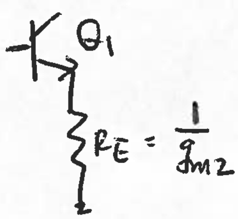
5. Compute the voltage gain and the input impedance of the circuits shown below. Assume $V_A = \infty$ and $\beta \gg 1$. Place your answers within the boxes.



$R_X = \frac{V_X}{I_X}$

$I_X = \frac{\frac{1}{g_{m2}} \times g_{m2}}{\frac{1}{g_{m2}} + \frac{1}{g_{m3}}}$

$R_X = \frac{1}{g_{m2}} + \frac{1}{g_{m3}}$



2

(a): $A_v = \frac{-R_C}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$

1

(a): $R_{in} = r_{\pi 1} + (1+\beta)\frac{1}{g_{m2}}$

2

(b): $A_v = \frac{-R_C}{R_E + \frac{1}{g_{m2}}} - g_{m1} R_C$

(by superposition)

1

(b): $R_{in} = r_{\pi 1} \,//\, [r_{\pi 2} + R_E(1+\beta)]$

2

(c): $A_v = \dfrac{-R_2}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}} + \frac{1}{g_{m3}}}$

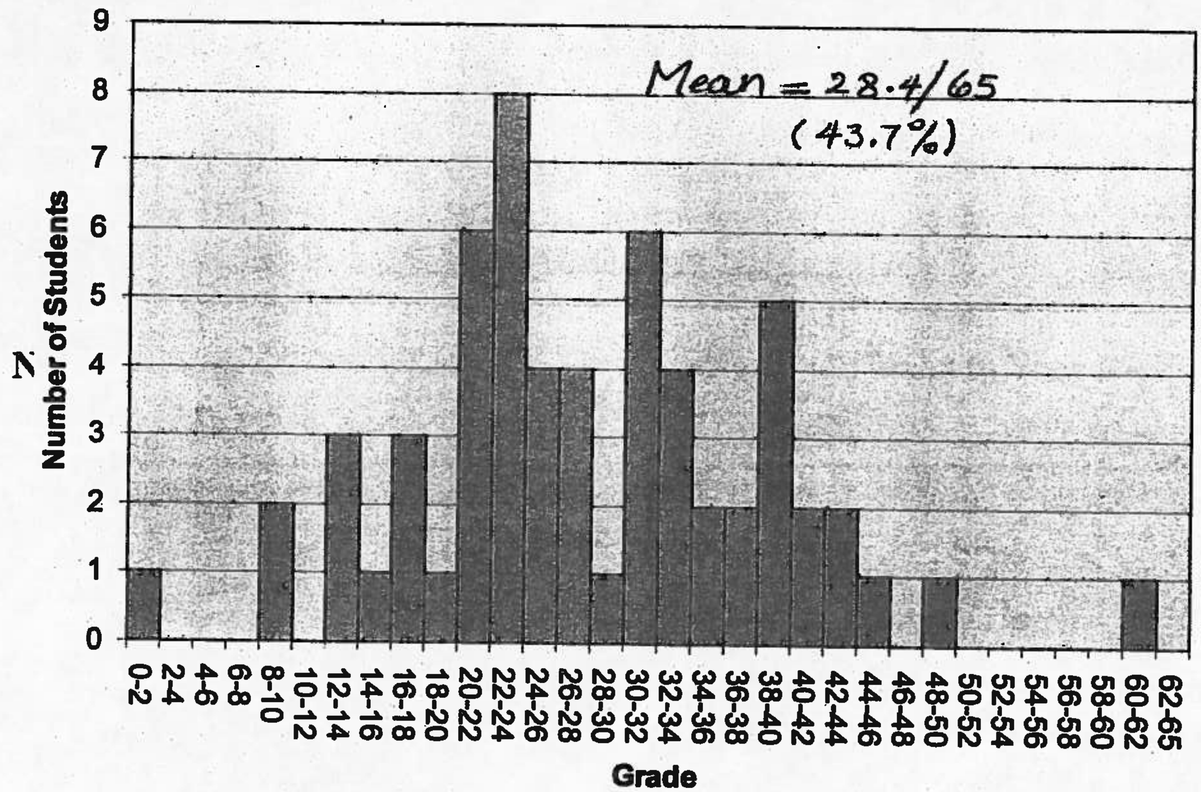
2

(c): $R_{in} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m2}} + \frac{1}{g_{m3}}\right)$

EE 115A

Midterm Exam
Winter 2008
Group I

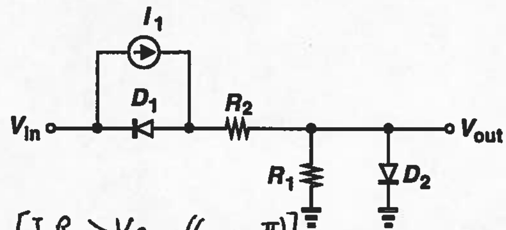
Solutions



1. 10
2. 10
3. 15
4. 15
5. 15

Total: 65

1. Assuming a constant-voltage model, plot V_{out} as a function of V_{in} if I_1 is a constant, positive, ideal current source. Show the details of your calculations.



Assume $I_1 R_1 > V_{D,ON}$ (Group I) $[I_1 R_2 > V_{D,ON}$ (Group II)$]$

D_1 and D_2 can't be OFF at the same time as the current passing through R_1 in Group I or R_2 in Group II will turn D_2 ON

$V_{in} \to -\infty$, D_1 is ON and D_2 is OFF

$$V_{out} = (V_{in} + V_{D,ON}) \frac{R_1}{R_1+R_2} \quad \text{(Group I)}$$

$$= (V_{in} + V_{D,ON}) \frac{R_2}{R_1+R_2} \quad \text{(Group II)}$$

D_2 turns ON when $V_{out} = V_{D,ON}$

for Group I : $V_{D,ON} = (V_{in} + V_{D,ON}) \frac{R_1}{R_1+R_2} \Rightarrow V_{in} = V_{D,ON} \frac{R_2}{R_1}$

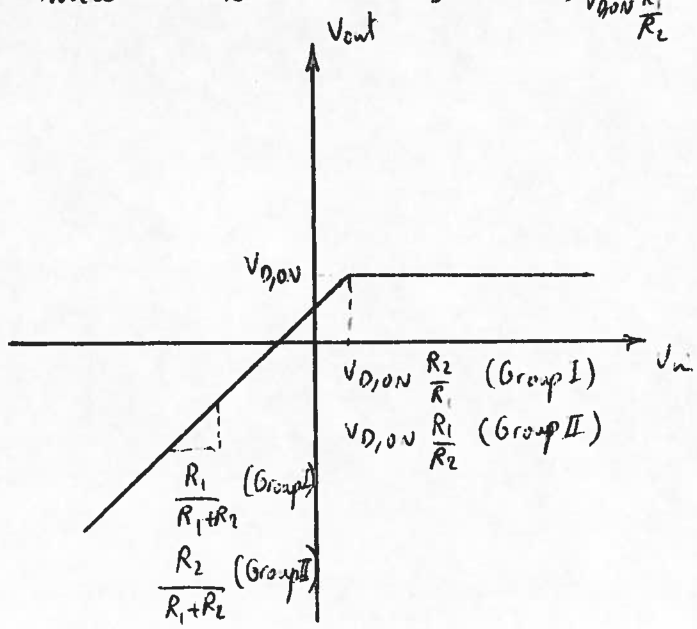
for Group II : $V_{D,ON} = (V_{in} + V_{D,ON}) \frac{R_2}{R_1+R_2} \Rightarrow V_{in} = V_{D,ON} \frac{R_1}{R_2}$

Afterwards $V_{out} = V_{D,ON}$ even when D_1 turns OFF

which happens when $V_{in} + V_{D,ON} = V_{D,ON} + I_1 \begin{cases} R_2 & \text{(Group I)} \\ R_1 & \text{(Group II)} \end{cases}$

i.e. $V_{in} = \begin{cases} I_1 R_2 & \text{(Group I)} \\ I_1 R_1 & \text{(Group II)} \end{cases}$

Notice that these values are larger than $\begin{cases} V_{D,ON} \frac{R_2}{R_1} \\ V_{D,ON} \frac{R_1}{R_2} \end{cases}$



Assume $I_1 R_1 < V_{D,ON}$ (Group I) $[I_1 R_2 < V_{D,ON}$ (Group II)$]$

D_2 can never be ON because the current through its parallel resistor is equal to I_1 if D_1 is OFF or smaller if D_1 is ON and so the voltage drop is not enough to turn D_2 ON

$V_{in} \to -\infty$, D_1 is ON

$$V_{out} = (V_{in} + V_{D,ON}) \frac{R_1}{R_1+R_2} \quad \text{(Group I)}$$

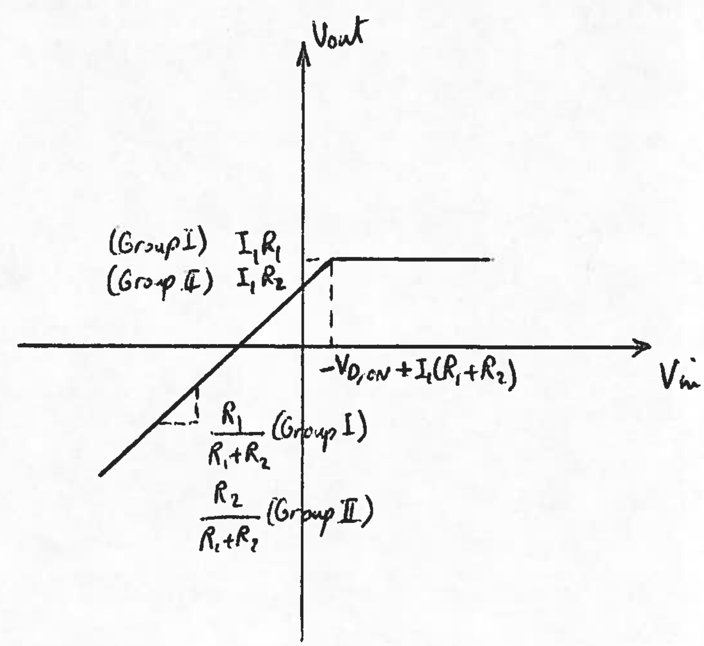
$$= (V_{in} + V_{D,ON}) \frac{R_2}{R_1+R_2} \quad \text{(Group II)}$$

D_1 turns OFF when $V_{in} + V_{D,ON} = I_1 (R_1 + R_2)$

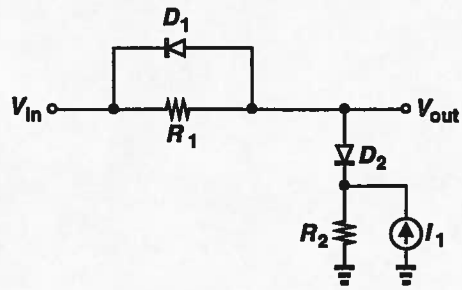
i.e. $V_{in} = -V_{D,ON} + I_1 (R_1 + R_2)$ (+ve or −ve)

After this $V_{out} = I_1 R_1$ (Group I)

$= I_1 R_2$ (Group II)



2. **Assuming a constant-voltage model, plot V_{out} as a function of V_{in} if I_1 is a constant, positive, ideal current source. Show the details of your calculations.**



D_1 Can never be ON as it requires a current flowing into V_{in} (left) in both the diode and its parallel resistor. This is not allowed by D_2

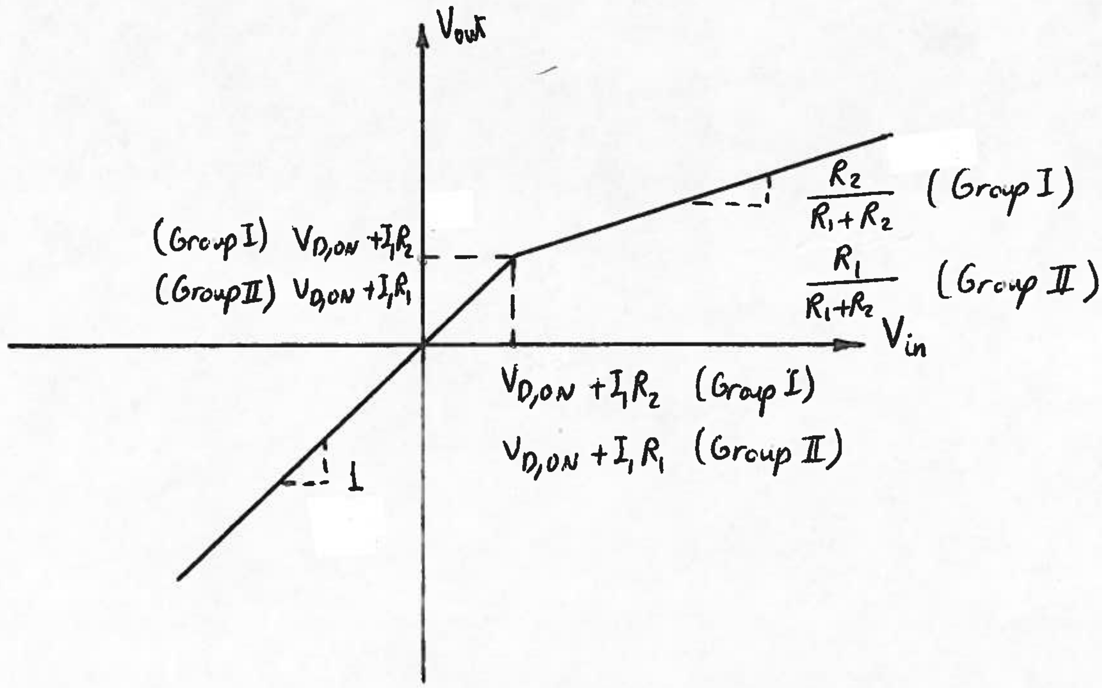
When $V_{in} \to -\infty$, D_2 is OFF and $V_{out} = V_{in}$

D_2 turns ON when $V_{out} - I_1 R_2 = V_{D,ON}$ (Group I) | $V_{out} - I_1 R_1 = V_{D,ON}$ (Group II)

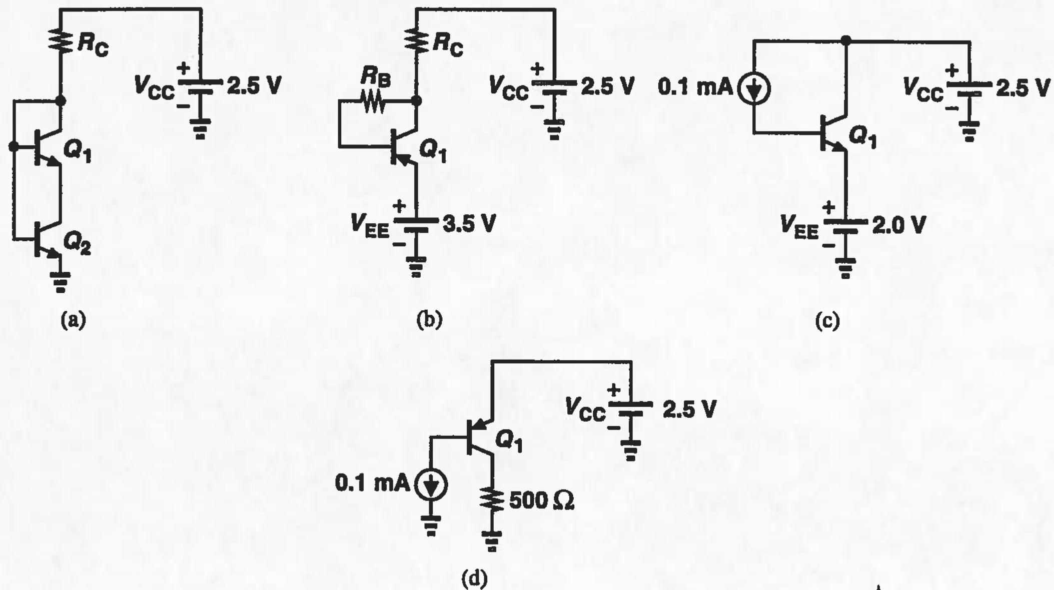
or $V_{in} = V_{D,ON} + I_1 R_2$ as $V_{out} = V_{in}$ (Group I) | $V_{in} = V_{D,ON} + I_1 R_1$ (Group II)

Afterwards $V_{out} = V_{in} \dfrac{R_2}{R_1+R_2} + V_{D,ON} \dfrac{R_1}{R_1+R_2} + I_1 (R_1 // R_2)$ (Group I)

$V_{out} = V_{in} \dfrac{R_1}{R_1+R_2} + V_{D,ON} \dfrac{R_2}{R_1+R_2} + I_1 (R_1 // R_2)$ (Group II)



3. Determine the region of operation of the transistors in each of the circuits shown below. Assume $I_S = 2 \times 10^{-16}$ A, $\beta = 100$, $V_A = \infty$. You need only show whether the transistor is in active region, saturated, at the edge of saturation, or off. Simply indicate which region and explain why.



a) For Q_1 to be ON, it requires a +ve V_{BE}. This will make V_{BC} of Q_2 +ve and thus Q_2 is in the saturation region.

∵ V_{BC} of Q_1 is zero

∴ Q_1 is at the edge of saturation.

Transistors are not OFF as the voltage drop is enough to have current through R_C.

∴ Q_1 is at the edge of saturation

& Q_2 is in the saturation region

b) If the transistor is ON, the base current will flow from base to collector through R_B producing positive V_{BC} and so it will be in the active region.

However we have to check if the transistor is ON or not.

V_{EB} is in the range of 0.75V. This will make the base voltage equal to 2.75V (Group I) or (2.95V Group II). This is enough to produce collector voltage larger than Vcc of 2.5 V

∴ Q_1 is in the active region

c) $V_{CE} = 0.5V$

∵ The base current is forced the transistor must be ON for it to be active it requires $V_{BE} < 0.5$ which is too small to support a current of βI_B

$I_C = \beta I_B = $ 10mA (Group I), 20mA (Group II)

requiring $V_{BE} = $ 0.82V (Group I), 0.84V (Group II)

∴ Q_1 is in the saturation region

d) The transistor is ON as the base current is forced. However, it is not active as $\beta I_B = $ 10mA, 20mA

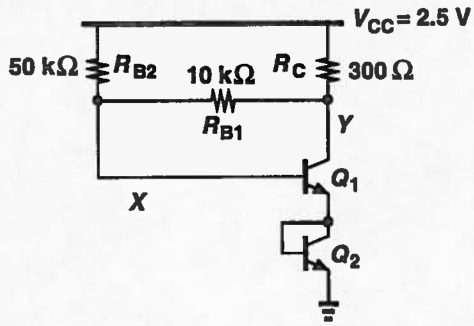
producing $V_C = $ 5 V (Group I), 10 V (Group II)

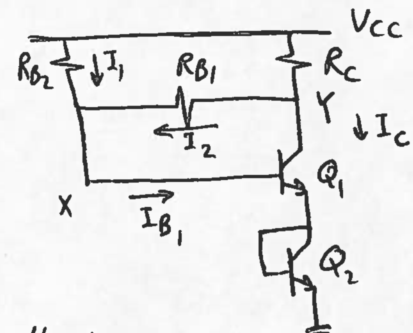
which is not possible as $V_{CC} = 2.5$

So it must be in the saturation region with $I_C < \beta I_B$

∴ Q_1 is in the saturation region

4. Consider the circuit shown below, where $I_S = 6 \times 10^{-16}$ A, $\beta = 100$, and $V_A = \infty$. Calculate the operating point of Q_1 and Q_2.





$\because \beta = 100$

$\therefore I_B << I_C$

$\therefore I_{C_1} \approx I_{C_2} = I_C$ & $I_{E_1} = I_{E_2} = I_E$ & $I_C = I_E$

$\therefore V_{BE_1} = V_{BE_2} = V_{BE}$, $V_X = 2V_{BE}$, $V_Y = V_{BE} + V_{CE}$

I_{B_1} is provided through R_{B_2} and R_{B_1}

$\therefore$ The current through R_{B_1} is smaller than I_{B_1} and much smaller than I_c

$\therefore$ The current through R_c is approximately I_c

$$V_Y \approx V_{CC} - I_C R_C$$

$$I_{B_1} = I_1 + I_2 = \frac{V_{CC} - 2V_{BE}}{R_{B_2}} + \frac{V_{CC} - I_C R_C - 2V_{BE}}{R_{B_1}}$$

but $I_{B_1} = \frac{I_C}{\beta}$

$$\therefore I_C \left(\frac{1}{\beta} + \frac{R_C}{R_{B_1}}\right) = \frac{V_{CC} - 2V_{BE}}{R_{B_2}} + \frac{V_{CC} - 2V_{BE}}{R_{B_1}}$$

$$I_C = \left[\frac{V_{CC} - 2V_{BE}}{R_{B_2}} + \frac{V_{CC} - 2V_{BE}}{R_{B_1}}\right] \Big/ \left(\frac{1}{\beta} + \frac{R_C}{R_{B_1}}\right) = \begin{cases} 0.0075 - 0.006\, V_{BE} & \text{(Group I)} \\ 0.007 - 0.0056\, V_{BE} & \text{(Group II)} \end{cases} \quad (1)$$

Also $V_{BE} = V_T \ln \frac{I_C}{I_S}$ (2)

Assume V_{BE} in (1) → Get I_C → Use (2) to get a new value for V_{BE} → iterate till convergence

starting from $V_{BE} = 0.75$, we converge at $V_{BE} = 759$ mV, $I_C = 2.9$ mA (Group I)

$V_{BE} = 758$ mV $I_C = 2.8$ mA (Group II)

$\therefore$ for Q_1: $V_{CE} = V_{CC} - I_C R_C - V_{BE} = $ 871 mV (Group I); 902 mV (Group II)

$\because V_{CE} > V_{BE}$ → Forward Active

V_{BE} = 759 mV; 758 mV

I_C = 2.9 mA; 2.8 mA

I_B = 29 μA (Group I); 28 μA (Group II)

for Q_2: $V_{BE} = V_{CE}$ = 759 mV (Group I); 758 mV (Group II)

I_C = 2.9 mA (Group I); 2.8 mA (Group II)

I_B = 29 μA (Group I); 28 μA (Group II)

at edge of saturation

5. Suppose the bipolar transistor shown in the circuit below exhibits the following hypothetical characteristic:

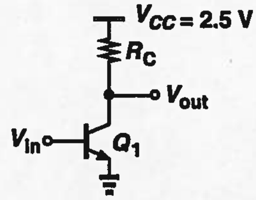
$$I_C = I_S \exp \frac{V_{BE}}{2V_T}, \quad (1)$$

and no Early effect. Assume $I_S = 5 \times 10^{-16}$.

(a) Assuming the transistor operates in the active region, calculate the required base-emitter voltage for a bias collector current of 1 mA.

(b) Choose the value of R_C to place the transistor at the edge of saturation.

(c) With the above choices, compute the small-signal voltage gain of the circuit.



a) $\because I_C = I_S \exp \frac{V_{BE}}{2V_T}$

$\therefore V_{BE} = 2V_T \ln \frac{I_C}{I_S} = \boxed{1.47\,V}$ for $I_C = 1\,mA$

b) At the edge of saturation $V_{BE} = V_{CE}$

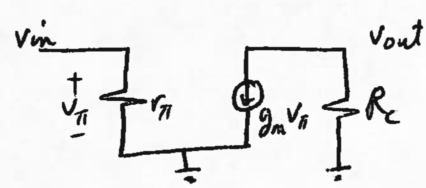
$\therefore V_{CE} = 1.47V$

$\therefore R_C = \frac{V_{CC} - V_{CE}}{I_C} =$

- $\frac{2.5 - 1.47}{1mA} = \boxed{1.03\,k\Omega}$ (Group I)
- $\frac{3 - 1.47}{1mA} = \boxed{1.53\,k\Omega}$ (Group II)

c) $g_m \triangleq \frac{\partial I_C}{\partial V_{BE}}$

$\therefore g_m = \frac{1}{2V_T} I_S \exp \frac{V_{BE}}{2V_T} = \frac{I_C}{2V_T} = 19.2\,mS$



$v_{out} = -g_m v_\pi R_C = -g_m v_{in} R_C$

$A_v = \frac{v_{out}}{v_{in}} = -g_m R_C =$

- $\boxed{-19.776} = 25.9\,dB$ (Group I)
- $\boxed{-29.376} = 29.4\,dB$ (Group II)