# EE 115A

# Final Exam
# Fall 2006

**Your Name:**

Solutions

**Name of Person to Your Left:** ____________

**Name of Person to Your Right:** ____________

**Time Limit: 3 Hours**

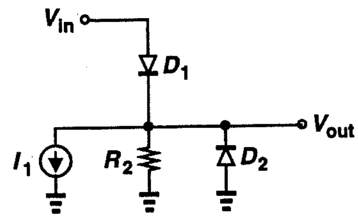
**Where applicable, place answers inside designated boxes.**

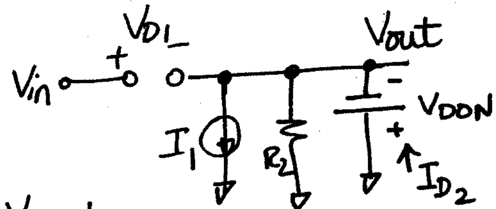
**Use all approximations specified in each problem.**

| | |
|---|---|
| **1.** 5 | **5.** 12 |
| **2.** 5 | **6.** 10 |
| **3.** 10 | |
| **4.** 8 | |

**Total:** 50

1. Plot $V_{out}$ and $I_{D2}$ as a function of $V_{in}$ as $V_{in}$ goes from $-\infty$ to $+\infty$. Determine the coordinates of each break point in the plots. Assume a constant-voltage diode model and $I_1 R_2 > V_{D,on}$.



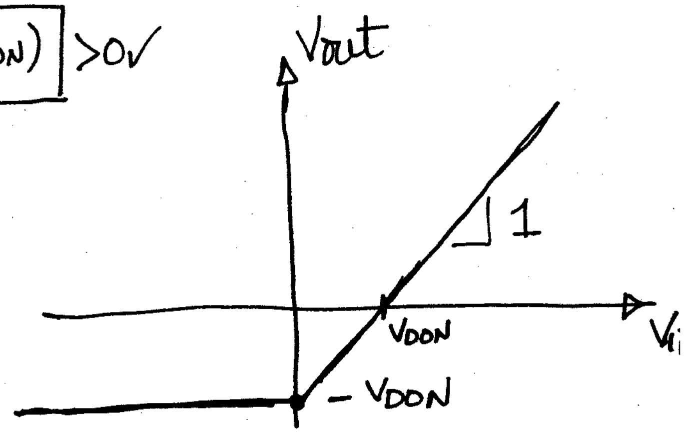

$V_{in} \rightarrow -\infty$

$D_1$ off $\boxed{V_{out} = -V_{DON}}$

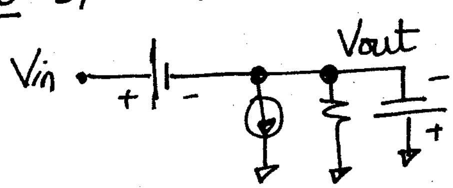
$D_2$ ON $\quad V_{D_1} < V_{DON} \rightarrow V_{in} - V_{out} < V_{DON}$

$\rightarrow \boxed{V_{in} < 0}$ 1

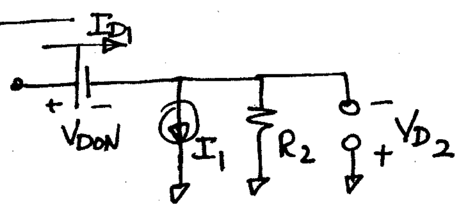
$I_1 = I_{D_2} + \frac{V_{DON}}{R_2} \rightarrow \boxed{I_{D_2} = \frac{1}{R_2}(I_1 R_2 - V_{DON})} > 0 V$ ✓




at $\underline{V_{in}=0}$ $D_1$ turns on:

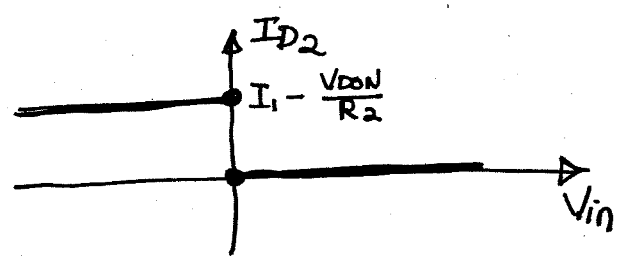



$V_{in} = 0$




$D_1$ : ON

$D_2$ : OFF




$\boxed{V_{out} = V_{in} - V_{DON}}$ $\boxed{I_{D_2} = 0}$

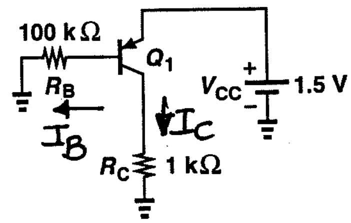
$V_{D_2} = -V_{out} < V_{DON} \rightarrow -V_{in} + V_{DON} < V_{DON}$

$\rightarrow \boxed{V_{in} > 0}$

$I_{D_1} = I_1 + \frac{V_{out}}{R_2} > 0$

$I_{D_1} = I_1 + \frac{V_{in}}{R_2} - \frac{V_{DON}}{R_2} > 0 \rightarrow V_{in} > \underbrace{-(I_1 R_2 - V_{DON})}_{\text{negative}}$

2. Determine the value of $\beta$ that places $Q_1$ at the edge of saturation. Assume $V_A = \infty$ and $I_S = 8 \times 10^{-16}$ A.




edge of saturation $\rightarrow$ $V_C = V_B$

$V_B = 100K \cdot I_B$

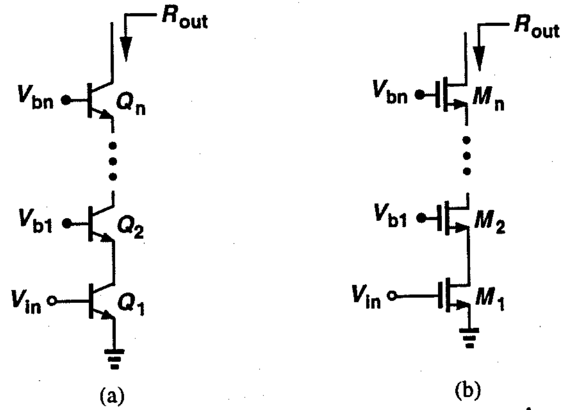
$V_C = 1K \cdot I_C$

$I_C = \beta \cdot I_B$

$\Rightarrow \quad 100K \frac{I_C}{\beta} = 1K\, I_C \Rightarrow \boxed{\beta = 100}$

$\beta = \boxed{100}$

3. Compute the output impedance of each circuit as $n \to \infty$. Assume $V_A < \infty$, $\lambda > 0$, and $\beta < \infty$.




a) $R_n$: Rout when we have 'n' transistors.

$R_1 = r_o \quad R_2 = r_o(1 + g_m(r_o \| r_\pi)) + r_o \| r_\pi, \ldots, R_n = r_o(1 + g_m(R_{n-1} \| r_\pi)) + R_{n-1} \| r_\pi$

as $n \to \infty$; $R_n = R_{n-1}$ so

$$R_\infty = r_o(1 + g_m(R_\infty \| r_\pi)) + R_\infty \| r_\pi$$

$$\Rightarrow R_\infty = r_o \cdot \frac{1 + \beta + \sqrt{(1+\beta)^2 + 4\frac{r_\pi}{r_o}}}{2} \simeq r_o(1+\beta)$$

b) $R_1 = r_o$ ; $R_2 = r_o(1 + g_m r_o) + r_o$ ; .... ; $R_n = r_o(1 + g_m R_{n-1}) + R_{n-1}$

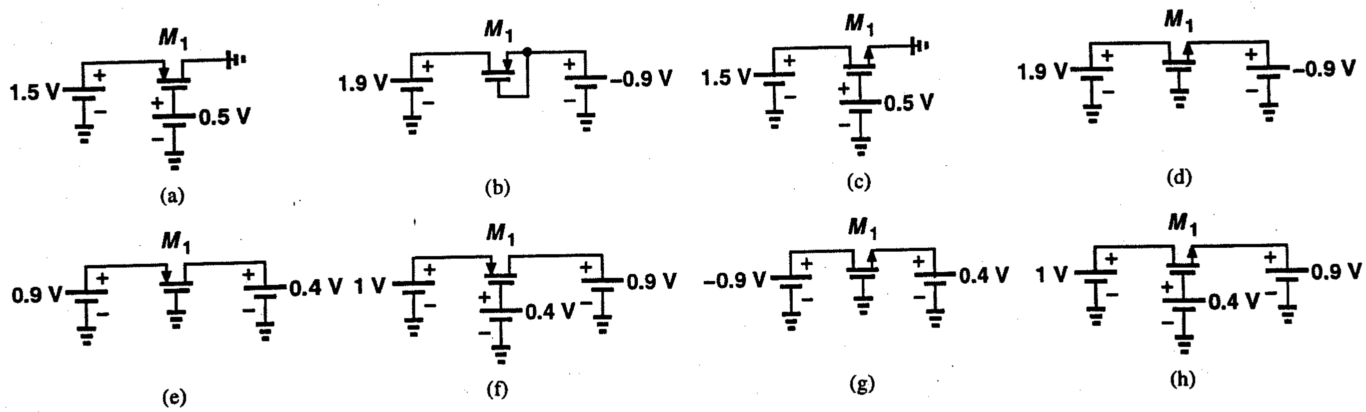
$R_n = r_o + (1 + g_m r_o) R_{n-1}$

Since $1 + g_m r_o > 1$ ; $R_n$ keeps increasing as $n \to \infty$

so $R_\infty = \infty$

(a): $R_{out} = \boxed{\simeq r_o(1+\beta)}$ (b): $R_{out} = \boxed{\infty}$

4. Determine the region of operation of each transistor (off, triode region, or saturation region). Assume a threshold voltage of 0.4 V for NMOS devices and −0.4 V for PMOS devices.




a) PMOS

$V_{GD} = 0.5 > V_{THP}$

$V_{GS} = -1 < V_{THP}$

SAT

b) PMOS

$V_{GD} = 0 > V_{THP}$

$V_{GS} = -1 < V_{THP}$

SAT

c) NMOS

$V_{GD} = -1 < V_{THN}$

$V_{GS} = 0.5 > V_{THN}$

SAT

d) NMOS

$V_{GD} = -1.9 < V_{THN}$

$V_{GS} = 0.9 > V_{THN}$

SAT

e) PMOS

$V_{GS} = -0.9 < V_{THP}$

$V_{GD} = -0.4 = V_{THP}$

Edge of Sat

f) PMOS

$V_{GS} = -0.5 < V_{THP}$

$V_{GD} = -0.5 < V_{THP}$

TRIODE

g) NMOS

$V_{GD} = -0.4 < V_{THN}$

$V_{GS} = 0.9 > V_{THN}$

SAT

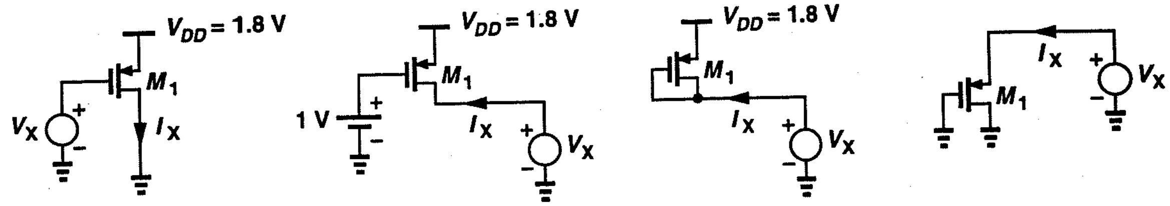
h) NMOS

$V_{GS} = -0.5 < V_{THN}$

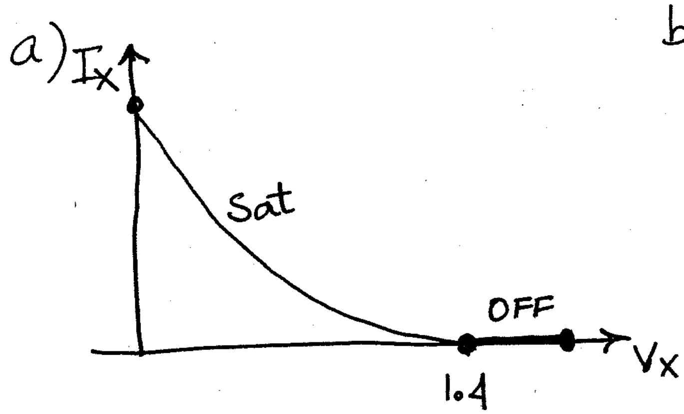
OFF

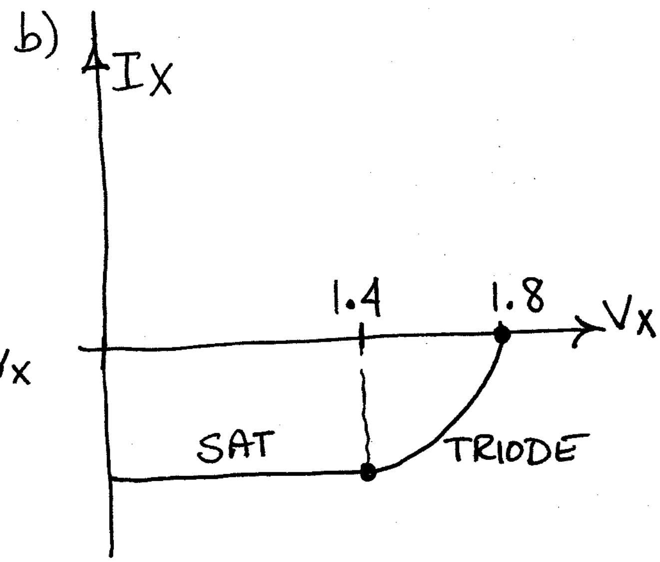
(a) SAT (b) SAT (c) SAT (d) SAT (e) EDGE (f) TRIODE (g) SAT (h) OFF

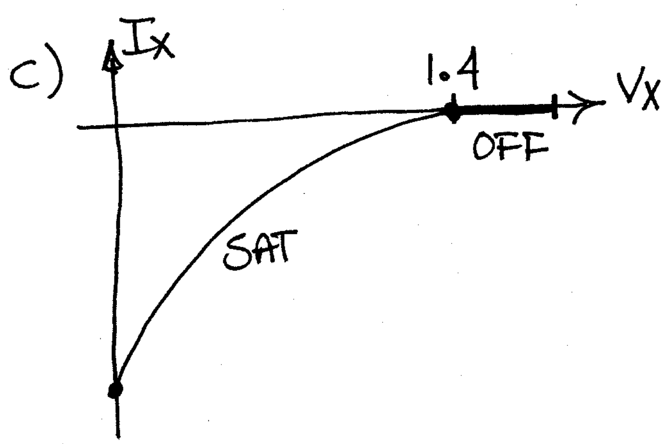
5. Plot $I_X$ as a function of $V_X$ as $V_X$ varies from 0 to 1.8 V. Assume $\lambda = 0$ and $V_{THP} = -0.4$ V. Determine at what value of $V_X$ the device changes its region of operation.

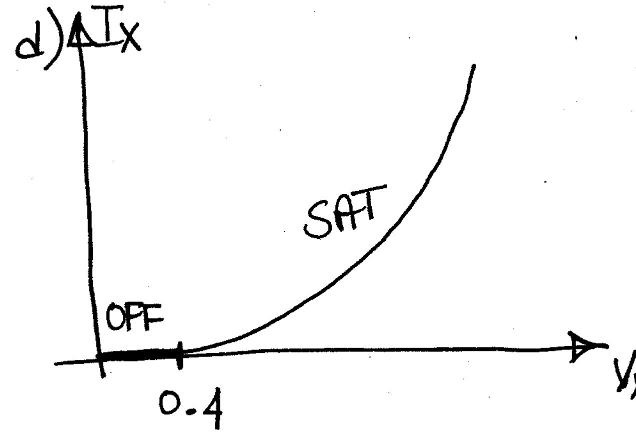



(a) (b) (c) (d)



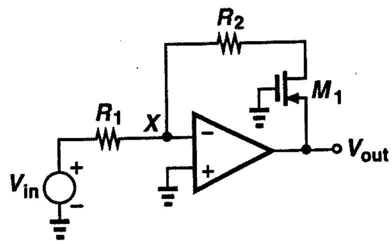



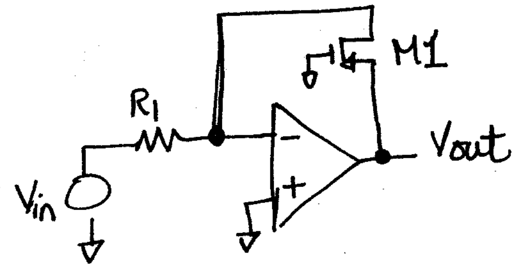
6. In the circuit shown below, assume the op amp is ideal and has an infinite gain. Also, $\lambda = 0$.

(a) Assuming $R_2 = 0$, determine $V_{out}$ as a function of $V_{in}$. For what range of $V_{in}$ does the circuit produce a meaningful output, i.e., $M_1$ ramains on?

(b) Now assume $R_2 > 0$ and calculate the value of $V_{in}$ that places $M_1$ at the edge of saturation.




a)



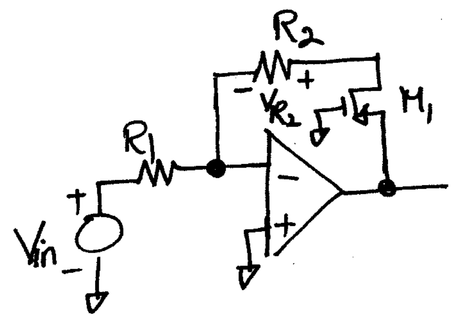

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG}-|V_{THP}|)^2 = -\frac{V_{in}}{R_1}$$

$$V_{SG} = V_{out}$$

$$\Rightarrow V_{out} = \sqrt{\frac{-2V_{in}}{R_1 \mu_p C_{ox}\frac{W}{L}}} + |V_{THP}|$$

$V_{in} \leq 0$

b)




$$V_{R_2} = -\frac{V_{in}}{R_1}\cdot R_2$$

EDGE OF SAT: $V_{R_2} = |V_{THP}|$

$$\Rightarrow V_{in} = -\frac{R_1}{R_2}|V_{THP}|$$

(a)$V_{out}$ = $\boxed{\sqrt{\frac{-2V_{in}}{R_1 \mu_p C_{ox}\frac{W}{L}}} + |V_{THP}| \quad V_{in} \leq 0}$ (b)$V_{in}$ = $\boxed{-\frac{R_1}{R_2}|V_{THP}|}$