EE 115A, Winter 2014, Final Exam – Feb 20, 2014

Instructions: This exam booklet consists of four problems, blank sheets for the solutions and additional blank sheets. Please follow these instructions while answering your exam:

1. You have 3 hours to finish your exam.
2. Write your solutions in the provided blank space after each problem.
3. The sheets marked "Scratch Paper" at the end of the blanket will NOT be graded. These sheets are provided for your rough calculations only.
4. Write your solutions clearly. Illegible solutions will NOT be graded.
5. Be brief.
7. Write your name and student identification number below.

NAME: _Solutions____

STUDENT ID: ____

NAME OF STUDENT ON LEFT:______________________________

NAME OF STUDENT ON RIGHT: ____________________________

Problem	Score
#1	/10
#2	/25
#3	/25
#4	/40
Total	/100

Problem 1: Consider the CE circuit shown in Figure 1. Draw approximate sketches of the base and collector voltage waveforms and the collector current waveform for the following case: $V_B = 0.7V$, $v_{in}(t) = 0.1\sin(2\pi t)$ for $t \geq 0$, $I_S = 10^{-17}A$, $V_T = 26mV$, $V_{CC} = 2.7V$, and $R_C = $ 2kiloOhm. You can ignore the base current and Early effect.
(10 points)

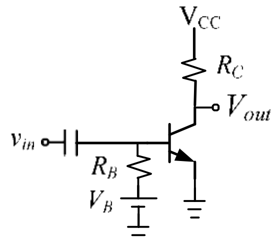
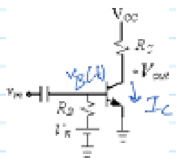


Figure 1

Solution:

Ignore I_b and r_o

(1)



$V_B = 0.7V$, $V_{cc} = 2.7V$, $V_T = 26\,mV$,
$I_S = 10^{-17}A$, $R_C = 2k\Omega$, $R_B = ?$

$v_{in}(t) = 0.1\sin(2\pi t)$ $\forall t \geq 0$

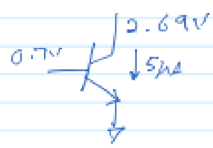
$V_B(t) = ?$, $V_{out}(t) = ?$ $I_C(t) = ?$

Solution:

Operating point: $I_E = I_C = I_S e^{V_{BE}/V_T} = 10^{-17} \times \left(e^{\frac{0.7V}{26mV}}\right) \simeq 5\mu A$

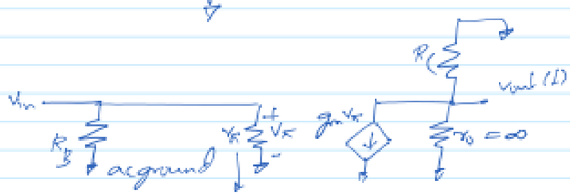
$\Rightarrow V_{out} = V_{CC} - I_C R_C = 2.7 - 5\mu A \times 2k\Omega = 2.7V - 10mV$
$= 2.69V$

Transistor's region of operation:



$\Rightarrow$ Active region

SSM:



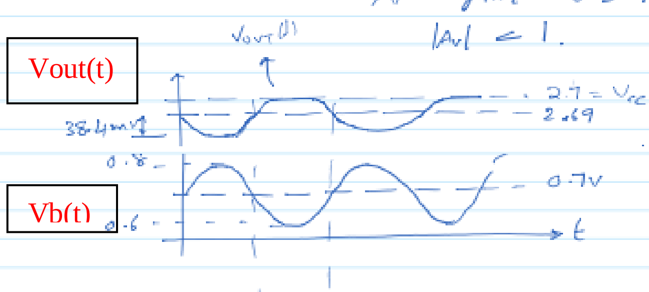
$r_\pi = \infty$
(we are ignoring I_b)

$g_m = \frac{I_C}{V_T} = \frac{5\mu A}{26mV} = 192\mu℧$

$\Rightarrow v_{out}(t) = -g_m R_C v_{in}(t)$

$A_v = -g_m R_C = -0.384$

$|A_v| < 1.$



Problem 2: Consider the amplifier schematic shown in Figure 2. This is called a folded cascode amplifier. Please use the following parameters wherever necessary for your calculations:

$I_B = 200\mu A$, $V_{DD} = 1.8V$, $\mu_n C_{ox} = 140\mu A/V^2$, $\mu_p C_{ox} = 60\mu A/V^2$, $V_{Tn} = 0.4V$, and $|V_{Tp}| = 0.5V$, $\lambda_n = 0.1V^{-1}$, and $|\lambda p| = 0.2V^{-1}$, unless otherwise specified.

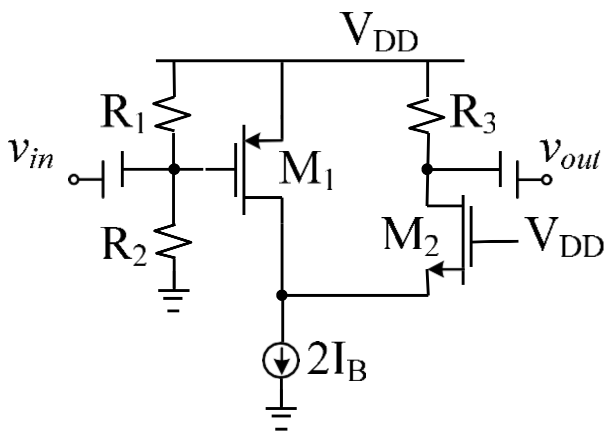


Figure 2

(a) Determine the smallest ratio R_2/R_1 that still keeps the transistor M_1 in the saturation region. Assume that the transistors have identical sizes, carry equal currents, and ignore channel length modulation.

(b) Determine the range of values of R_3 for which transistor M_2 stays in saturation. Assume that the transistors are identical, carry equal currents, and ignore channel length modulation.

(c) Draw a small signal model of the amplifier. Use the subscript “1” or “2” on g_m and r_o to identify which transistor they correspond to. Do not ignore channel length modulation.

(d) Derive symbolic expressions for the gain, input impedance, and the output impedance of this amplifier. Ignore channel length modulation for both transistors. Assume that the capacitors are shorted.

(e) Calculate the output impedance if channel length modulation is not ignored.

(10 + 5 + 6 + 12 + 7 = 40 points)

Solution:

(a) What is the smallest $\frac{R_2}{R_1}$ such that M1 is in saturation.
All transistors have same W/L, carry same current.

Solution $2I_B = 400\mu A \Rightarrow I_{M_2} = I_{M_1} = 200\mu A.$

$$I_{M_2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD}-V_S-V_{Tn})^2 \quad \text{...}(1)$$ (No channel length modulation)

and $$I_{M_1} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{DD}-V_G-|V_{Tp}|)^2 \quad .(2)$$

$$(1) = (2) \Rightarrow \mu_n C_{ox}(V_{DD}-V_S-V_{Tn})^2 = C_{ox}\mu_p(V_{DD}-V_G-|V_{Tp}|)^2$$

$$\frac{140\mu A}{V^2}(1.8-V_S-0.4)^2 = \frac{60\mu A}{V^2}(1.8-V_G-0.5)^2$$

$$1.527(1.8-V_S-0.4) = (1.8-V_G-0.5)$$

$\Rightarrow$ Relation between V_G and V_S. ...(3)

We also know that M1 has to be in saturation.

ie $V_S \leq V_G + |V_{Tp}|$ ...(4)

From (3) and (4), we get a bound on V_G

$$(3) \Rightarrow 0.8378 - 1.527V_S = -V_G$$

$$\Rightarrow V_G = 1.527V_S - 0.8378 \quad \text{--- } (3)'$$

Plug (3)' into (4):

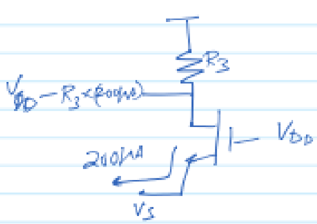
$$\frac{V_G + 0.8378}{1.527} \leq V_G + 0.5$$

$$0.0743 \leq 0.527V_G \Rightarrow V_G \geq 0.14$$

But $V_G = \frac{R_2}{R_1+R_2}V_{DD} \Rightarrow$ min value of $\frac{R_2}{R_1}$

$(R2/R1)_{min}$=0.0843

(b) Range of acceptable values of R_3 for which M_2 stays in saturation

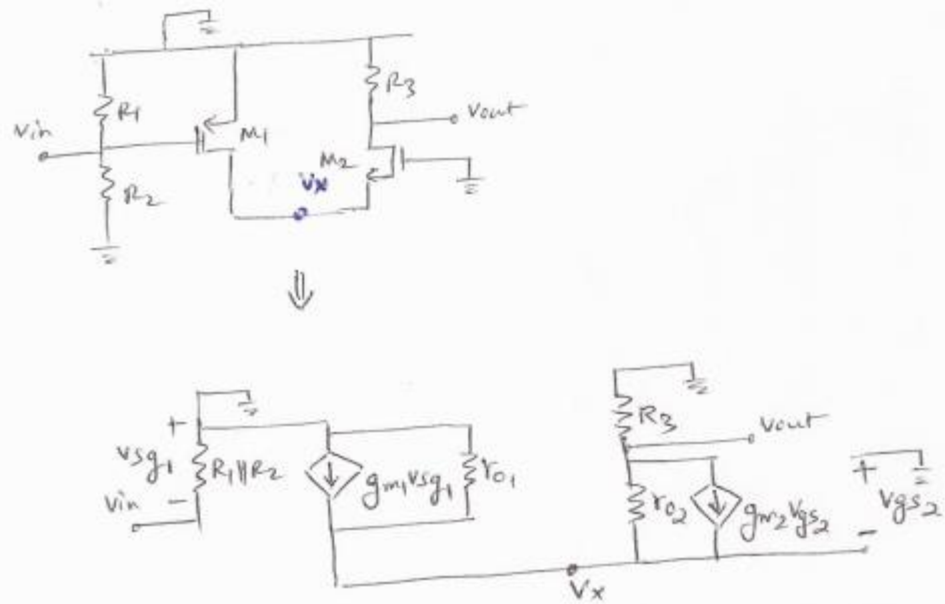


Want: $V_{DD} - 200\mu A \times R_3 \geq V_{DD} - V_{Tn}$

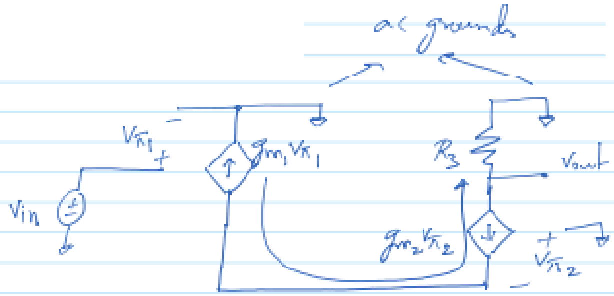
$$\Rightarrow R_3 \leq \frac{V_{Tn}}{200\mu A} = \frac{0.4}{200\mu A}$$

ie $R_3 \leq 2k\Omega$

Part (c) In SSM.


R1
R2
R3
Vin
Vout
M1
M2
Vx
vsg1
R1||R2
gm1vsg1
ro1
vin
R3
Vout
ro2
gm2vgs2
vgs2
Vx

(d)



$$V_{out} = (-g_{m_1} V_{in}) R_3 = -g_{m_1} R_3 V_{in}$$

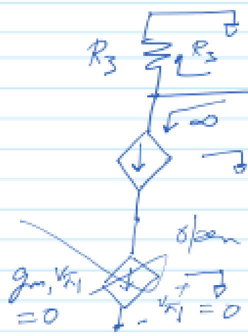
$$A_v = -g_{m_1} R_3$$

$R_{in} = \infty$

without channel length modulation

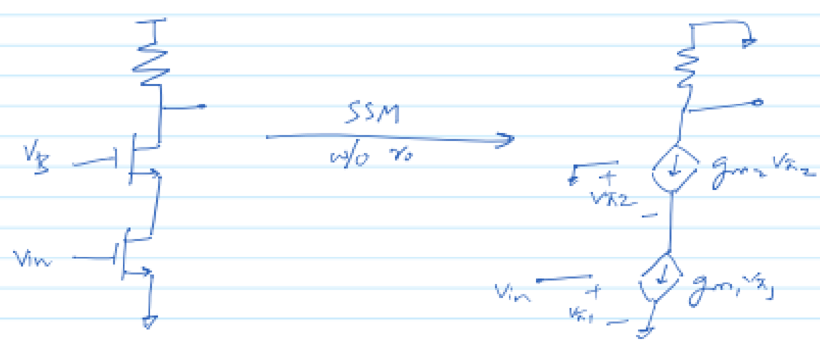
$R_{out} = ?$

Redraw:

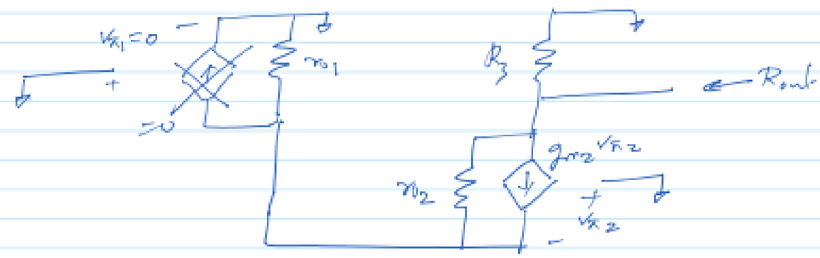


$R_{out} = R_3$

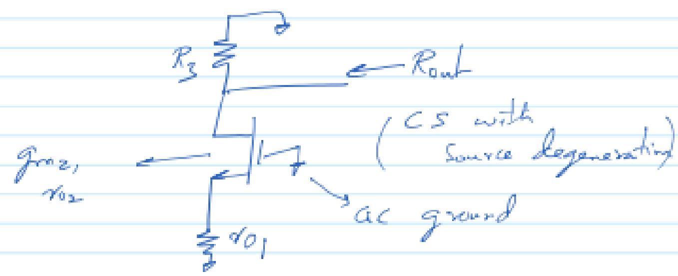
Recall:



ⓔ When $r_o < \infty$



as if



$$R_{out} = R_3 \,||\, (r_{o1} + r_{o2} + g_{m2} r_{o2} r_{o1})$$

Problem 3: Consider the OpAmp based circuit shown in the Figure 3 (a).

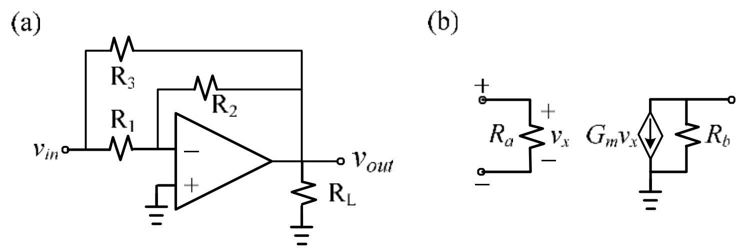
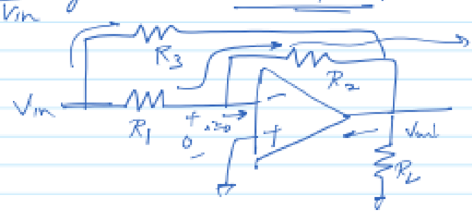


Figure 3

(a) Derive a symbolic expression for v_{out}/v_{in} assuming an ideal OpAmp.
(b) Derive symbolic expressions for the input and output impedances of the circuit assuming an ideal OpAmp. Please do not include R_L in the calculation of the output impedance.
(c) Derive a symbolic expression for the total current delivered by the amplifier assuming an ideal OpAmp.
(d) A student like yourself designed the circuit shown in Figure 3(b) for use as an OpAmp. Assuming R_a, G_m, and R_b are all very large, would it serve the purpose in Figure 3(a)? Give reasons in support of your answer. Assume $R_3 = R_L = \infty$ to keep things simple. Note: Do not rush into an answer. The question is not whether Figure 3(b) is an ideal OpAmp or not. The question is whether Figure 3(a) will still result in approximately the same transfer function as in part (a).

(3 + 6 + 6 + 10 = 25 points)

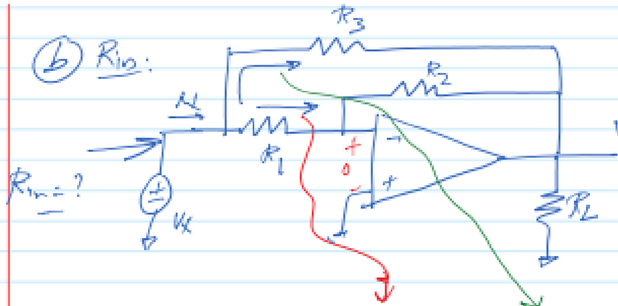
(a) $\frac{V_{out}}{V_{in}}$ for an ideal op amp.



$$\frac{V_{in} - 0}{R_1} = \frac{0 - V_{out}}{R_2}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

(b) R_{in}:



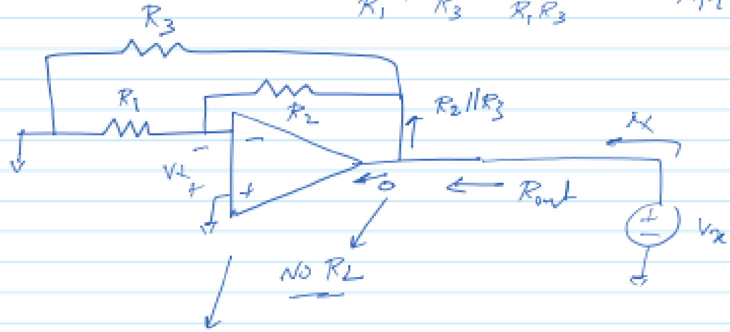
$V_{out} = -\frac{R_2}{R_1} V_{in} = -\frac{R_2}{R_1} V_x$

$$R_{in} = \frac{V_x}{i_x}, \quad i_x = \frac{V_x}{R_1} + \frac{V_x - V_{out}}{R_3}$$

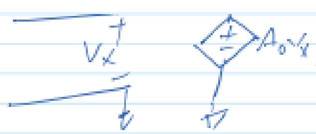
$$i_x = \frac{V_x}{R_1} + \frac{V_x + \frac{R_2}{R_1} V_x}{R_3} = \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{R_2}{R_1 R_3}\right) v_x$$

$$\Rightarrow R_{in} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_3} + \frac{R_2}{R_1 R_3}} = \frac{R_1 R_3}{R_1 + R_3 + R_2}$$

R_{out}:

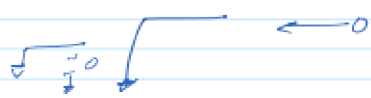


NO R_L



→ dependent voltage source

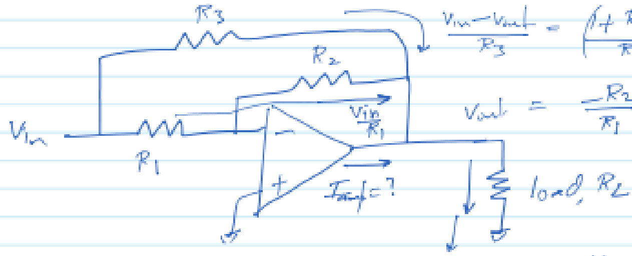
if $v_d = 0$ ⇒



⇒

$$R_{out} = 0 \,\|\, (R_2 \| R_3) = 0.$$

(c)



$$\frac{V_{in} - V_{out}}{R_3} = \frac{(1 + R_2/R_1)\, V_{in}}{R_3}$$

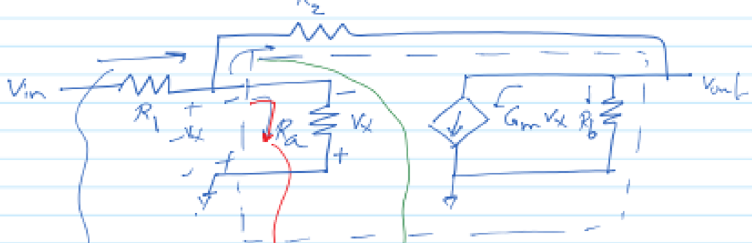
$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

$$\frac{V_{out}}{R_L} = -\frac{R_2}{R_1}\frac{V_{in}}{R_L}$$

$$\Rightarrow I_{amp} = -\frac{R_2}{R_1}\frac{V_{in}}{R_L} - \left[\frac{V_{in}}{R_1} + \left(\frac{1 + R_2/R_1}{R_3}\right) V_{in}\right]$$

$$= -\left[\frac{R_2}{R_1 R_L} + \frac{1}{R_1} + \frac{1 + R_2/R_1}{R_3}\right] V_{in}$$

(d)



R_a, G_m, R_b are very large

Note: Gut reaction: o/p impedance of opamp = R_b is very large

$\Rightarrow$ NOT a very good "op amp".

$$\text{KCL:} \quad \frac{V_{in} - (-V_x)}{R_1} = \frac{-V_x}{R_a} + \frac{-V_x - V_{out}}{R_2} \quad \ldots ①$$

Also,

KCL @ output node:

$$G_m v_x + \frac{V_{out}}{R_b} = \frac{-V_x - V_{out}}{R_2} \quad \cdots (2)$$

$$\Rightarrow \left(G_m + \frac{1}{R_2}\right) V_x = -\frac{V_{out}}{R_2} - \frac{V_{out}}{R_b} \quad \cdots (2)'$$

$$= -\left(\frac{1}{R_2} + \frac{1}{R_b}\right) V_{out}$$

$$= -\frac{V_{out}}{R_2 \| R_b} \quad \cdots (2)'$$

Rewrite (1):

$$\frac{V_{in}}{R_1} = \underbrace{-\left(\frac{1}{R_a} + \frac{1}{R_1} + \frac{1}{R_2}\right) V_x}_{} - \frac{V_{out}}{R_2} \quad \cdots (1)'$$

$$= \frac{-V_x}{R_a \| R_1 \| R_2}$$

Plug (2)' into (1)' $\Rightarrow$

$$\frac{V_{in}}{R_1} = -\frac{1}{R_a \| R_1 \| R_2} \cdot \frac{-V_{out}}{R_2 \| R_b} \left(\frac{1}{G_m + \frac{1}{R_2}}\right) - \frac{V_{out}}{R_2}$$

$$\xrightarrow{R_a, R_b, G_m \to \infty} \underbrace{+\frac{1}{R_1 \| R_2} \frac{V_{out}}{R_2} \left(\frac{1}{\infty + 1/R_2}\right)}_{\longrightarrow 0} - \frac{V_{out}}{R_2}$$

$$\Rightarrow V_{out} = -\frac{R_2}{R_1} V_{in} \quad \checkmark$$

Problem 4: Consider the amplifier shown in Figure 4.
(a) Draw a small signal model of the circuit. Assume that all the capacitors act as shorts for signals of interest. Do **not** ignore channel length modulation for this part.
(b) Derive symbolic expressions for the voltage gain of the circuit. You can ignore channel length modulation for this part.
(c) Repeat part (b) but consider that the input has a source resistance, R_S. You can ignore channel length modulation for this part.
(c) Derive symbolic expressions for the input and output impedances of the amplifier. You can ignore channel length modulation for this part.
(4 + 4 + 7 + 10 = 25 points)

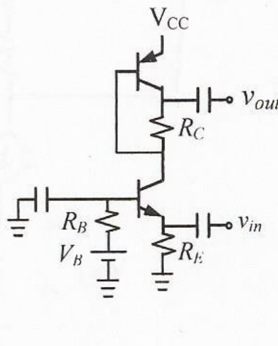
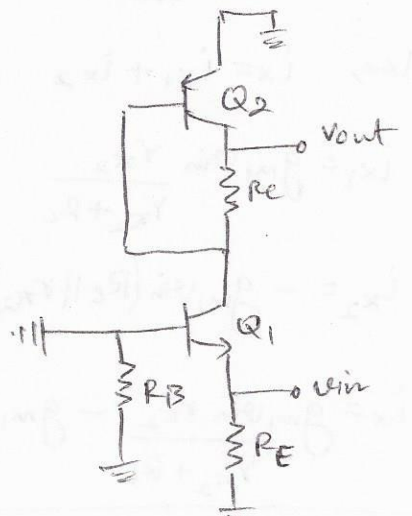


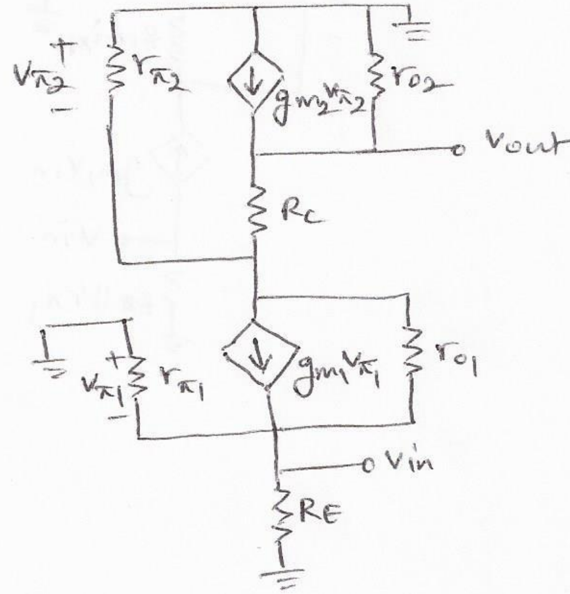
Figure 4

Solution:

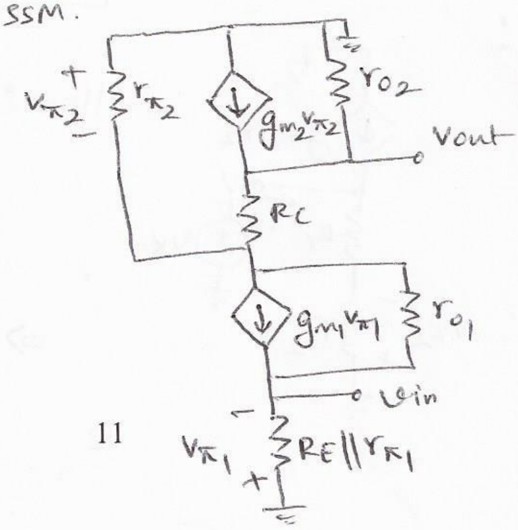
Part(a)



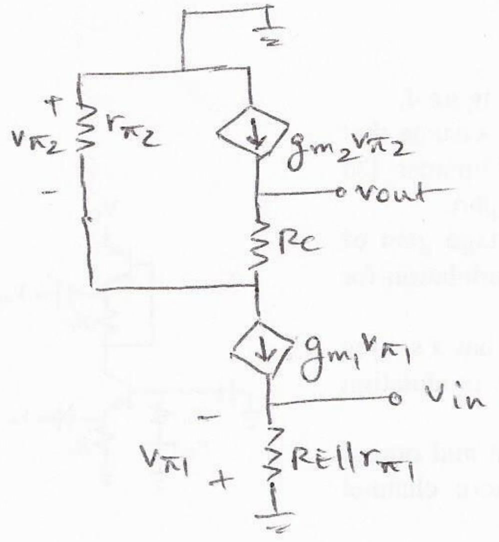
SSM ⇒



Part (b) Simplifying the SSM.

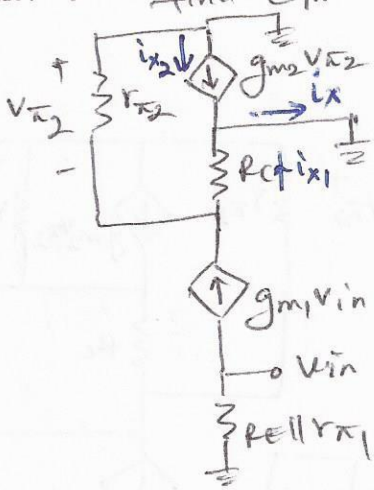


No CLM in this case :



Here, $v_{\pi 1} = -v_{in}$

Approach 1 : find G_m.



Note: $G_m = \dfrac{i_x}{v_{out}}$

Also, $i_x = i_{x1} + i_{x2}$

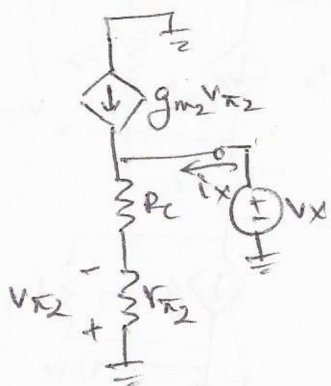
$$i_{x1} = g_{m1} v_{in} \frac{r_{\pi 2}}{r_{\pi 2} + R_C}$$

$$i_{x2} = -g_{m1} v_{in} (R_C \| r_{\pi 2}) g_{m2}$$

$$\therefore \quad i_x = \frac{g_{m1} v_{in} r_{\pi 2}}{r_{\pi 2} + R_C} - g_{m1} v_{in} g_{m2} \frac{R_C r_{\pi 2}}{R_C + r_{\pi 2}}$$

$$\Rightarrow \boxed{G_m = \frac{g_{m1} r_{\pi 2}}{r_{\pi 2} + R_C} (1 - g_{m2} R_C)}$$

find R_{out} :



Here: $\dfrac{v_x}{i_x} = (R_C + r_{\pi 2}) \| \left(\dfrac{v_x}{-g_{m2} v_{\pi 2}} \right)$

$$v_{\pi 2} = -v_x \cdot \frac{r_{\pi 2}}{r_{\pi 2} + R_C}$$

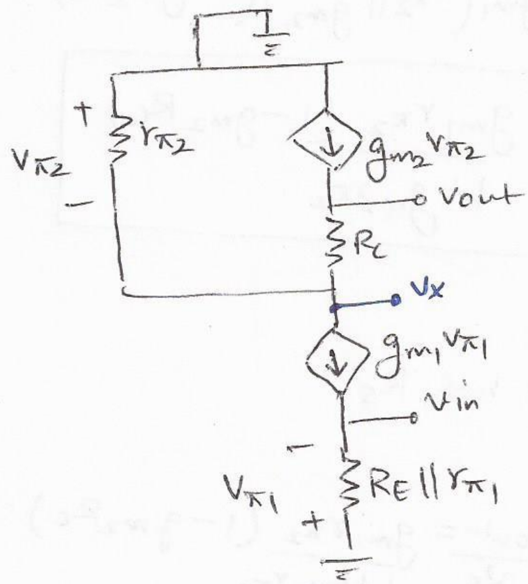
$$\Rightarrow R_{out} = (R_C + r_{\pi 2}) \| \left(\frac{R_C + r_{\pi 2}}{g_{m2} r_{\pi 2}} \right)$$

$$\boxed{R_{out} = \frac{R_C + r_{\pi 2}}{1 + g_{m2} r_{\pi 2}}}$$

Scratch Paper (Will Not Be Graded)

$$\therefore \quad A_v = G_m R_{out} = \frac{g_{m1} r_{\pi 2}}{1 + g_{m2} r_{\pi 2}} (1 - g_{m2} R_c)$$

Approach 2 :



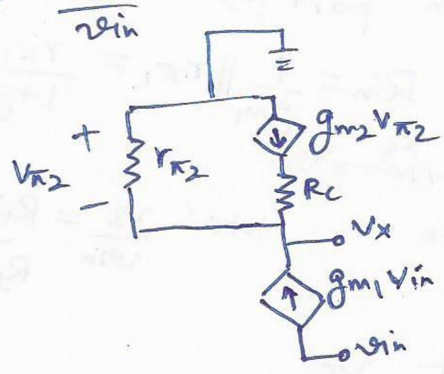
Here, $v_{\pi 1} = -v_{in}$

find $\frac{v_x}{v_{in}}$ & $\frac{v_{out}}{v_x}$

then

$$\frac{v_{out}}{v_{in}} = \left(\frac{v_x}{v_{in}}\right) \cdot \left(\frac{v_{out}}{v_x}\right)$$

find $\frac{v_x}{v_{in}}$

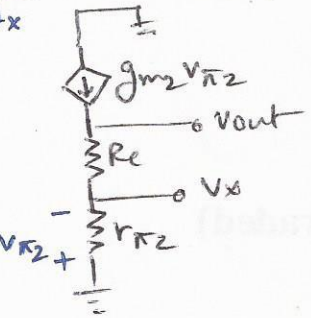


$$v_x = g_{m1} v_{in} \left(r_{\pi 2} \| \frac{v_x}{(-g_{m2} v_{\pi 2})} \right)$$

Also, $v_{\pi 2} = -v_x$

$$\therefore \quad \boxed{\frac{v_x}{v_{in}} = g_{m1} \left(r_{\pi 2} \| \frac{1}{g_{m2}} \right)}$$

find $\frac{v_{out}}{v_x}$



Here, $v_{\pi 2} = -v_x$.

Also, using KCL @ Vout

$$\Rightarrow \frac{V_x - V_{out}}{R_C} = g_{m2} V_x$$

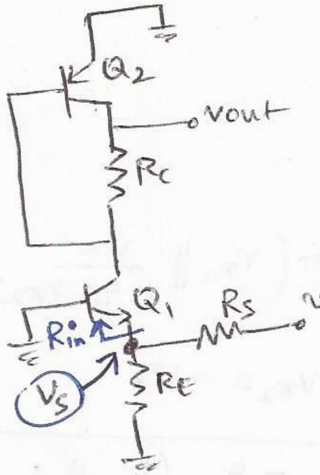
$$\therefore V_{out} = v_x (1 - g_{m2} R_C)$$

$$\Rightarrow \frac{V_{out}}{V_x} = (1 - g_{m2} R_C)$$

$$\therefore \frac{V_{out}}{v_{in}} = \left(\frac{V_x}{v_{in}}\right) \cdot \left(\frac{V_{out}}{v_x}\right) = g_{m1}\left(r_{\pi 2} \,\|\, \frac{1}{g_{m2}}\right)(1 - g_{m2} R_C)$$

$$\boxed{\frac{V_{out}}{v_{in}} = \frac{g_{m1} r_{\pi 2} (1 - g_{m2} R_C)}{1 + g_{m2} r_{\pi 2}}}$$

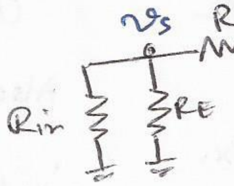
Part (c) find $\frac{v_{out}}{v_{in}}$ & consider I/p has R_s.



Here, $\frac{v_{out}}{v_s} = \frac{g_{m1} r_{\pi 2}}{1 + g_{m2} r_{\pi 2}} (1 - g_{m2} R_C)$

from part (b)

Note: $R_{in} = \frac{1}{g_{m1}} \,\|\, r_{\pi 1} = \frac{r_{\pi 1}}{1 + g_{m1} r_{\pi 1}}$

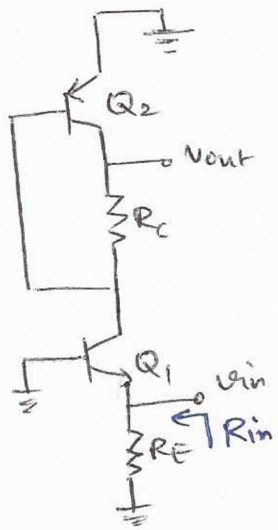


Note: $\frac{v_s}{v_{in}} = \frac{R_{in} \,\|\, R_E}{R_s + R_{in} \,\|\, R_E}$

Also, $\frac{V_{out}}{v_{in}} = \frac{v_{out}}{V_s} \cdot \frac{v_s}{v_{in}}$

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{g_{m1} r_{\pi 2} (1 - g_{m2} R_C)}{1 + g_{m2} r_{\pi 2}} \cdot \frac{R_E \,\|\, \left(\frac{r_{\pi 1}}{1 + g_{m1} r_{\pi 1}}\right)}{R_s + R_E \,\|\, \left(\frac{r_{\pi 1}}{1 + g_{m1} r_{\pi 1}}\right)}}$$

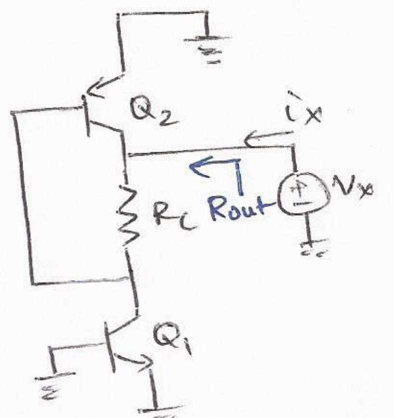
Part (d)



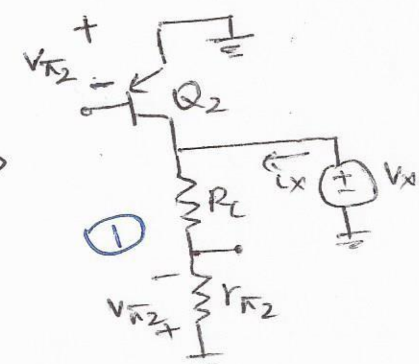
find R_{in} Here, $R_{in} = R_E \,||\, r_{\pi 1} \,||\, \frac{1}{g_{m1}}$

$$= R_E \,||\, \left(\frac{r_{\pi 1}}{1+g_{m1}r_{\pi 1}}\right)$$

find R_{out}.



$\Rightarrow$



Here, $R_{out} = (R_C + r_{\pi 2}) \,||\, \left(\frac{V_x}{-g_{m2}V_{\pi 2}}\right)$

$v_{\pi 2} = -V_x\left(\frac{r_{\pi 2}}{r_{\pi 2}+R_C}\right)$ from voltage division in Branch 1

$$\therefore \quad R_{out} = (R_C + r_{\pi 2}) \,||\, \left(\frac{r_{\pi 2}+R_C}{g_{m2}r_{\pi 2}}\right) = \boxed{\frac{R_C + r_{\pi 2}}{1+g_{m2}r_{\pi 2}}}$$