Name: [redacted]
Student ID: [redacted]

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Name (person on left): [redacted] Name (person on right): [redacted]

(3 points for name on all pages, 2 points for names of persons on left and right)

Note: For all problems, assume $\lambda = 0$ (infinite output resistance), each discrete capacitor has $C = \infty$, and the body effect is negligible. Assume the thermal voltage is 0.025 V.

Problems 1 - 2:

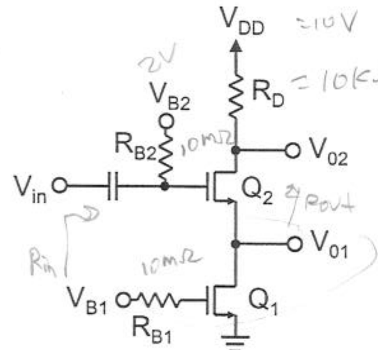

Figure P1.

With some thoughtful design, the circuit in Figure P1 can be used as either a linearized common-source amplifier or source follower. Assume each transistor has identical $k_n'(W/L) = 1\ mA/V^2$ and $V_t = 1.1$ V. $R_D = 10\ k\Omega$, $R_{B1} = R_{B2} = 10\ M\Omega$ and $V_{DD} = 10$ V. You have access to a 5V source and a 2V source (as well as ground and the 10V supply) to use as biases.

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Problem 1 (20 points): 19/20 good job!

Assume you want to use the circuit in Figure P1 as a source follower.

a) (5 pts): From the following regions of operation of the MOS (off, deep triode, triode and saturation), which operating region is most desirable for Q_1? Why? → out of Q_2's source

Saturation, Since we want a constant current flowing from $Q_2 \to Q_1$. Since, current is more stable after pinchoff, I_D won't fluctuate as widely as in triode or not at all when off.

5/5

V_{o2} output

b) (5 pts): What bias(es) would work for V_{B1} and V_{B2}? Why?

$V_{B2} = 5V$ $V_{B1} = 2V$

Because to keep both MOS's in saturation, $V_{DS} \geq V_{GS} - V_t$. Therefore in general, the drain voltage has to be > the gate voltage. If we use any other values, we will get conflicting current on the same line or $I = 0$.

5/5 good!

Assume you want to use the circuit in Figure P1 as a linearized common-source amplifier.

c) (5 pts): From the following regions of operation of the MOS (off, deep triode, triode and saturation), which operating region is most desirable for Q_1? Why?

~~Saturation~~ Triode. Since we need maximum output swing across the drain at Q_2, and want Q_1 to act like a linear resistor to stabilize the current by way of feedback. then Q_1 becomes

$$r_{DS} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_t)}$$

4/5 deep

V_{DD} 10, max, $V_{GS} - V_t$ @Q_2

V_{o2} output

d) (5 pts): Assume you select $V_{B2} = 2V$. What bias should you use for V_{B1} to maximize the gain? Why?

The smaller $r_{DS} = R_S$ is the greater the gain is $\left(A_v = \frac{-g_m R_D}{1 + g_m R_S}\right)$

Therefore $V_{B1} = V_G$ should be the highest voltage value possible, $= 10V = V_{DD}$, to minimize r_{DS} and max gain.

5/5

~~10/10~~

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Problem 1 (20 points):

Assume you want to use the circuit in Figure P1 as a source follower.

a) (5 pts): From the following regions of operation of the MOS (off, deep triode, triode and saturation), which operating region is most desirable for Q_1? Why?

b) (5 pts): What bias(es) would work for V_{B1} and V_{B2}? Why?

Assume you want to use the circuit in Figure P1 as a linearized common-source amplifier.

c) (5 pts): From the following regions of operation of the MOS (off, deep triode, triode and saturation), which operating region is most desirable for Q_1? Why?

d) (5 pts): Assume you select $V_{B2} = 2V$. What bias should you use for V_{B1} to maximize the gain? Why?

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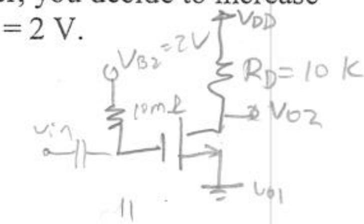
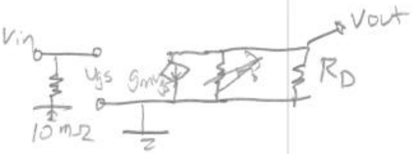
Problem 2 (15 points):
When using the circuit in Fig. P1 as a common-source amplifier, you decide to increase the gain further by connecting V_{01} directly to ground. Let $V_{B2} = 2$ V.



a) (5 pts) Calculate the gain at the quiescent point.

$A_v = \frac{v_{out}}{v_{in}} = \frac{g_m v_{gs} R_D}{v_{gs}}$ $v_{in} = v_{gs}$



4/5

$A_v = g_m R_D$

$\approx 0.9\frac{mA}{V}(10K) = -9$ V/V

$I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS}-V_t)^2 = \frac{1}{2}(\frac{1mA}{V^2})(2-0-1.1)^2$

$g_m = k_n'\frac{W}{L}V_{ov} = (1mA/V^2)(2-0-1.1)$

$= 0.9$ mA/V

✓

b) (10 points) Calculate the maximum input voltage swing for which Q_2 is still saturated.

$V_{DD} = 10$

7/10

$V_{OD} = 5.95$V

$V_{gs} - V_t = V_g - V_t = 0.9$

$V_D = V_{GS} - V_t$

$I_D = \frac{V_{DD}-V_D}{10K} = \frac{1}{2}(\frac{1mA}{V^2})(0.9)^2$

$\frac{10 - V_D}{10K} = 0.405$mA

$V_D = 5.95$ V

$V_{OD} - V_{ov} = 5.05$ V output swing

$A_v = \frac{-2(V_{DD} - V_{OD})}{V_{OV}} = \frac{-2(10-5.95)}{0.9} = 9$

$A_v = \frac{v_o}{v_i} =$

$V_{in} = \frac{V_{out}}{A_v} = \frac{\pm 5.05 V}{9} = \pm 0.561$ V input swing

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Problem 3 (30 pts, 10 points each for a, b, c):
Find the gain, input and output resistances for these circuits. Assume Q_2 is in saturation.

Hint: For (c), $R_{in} = R_B \,||\, R_{in}$ (from all other elements).

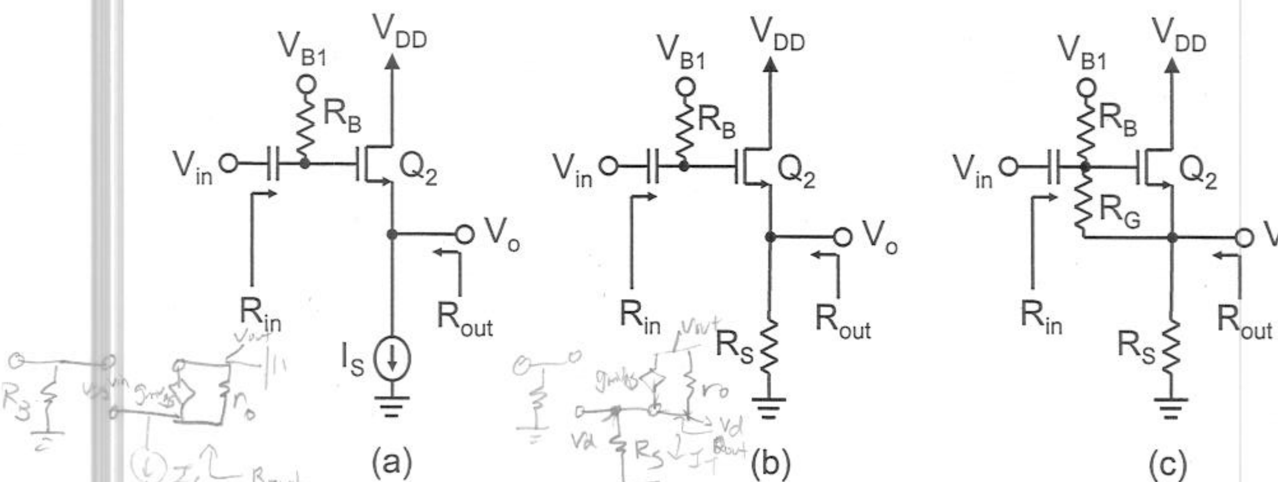
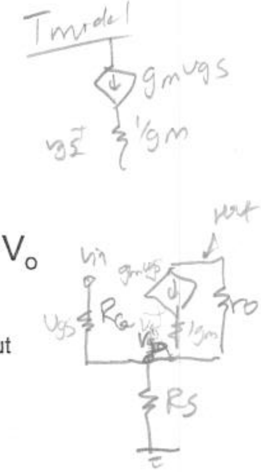


Figure P2.



a) $R_{in} = R_B$

$R_{out} = r_o$

3/10

$A_v = \frac{v_o}{v_i} = \frac{g_m v_{gs} r_o}{g_m v_{gs}}$

$A_v = g_m r_o$

$\frac{1}{g_m} = r_o$

b) $R_{in} = R_B$ ✓

$R_{out} = R_S \,||\, 1/g_m$ ✓

$A_v = \frac{v_o}{v_i} = \frac{g_m v_{gs} (\quad)}{v_{gs}}$

$I_t = g_m v_{gs} + \frac{V_{out} - V_d}{r_o} = \frac{V_d}{R_S}$

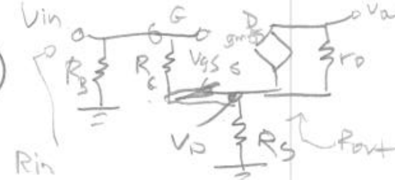
$V_d = I_t R_S$

$I_t = g_m v_{gs} + \frac{V_{out} - I_t R_S}{r_o}$

$A_v = g_m r_o$

6/10

c)



$R_{in} = R_B \,||\, R_{in}$

$R_{in} = R_B \,||\, (R_G + R_S)$

$R_{out} = \frac{1}{g_m} \,||\, R_G \,||\, R_S$

$A_v = \frac{v_{out}}{v_{in}} = g_m (r_o \,||\, R_G \,||\, R_S)$

0

$\frac{V_{out} - V_{gs}}{1/g_m} = g_m v_{gs}$ $V_{out} = 2 v_{gs}$

$V_{in} - V_D = V_{GS}$

$\frac{V_D}{R_S} = g_m v_{gs} + \frac{V_{out} - V_D}{r_o} + \frac{V_{in}}{R_G}$

$V_{in} - V_D = V_{gs}$

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Problem 4: (20 pts): 18/20

Diode D_1 has $I_S = 1 \times 10^{-15}$ and $n = 1.1$. Find I_1 for V_{DD} = (a) 10 V, and (b) 1 V. Points are awarded based on error from the exact answer.

10V 1V

$I = I_S e^{V_D/nV_T}$

$= 1\times10^{-15} e^{V_D/(1.1)(25mV)}$

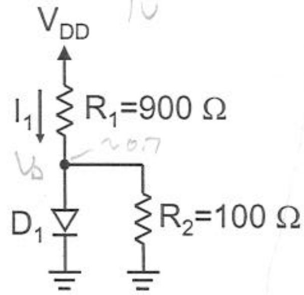

Figure P3.

a) $\frac{V_{DD}-V_D}{900\Omega} = I_1 \qquad I_1 = I_{D_1} + I_{R2}$

$\frac{V_{DD}-V_D}{900\Omega} = 1\times10^{-15} e^{V_D/(1.1)(25mV)} + \frac{V_D}{100\Omega}$

$\frac{10-V_D}{900\Omega} = 10^{-15} e^{V_D/0.0275V} + \frac{V_D}{100\Omega}$

$\frac{10-V_D}{900} - \frac{9V_D}{900} = 10^{-15} e^{V_D/0.0275V}$

$\frac{1-V_D}{90} = 10^{-15} e^{V_D/0.0275}$

$1 - 90\times10^{-15} e^{V_D/0.0275} = V_D$ → use iteration

assume $V_D = 0.7$ V

$1 - 90\times10^{-15} e^{0.7/0.0275} = V_D = 0.989 = V_D$ ← need to consider I_2 = I (through R_2)

$I_1 = \frac{V_{DD}-V_D}{900\Omega} = \frac{10V - 0.989V}{900\Omega} = 9.998\ mA$

8/10

b) $\frac{1-V_D}{900} = 10^{-15} e^{V_D/0.0275} + \frac{V_D}{100}$

$\frac{1-10V_D}{900} = 10^{-15} e^{V_D/0.0275}$

$\frac{1-900\times10^{-15} e^{V_D/0.0275}}{10} = V_D$

set $V_D = 0.7V$

$V_D = 0.0897$ V ~? – can't be real

if on $V = 0.7V$

$\frac{V_{DD}-0.7V}{900\Omega} = I_1 = \frac{1-0.7}{900}$

$I = 0.33$ mA

if off

$\frac{V_{DD}-0}{900+100} = I_1 = \frac{1}{1000}$ =

$I = 1mA$

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Problem 5 (10 points):
Assume you have a transconductor with the current vs. voltage plot shown in Figure P4.

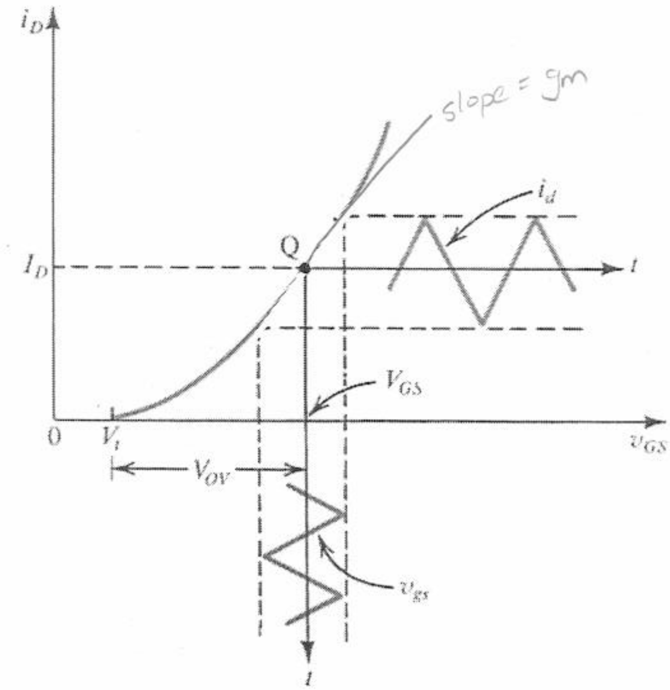

Figure P4.

a) (3 points) Express the transconductance as a function of the small-signal current i_d and small-signal voltage v_{gs}.

$g_m = \frac{i_d}{v_{gs}}$ = ~~$k_n' \frac{W}{L}(V_{GS} - V_t)$~~

3/3

b) (3 points) Express the transconductance as a function of the large-signal current i_D and large-signal voltage v_{GS}.

$g_m = \frac{\partial i_D}{\partial v_{GS}} \Big|_{v_{GS} = V_{GS}}$

3/3

c) (4 points) Do the definitions lead to the same or different results at the bias point? Why?

4/4

Yes. (sort of) the output current and input voltage swing at point Q, whether at small or large signal, swings off the slope (=gm) at point Q, If large signal gets too large, the slope is no longer linear everywhere and starts differing from small signal)

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Problem 1:

a) A source Follower (output taken at V_{o1}) works best when the source is connected to an ideal current source - the gain is then ≈ 1. If Q_1 is saturated, it acts as a current source with the current set by V_{B1}.

[Saturation]

b) Since Q_1 should be saturated, $V_{o1} \geq V_{B1} - V_t$.
This means $V_{B2} - V_{GS2} = V_{o1} \geq V_{B1} - V_t$, or
$V_{B2} \geq V_{B1} + V_{GS2} - V_t$. Since current is flowing through the transistors, $V_{GS2} > V_t$ and thus $V_{B2} > V_{B1}$. We then try various values of V_{B1}.

V_{B1}	$V_{o2} = V_{DD} - \frac{1}{2} k_n' \frac{W}{L} (V_{B1} - V_t)^2 R_D$	
0	(off → circuit doesn't work)	"
2	4.5	5.95
5	-78	-66
	↑ Version 1	↑ Version 2

In either case, if $V_{B2} = 5V$, $V_{o2} > V_{B2} - V_t$ and Q_2 is saturated.

Thus [$V_{B1} = 2V$, $V_{B2} = 5V$]

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Problem 1 (con't)

c) To operate as a common-source amp, take the output at V_{o2}. To linearize the amp, a resistor should be placed between the source of Q_2 and ground. When a MOS is in deep triode, it acts as a voltage-controlled resistor.

[Deep triode]

d) In deep triode, $I_D \approx k_n' \frac{W}{L} (V_{GS} - V_t) V_{DS}$

so $R_{DS} = \frac{V_{DS}}{I_D} \approx \frac{1}{k_n' \frac{W}{L} (V_{GS} - V_t)}$ ← V_{B1}

To increase the gain, you need to decrease R_{DS} so V_{GS} must be as high as possible.

[$V_{B1} = 10V$]

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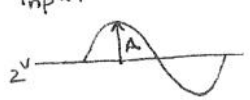
Problem 2) **Ver. 1**

a) $g_m = 1.1^{\,mA/V^2} \times (2-1) = \boxed{1.1\ mA/V}$

$A_V = -g_m R_D = -1.1 \times 10^K = \boxed{-11\ V/V}$

b) input:



$V_{out} = V_{DD} - R_D \frac{K_n'}{2}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$

@ the edge of sat. $V_{out} = V_{GS} - V_t$

$\Rightarrow V_{out} = V_{DD} - R_D \frac{K_n'}{2}\left(\frac{W}{L}\right) V_{out}^2$

$10^k \times \frac{1.1}{2} V_{out}^2 + V_{out} - 10 = 0 \quad \rightarrow 5.5 V_{out}^2 + V_{out} - 10 = 0$

$V_{out} = \frac{-1 + \sqrt{1 + 40 \times 5.5}}{2 \times 5.5} = 1.26$

$\Rightarrow V_{GS_{Max}} = V_{out} + V_t = 1 + 1.26 = 2.26$

$\rightarrow \boxed{A_{in} = V_{GS_{Max}} - 2 = 0.26^V}$

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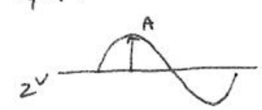
problem 2) **Ver. 2**

a) $g_m = 1^{\,mA/V^2} \times (2 - 1.1) = \boxed{0.9\ mA/V}$

$A_V = -g_m R_D = -0.9^{mS} \times 10^K = \boxed{-9\ V/V}$

b) input:



$V_{out} = V_{DD} - R_D \frac{K_n'}{2}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$

@ the edge of sat. $V_{out} = V_{GS} - V_t$

$\Rightarrow V_{out} = V_{DD} - R_D \frac{K_n'}{2}\left(\frac{W}{L}\right) V_{out}^2$

$10 \times \frac{1^{mA/V^2}}{2} V_{out}^2 + V_{out} - 10 = 0$

$V_{out} = \frac{-1 \pm \sqrt{1 + 40 \times 5}}{10} = 1.32$

$\rightarrow V_{GS_{Max}} = V_{out} + V_t = 1.32 + 1.1 = 2.42$

$\boxed{A_{in} = V_{GS_{Max}} - 2^V = 0.42^V}$

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Problem 3)

a) $R_{in} = R_B$

$A_V = 1$

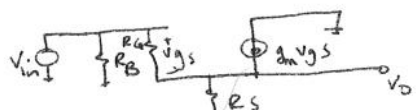
$R_{out} = \frac{1}{g_{m2}}$

b) $R_{in} = R_B$

$A_V = \frac{g_{m2} R_S}{1 + g_{m2} R_S}$

$R_{out} = 1/g_{m2} \| R_S$

c) gain



$V_{in} = V_{gs} + V_o \rightarrow \boxed{V_{gs} = V_{in} - V_o}$

$-g_m V_{gs} + V_o/R_S - V_{gs}/R_G = 0$

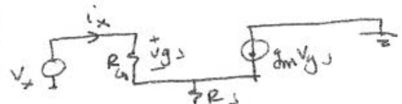
$\Rightarrow \frac{V_{gs}}{1/g_m \| R_G} = V_o/R_S \Rightarrow \frac{V_{in} - V_o}{1/g_m \| R_G} = V_o/R_S$

$\Rightarrow \frac{V_{in}}{1/g_m \| R_G} = \frac{V_o}{1/g_m \| R_G \| R_S} \rightarrow \boxed{\frac{V_o}{V_{in}} = A_V = \frac{1/g_m \| R_G \| R_S}{1/g_m \| R_G}}$

Rout:

$\boxed{R_{out} = R_S \| R_G \| 1/g_m}$

Rin



$i_x = V_{gs}/R_G$, $V_x = V_{gs} + \left(\frac{V_{gs}}{R_G} + g_m V_{gs}\right) R_S = V_{gs}\left[1 + g_m R_S + R_S/R_G\right]$

$V_x = R_G (1 + g_m R_S + R_S/R_G)$ | $R_{in} = R_G \| R_G (1 + g_m R_S + R_S/R_G)$

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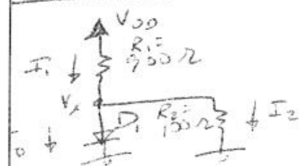
Problem 4:



a) $V_{DD} = 10\,V$. To get first approximation, use constant voltage drop model. If we assume D_1 is OFF, $V_x = 1\,V$ so its safe to assume D_1 is on and $V_x = 0.7\,V$.

Thus $I_1 = \frac{V_{DD} - 0.7}{R_1} = 10.3\,mA$.

Upon further iteration, one would get

V_x	I_1	$I_2 = \frac{V_x}{R_2}$	$I_D = I_1 - I_2$
0.7	10.33 mA	7 mA	3.33 mA
0.718	10.31 mA	7.18 mA	3.12 mA
0.717	10.31 mA	7.17 mA	3.14 mA

Version 1

V_x	I_1	$I_2 = \frac{V_x}{R_2}$	$I_D = I_1 - I_2$
0.7	10.33 mA	7 mA	3.33 mA
0.793	10.23 mA	7.93 mA	2.3 mA
0.783	10.24 mA	7.83 mA	2.41 mA

Version 2

b) $V_{DD} = 1\,V$: If we assume D_1 is OFF, $V_x = 0.1\,V$, which is consistent. If we assume D_1 is on, $I_2 > I_1$, which is impossible. (Note this assumes constant-voltage drop model. For an ideal model, the turn-on voltage $= 0\,V$, so D could be on if you stuck with an ideal model.) Then

$I_D = \frac{1\,V}{900 + 100} = 1\,mA$.

How much current goes through D_1? For $V_x = 0.1\,V$, For $V_x \gg V_T$

$I_D = I_s(e^{V_x/nV_T} - 1) \simeq I_s e^{V_x/nV_T}$

$\simeq \begin{cases} 1.4\,e^{-14}\,A & \text{Version 1} \\ 2.2\,e^{-14}\,A & \text{Version 2} \end{cases}$

These currents are negligible so $\boxed{I_D = 1\,mA}$

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problem 5)

a) $g_m = \frac{i_d}{v_{gs}}$

b) $g_m \triangleq \frac{\partial i_D}{\partial v_{GS}}$

c) if v_{gs} is small enough sothat the slope is roughly constant, both lead to the same result.