ECE115A **Midterm Exam** **Fall 2019**

Name: [redacted]

UID: [redacted]

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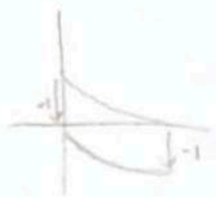
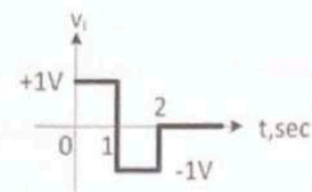
Total of 3 questions, 120 minutes.

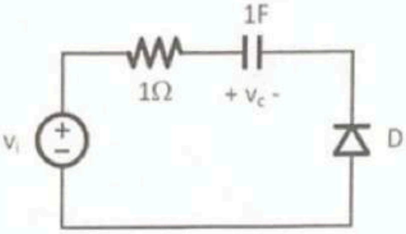
P1 (30)	30
P2 (30)	30
P3 (40)	40
Total (100)	100

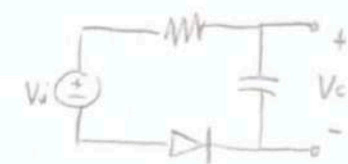
1. The circuit shown below is in zero state at $t = 0$. Calculate and plot the capacitor voltage ($v_c(t)$) for $t \geq 0$ given the input signal shown. The diode is ideal.





$e^{-\frac{t}{\tau}}$





At $t=0$:

D is off due to reverse-active:

→ no current flows, thus $V_c(t)=0$

At $t=1$:

D is on due to forward active:

→ a simple charging capacitor

$$V_c(t) = -1(1-e^{-\frac{t}{\tau}}) \quad \tau = RC = 1$$

$$= -(1-e^{-t}) \qquad t=1: e^{0}-1$$

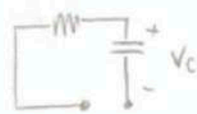
$$= (e^{-t}-1) \qquad t=2: e^{-1}-1$$

$$\rightarrow V_c(t) = e^{-(t-1)}-1$$

(account for the time when it actually starts charging)

At $t=2$:

D is off due to no bias (not forward active).

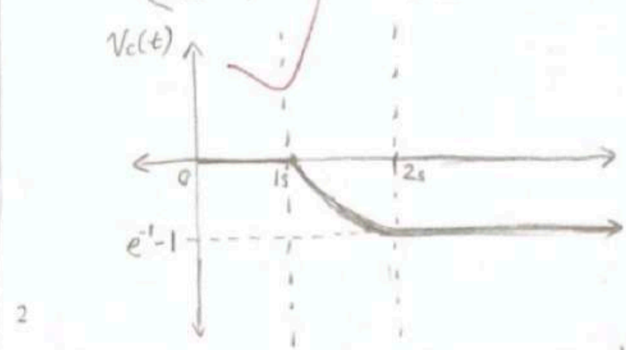


→ again, no current flows, meaning capacitor is unable to discharge:

→ maintains voltage from before:

$V_c(t) = (e^{-1}-1)$ ← here, $t=1$ since it only charged for $t=1$ seconds

$$V_c(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 1 \\ e^{-(t-1)}-1 & \text{for } 1 \leq t \leq 2 \\ e^{-1}-1 & \text{for } t \geq 2 \end{cases}$$



no early effect

2. For the circuit below, let $I_{ES} = 1pA$, $V_A = \infty$, $\beta = 100$, $V_{CE,SAT} = 0.2V$, $R_B = 89k\Omega$.

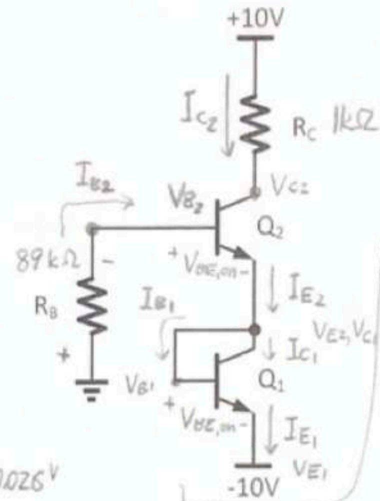
   a. For $R_C = 1k\Omega$, find the exact transistors operating point and the region of operation. $V_T = 0.026^V$

   b. Using $V_{BE,ON} = 0.6V$ approximation, find the maximum value of R_C that puts Q_2 on the edge of saturation.

a)

$I_{ES} = 1pA = 1\times10^{-12}A$

$$I_E = I_{ES}\left(\exp\left(\frac{V_{BE,on}}{V_T}\right)-1\right)$$



KVL:

$$-10 + 2V_{BE,on} + (89k)I_{B2} = 0 \quad \cdots ①$$

Assume forward active:

$I_C = \alpha I_E$, $I_C = \beta I_B$ $\}$ $\left(\frac{\beta}{\beta+1}\right)I_E = \beta I_B$

$$I_B = \frac{I_E}{\beta+1}$$

$V_T = 0.026^V$

$$I_{B2} = \frac{I_{ES}\left(\exp\left(\frac{V_{BE,on}}{V_T}\right)-1\right)}{\beta+1} \quad \cdots ②$$

Using ① & ②:

$$-10 + 2V_{BE,on} + 89000\left(\frac{10^{-12}\left(\exp\left(\frac{V_{BE,on}}{0.026}\right)\right)}{101}\right) = 0$$

→ solve non-linear equation:

$V_{BE,on}$:	Result: ← trying to get 0
0.6	+0.4736
0.65	+54.7499
0.595	−1.1588
0.598	−0.21699
0.599	+0.1217

$\underline{V_{BE,on} = 0.599V}$

Check: $V_{CE_1} = (-9.4010V) - (-10V) = 0.599V > V_{CE,SAT}$ (0.2V) ✓

→ Q_1 also confirmed forward active

$$\boxed{I_{E_1} = I_{E_2} = 9.9888\times10^{-3}A}$$

$$I_{C_1} = \alpha I_{E_1} = \frac{100}{101}I_{E_1}$$

$$\boxed{I_{C_1} = 9.8899\times10^{-3}A}$$

$$I_{B_1} = I_{E_1} - I_{C_1}$$

$$\boxed{I_{B_1} = 9.89\times10^{-5}A}$$

a) continued:

$$-10 + 2(0.599v) + (89k)I_{B2} = 0$$

$$89000\, I_{B_2} = 8.802$$

$$\boxed{I_{B2} = 9.8899\times10^{-5}A}$$

$$I_{C_2} = \beta I_{B_2}$$

$$\boxed{I_{C2} = 9.8899\times10^{-3}A}$$

$$I_{E_2} = (\beta+1)I_{B_2}$$

$$\boxed{I_{E2} = 9.9888\times10^{-3}A}$$

$V_{C_2} = 10 - (1k\Omega)I_{C2}$ $\quad$ $V_{E2} = -10 + V_{BE,on}$

$\boxed{V_{C2} = 0.1101V}$ $\quad$ $\boxed{V_{E2} = -9.4010V}$

Check: $V_{CE_2} = \underset{V_{C2}}{(0.1101V)} - \underset{V_{E2}}{(-9.4010V)} = 9.5111V > V_{CE,SAT}$ (0.2V) ✓

→ Q_2 confirmed forward active

$\boxed{V_{C_1} = V_{B_1} = V_{E2} = -9.4010V}$ $\quad$ $\boxed{V_{E_1} = -10V}$

$$V_{B2} = -89k(I_{B2})$$

$$\boxed{V_{B2} = -8.8020V}$$

$$g_m = \frac{I_C}{V_T}$$

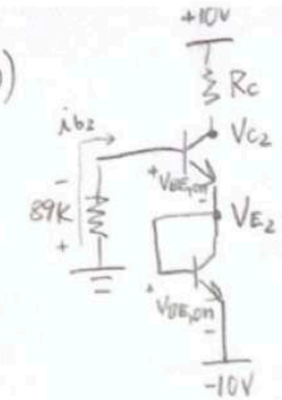
$$\boxed{g_m = 0.3804\ 1/\Omega}$$

$$r_\pi = \frac{\beta}{g_m}$$

$$\boxed{r_\pi = 262.8945\Omega}$$

b) on the back →

b)



$-10 + 2(0.6) + 89k(I_{B2}) = 0$

$89000(I_{B2}) = 8.8v$

$I_{B2} = 9.8876 \times 10^{-5} A$

Edge of saturation:

$I_{C2} = \beta I_{B2}$ ← still holds

$I_{C2} = (100) I_{B2}$

$= 9.8876 \times 10^{-3} A$

$V_{C2} = 10 - I_{C2} R_C \cdots ①$

$V_{E2} = -10 + V_{BE,on} = -9.4V \cdots ②$

Using ① & ②

$V_{CE} = V_{C2} - V_{E2}$

$V_{CE} = (10^v - I_{C2} R_C) - (-9.4^v) \geq 0.2^v$

in order to be on edge of saturation

$-I_{C2} R_C \geq -19.2 v$

$R_C \leq \frac{19.2}{9.8876 \times 10^{-3}}$

$\boxed{R_C = 1941.8261 \Omega}$ ← max Rc value that puts Q2 in edge of saturation

3. In the common-emitter amplifier below, $V_{BE,ON} = 0.6V$, $V_A = \infty$, $\beta = 100$, $V_{CE,SAT} = 0.2V$. $V_T = 0.026V$

a. Find the DC operating point and the transistor region of operation.

b. Calculate the amplifier small signal voltage gain ($\frac{v_o}{v_s}$).

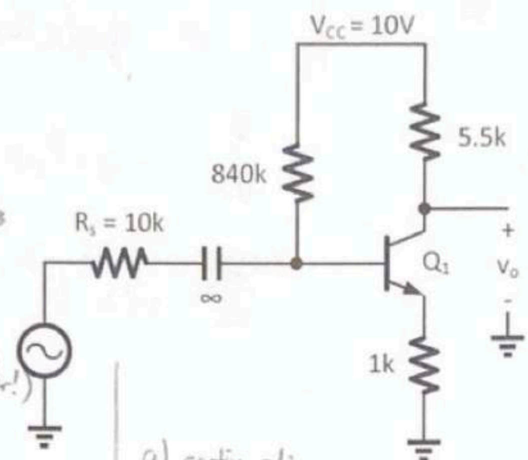
$I_c = \alpha I_E$

$I_c = \beta I_B$

$\alpha I_E = \beta I_B$

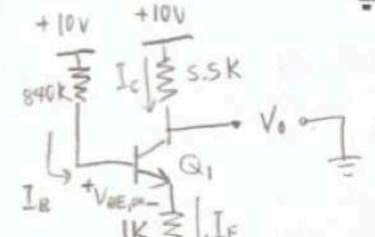
$\frac{\beta}{\beta+1} I_E = \beta I_B$

$I_E = (\beta+1) I_B$



→ Assume forward active: (check later!)

a) DC:



KVL:

$10 - 840000(I_B) - V_{BE,on} - 1000(I_E) = 0$

$10 - 840000(I_B) - 0.6 - 1000(101 \cdot I_B) = 0$

$9.4 = I_B(840000 + 101000)$

$9.4 = I_B(941000)$

$\boxed{I_B = 9.9894 \times 10^{-6} A}$

$\boxed{I_C = \beta I_B = 9.9894 \times 10^{-4} A}$

$\boxed{I_E = (\beta+1) I_B = 1.0089 \times 10^{-3} A}$

a) continued:

$V_E = I_E(1k)$

$\boxed{V_E = 1.0089 V}$

$V_C = 10 - I_c(5.5k)$

$\boxed{V_C = 4.5058 V}$

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Check:

$V_{CE} = V_C - V_E = (4.5058) - (1.0089)$

$= 3.4969 > V_{CE,SAT}$

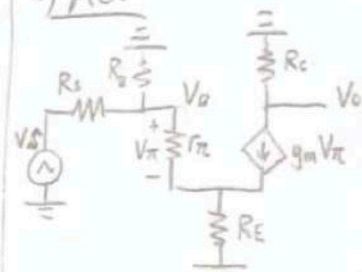
→ Q_1 confirmed to be forward-active

$V_B = 10 - I_B(840K)$

$V_B = 10 - 8.3911$

$\boxed{V_B = 1.6089 V}$

b) AC:



$V_B = \frac{R_B \| (r_\pi + (\beta+1)R_E)}{R_B \| (r_\pi + (\beta+1)R_E) + R_S} V_S$

$\underbrace{r_\pi + (\beta+1)R_E}_{R_{tot}} = 103602.7589 \Omega$

$R_B \| R_{tot} = 92227.7056 \Omega$

$V_B = 0.9021791603 V_S$

$\frac{V_B}{V_S} = 0.9021791603$

→ see backside

$g_m = \frac{I_c}{V_T}$, $r_\pi = \frac{\beta}{g_m}$

$V_T = 0.026 V$

$g_m = 0.03842076923$

$\boxed{g_m = 0.0384 \ 1/\Omega}$

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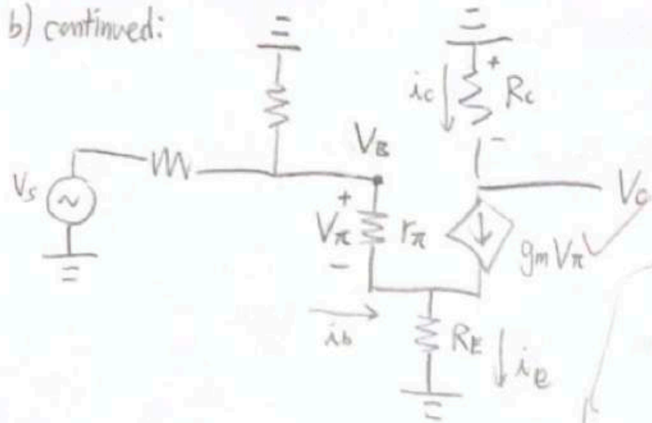
$\boxed{r_\pi = 2602.7589 \Omega}$

b) continued:



$$\frac{V_o}{V_s} = \frac{V_B}{V_s} \cdot \frac{V_\pi}{V_B} \cdot \frac{V_o}{V_\pi}$$

KVL:

$$V_B = R_E i_e + V_\pi \cdots (1)$$

KCL:

$$\frac{V_\pi}{r_\pi} + g_m V_\pi = i_e$$

$$V_\pi\left(\frac{1}{r_\pi} + g_m\right) = i_e \cdots (2)$$

Using (1) & (2):

$$V_B = R_E\left(V_\pi\left(\frac{1}{r_\pi} + g_m\right)\right) + V_\pi$$

$$V_B = V_\pi\left(R_E\left(\frac{1}{r_\pi} + g_m\right)\right) + V_\pi$$

$$V_B = V_\pi\left(\frac{R_E}{r_\pi} + g_m R_E + 1\right)$$

$$\frac{V_B}{V_\pi} = \left(\frac{1000\,\Omega}{2602\,\Omega} + (0.0384)(1000\,\Omega) + 1\right)$$

$$\frac{V_B}{V_\pi} = 39.7842$$

$$\frac{V_\pi}{V_B} = 0.02513560174$$

$$V_o = -i_c R_c$$

$$= -g_m V_\pi R_c$$

$$\frac{V_o}{V_\pi} = -g_m R_c$$

$$\frac{V_o}{V_\pi} = -211.2$$

$$\rightarrow \frac{V_o}{V_s} = \frac{V_B}{V_s} \cdot \frac{V_\pi}{V_B} \cdot \frac{V_o}{V_\pi}$$

$$= (0.9022)(0.0251)(-211.2)$$

$$\boxed{= -4.7827}$$

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Check:

$$\frac{V_o}{V_i} = -\alpha \frac{R_c}{R_E + \frac{r_\pi + R_s}{\beta + 1}}$$

$$= -\left(\frac{100}{101}\right)\frac{(5500)}{(1000) + \frac{(2602)+(10000)}{101}}$$

$$= -4.8414$$

$$\approx -4.7827$$