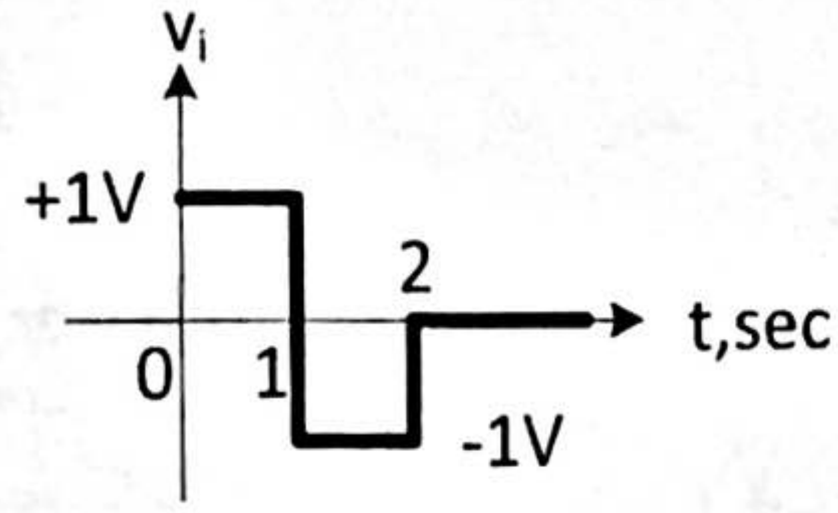
Name: [redacted]

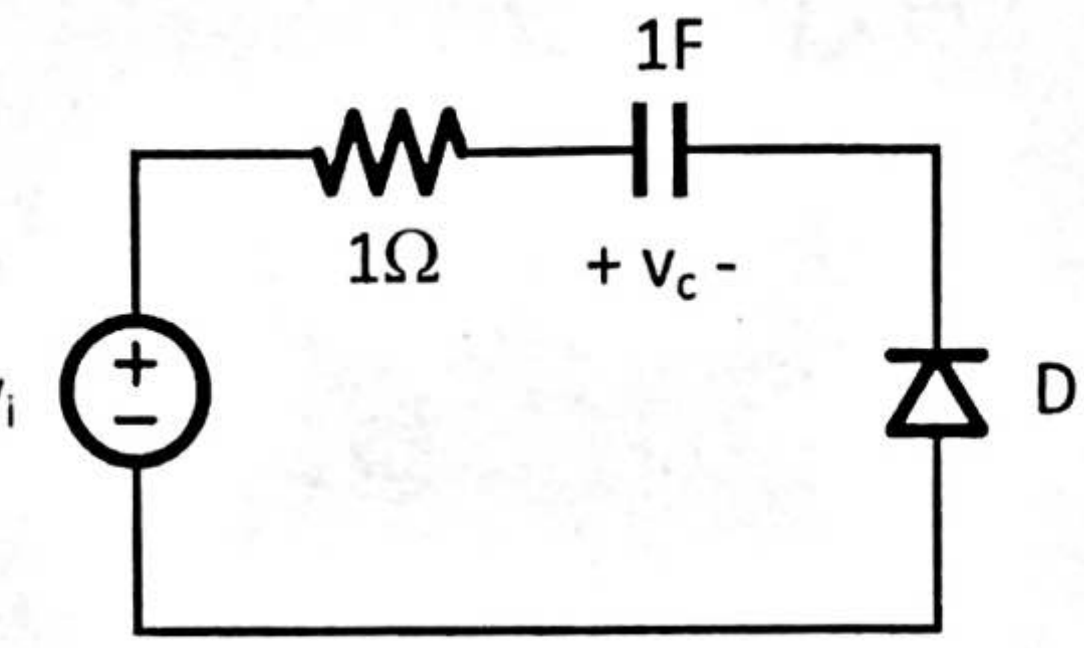
UID: [redacted]

Total of 3 questions, 120 minutes.

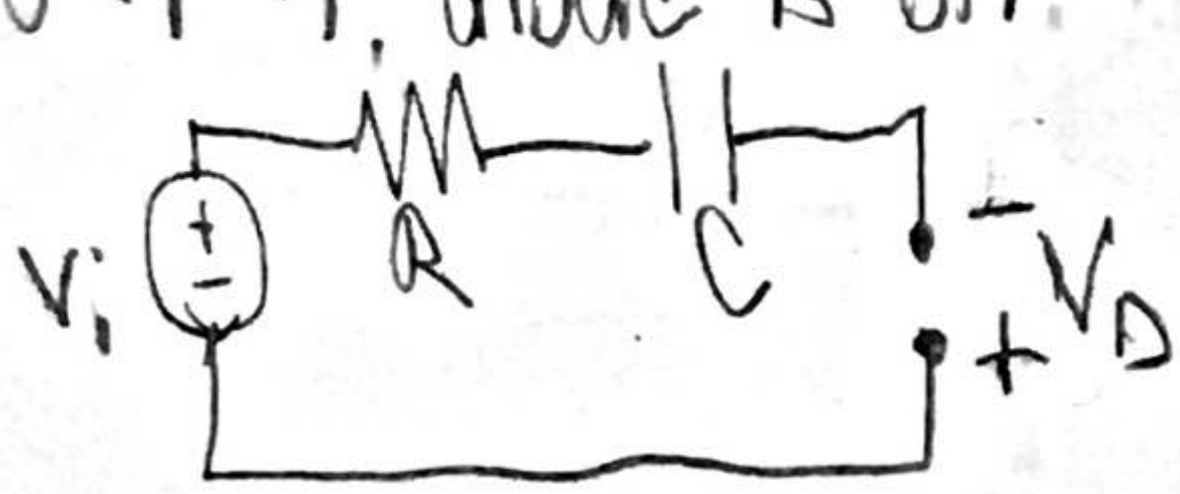
P1 (30)	30
P2 (30)	30
P3 (40)	40
Total (100)	100

1. The circuit shown below is in zero state at $t = 0$. Calculate and plot the capacitor voltage ($v_c(t)$) for $t \geq 0$ given the input signal shown. The diode is ideal.





for $0<t<1$, diode is off.

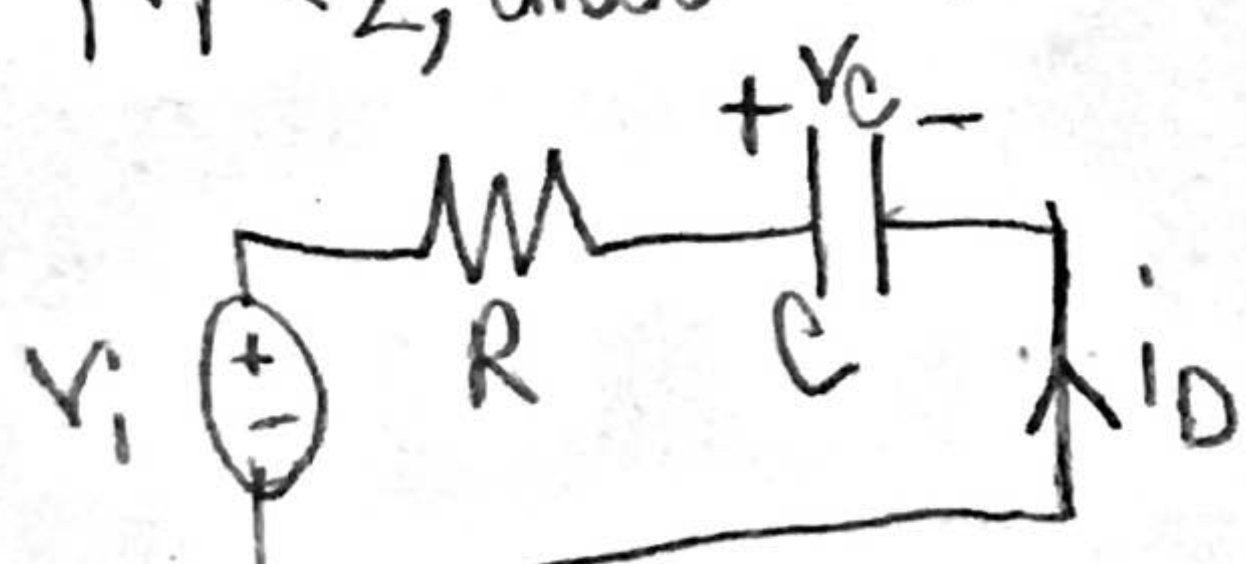


$\underline{V_c \stackrel{?}{=} 0}$

$V_i = -V_D$ $V_D = -1$ so diode must be off

diode turns on when $v_i < 0$

for $1<t<2$, diode is on

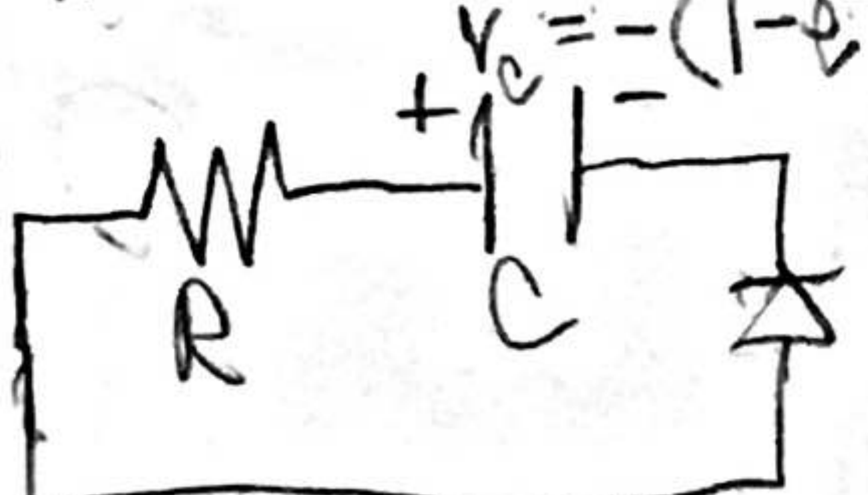


$v_c(t) = (v_i - v_f)e^{-t/RC} + v_f$ $\quad RC = 1$, $v_i = 0$, $v_f = v_i = -1^V$ (zero-state)

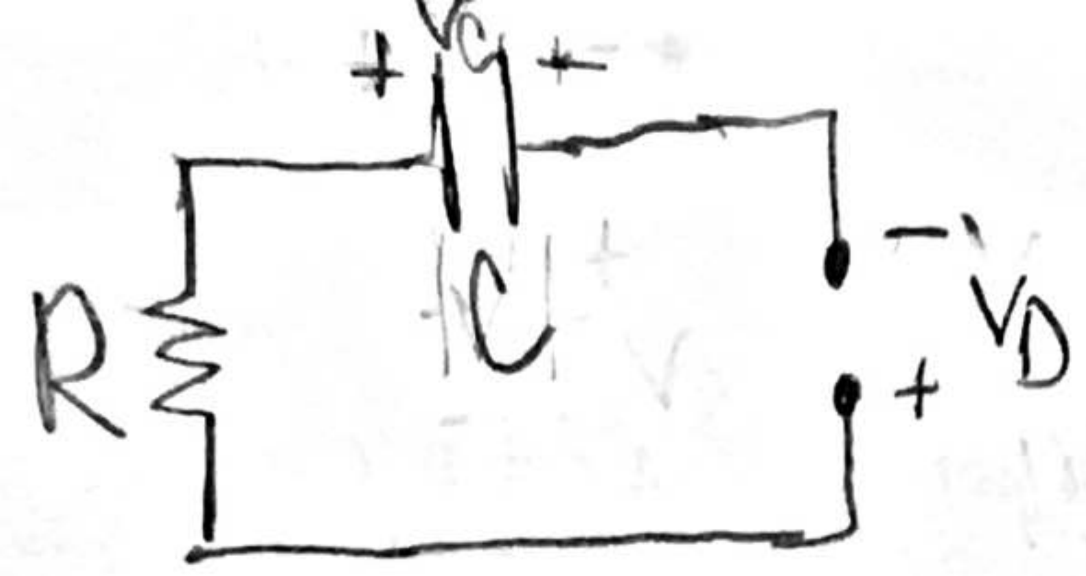
$= e^{-t} - 1 = -(1-e^{-t})$

needs to be time shifted: $\underline{-(1-e^{-(t-1)}) = v_c(t)}$

for $t>2$, $v_i = 0$



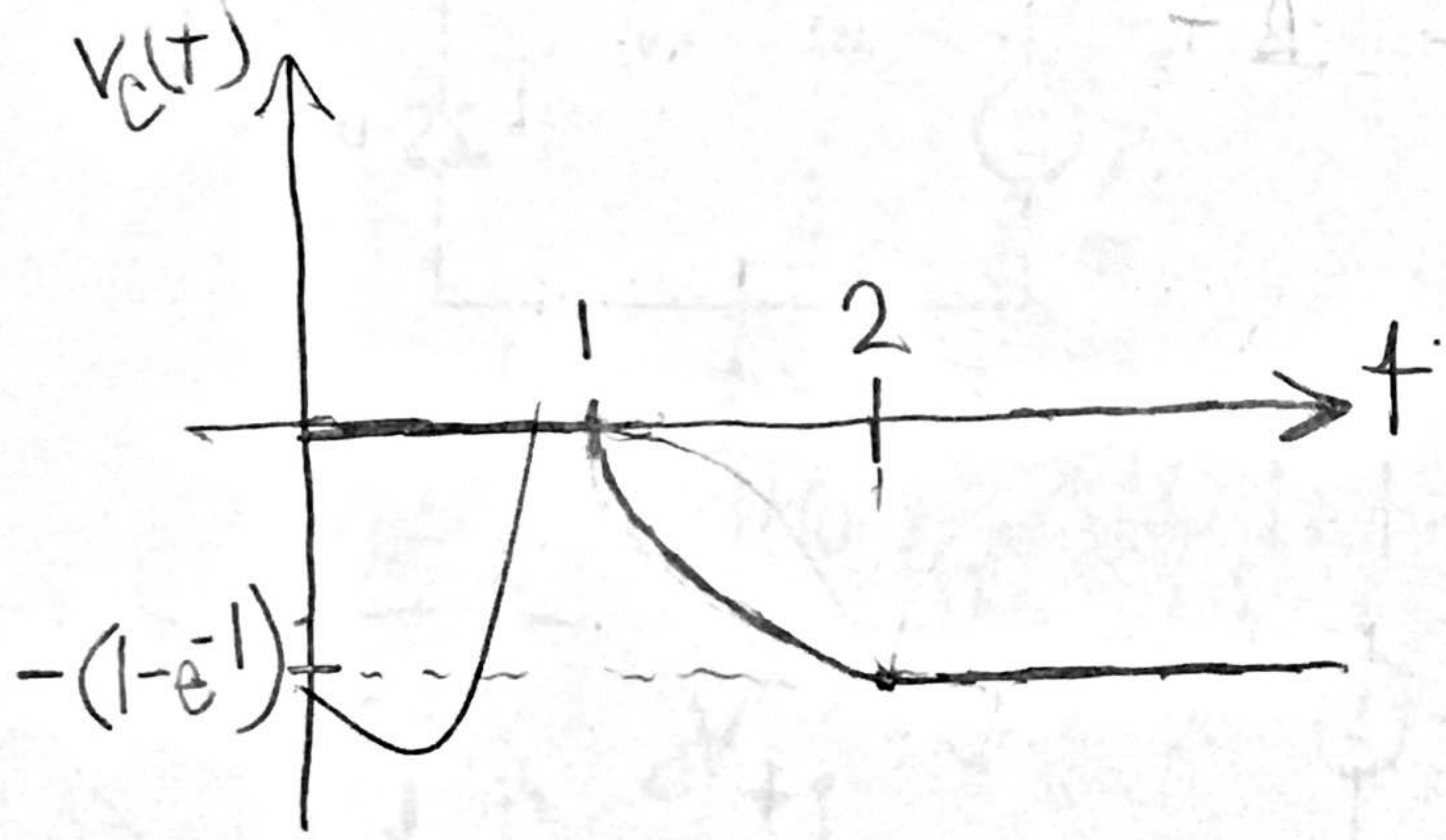
$v_c < 0$, so current flows clockwise, so diode is off equivalently,



$v_C = v_D = -(1-e^{-1})$

$v_D < 0$ so diode must be off

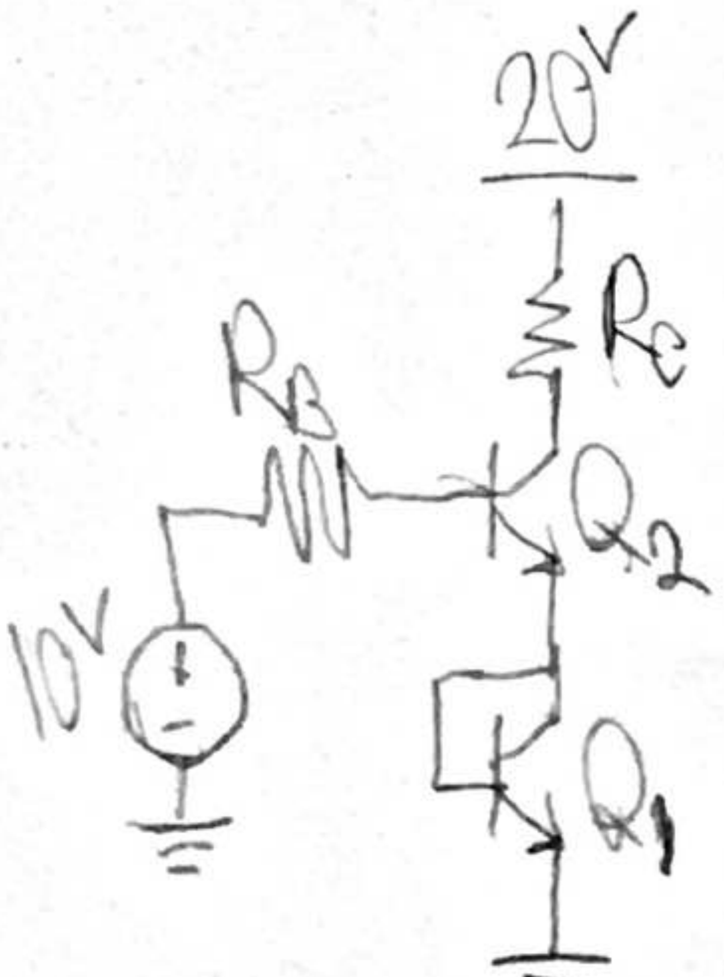
$$v_C(t) = \begin{cases} 0 & 0 < t \leq 1 \\ -(1-e^{-(t-1)}) & 1 < t \leq 2 \\ -(1-e^{-1}) & t > 2 \end{cases}$$



2. For the circuit below, let $I_{ES} = 1pA$, $V_A = \infty$, $\beta = 100$, $V_{CE,SAT} = 0.2V$, $R_B = 89k\Omega$.

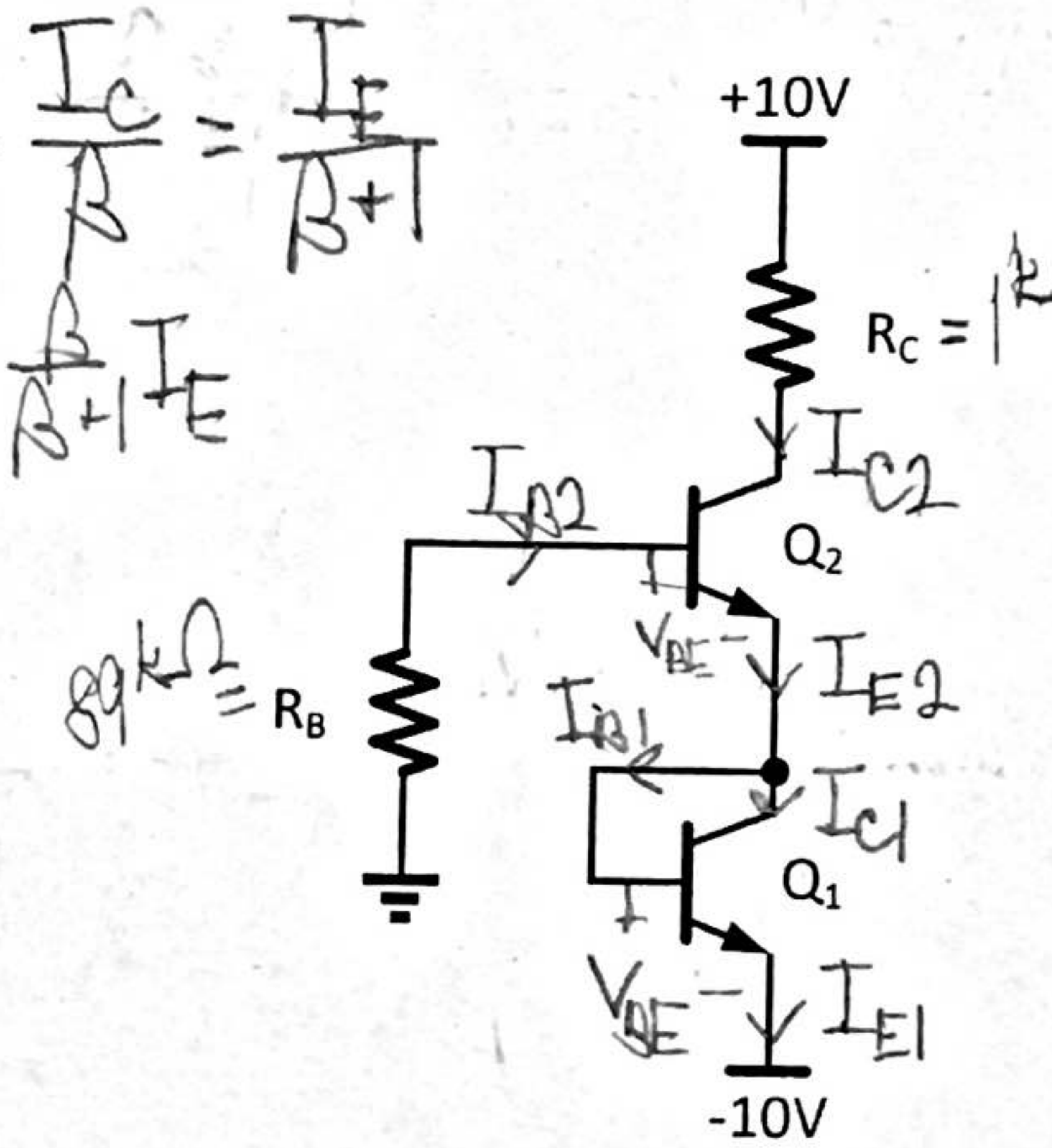
   a. For $R_C = 1k\Omega$, find the exact transistors operating point and the region of operation.

   b. Using $V_{BE,ON} = 0.6V$ approximation, find the maximum value of R_C that puts Q_2 on the edge of saturation.



$I_B = \frac{I_C}{\beta} = \frac{I_E}{\beta+1}$

$I_C = \frac{\beta}{\beta+1} I_E$



$I_C = I_{CS}(e^{V_{BE}/V_T} - 1)$

$= \frac{\beta}{\beta+1} I_{ES}(e^{V_{BE}/V_T} - 1)$

a) $10^V - I_C R_C - V_{CE,2} - V_{BE,on} = -10^V$ not used ~~$10^V - I_C R_C - V_{BE} - I_B R_B = 0$~~

KVLs $\longrightarrow 0 - I_{B2} R_B - V_{BE,on} - V_{BE,on} = -10^V$

$I_{B2} = \frac{I_{C2}}{\beta}$ $\quad \frac{10 - I_{B2} R_B}{2} = V_{BE,on} = 5 - \frac{I_{ES}(e^{V_{BE,on}/V_T} - 1) R_B}{(\beta+1) \cdot 2}$

$5 - \frac{I_{ES}(e^{V_{BE,on}/V_T} - 1) R_B}{2(\beta+1)} - V_{BE,on} = 0$ graph and find zero

$V_{BE,on} = 0.596356 \approx 0.596^V$

$I_{C2} = \frac{\beta}{\beta+1} I_{ES}(e^{V_{BE,on}/V_T} - 1) \Rightarrow \boxed{I_{C2} = 9.90 \text{ mA}}$

$I_B = \frac{I_C}{\beta}$ $\quad I_{C1} = I_{E2}$ b/c same $V_{BE,on}$ so $\boxed{I_{C1} = 9.90 \text{ mA}}$

$I_E = \frac{\beta+1}{\beta} I_C$ $\quad I_{B1} = I_{B2} = 99.0\ \mu A$ $\quad I_{E1} = I_{E2} = 9.99 \text{ mA}$

KVL: $10^V - I_{C2}R_C - V_{CE,2} - V_{BE,on} = -10^V$

$V_{CE,2} = 20^V - I_{C1}R_C - V_{BE,on} = 9.51^V > V_{CE,sat}$

$\boxed{Q_2 \text{ is in forward active}}$

KVL: $-I_{B2}R_B - V_{BE,on} - V_{CE,1} = -10^V$

$10^V - I_{B2}R_B - V_{BE,on} = V_{CE,1} = 0.596^V > V_{CE,sat}$

$\boxed{Q_1 \text{ is in forward active}}$

b) for edge of saturation, $V_{CE,2} = V_{CE,sat}$

$10^V - I_{C2}R_{C,max} - V_{CE,sat} - V_{BE,on} = -10^V$

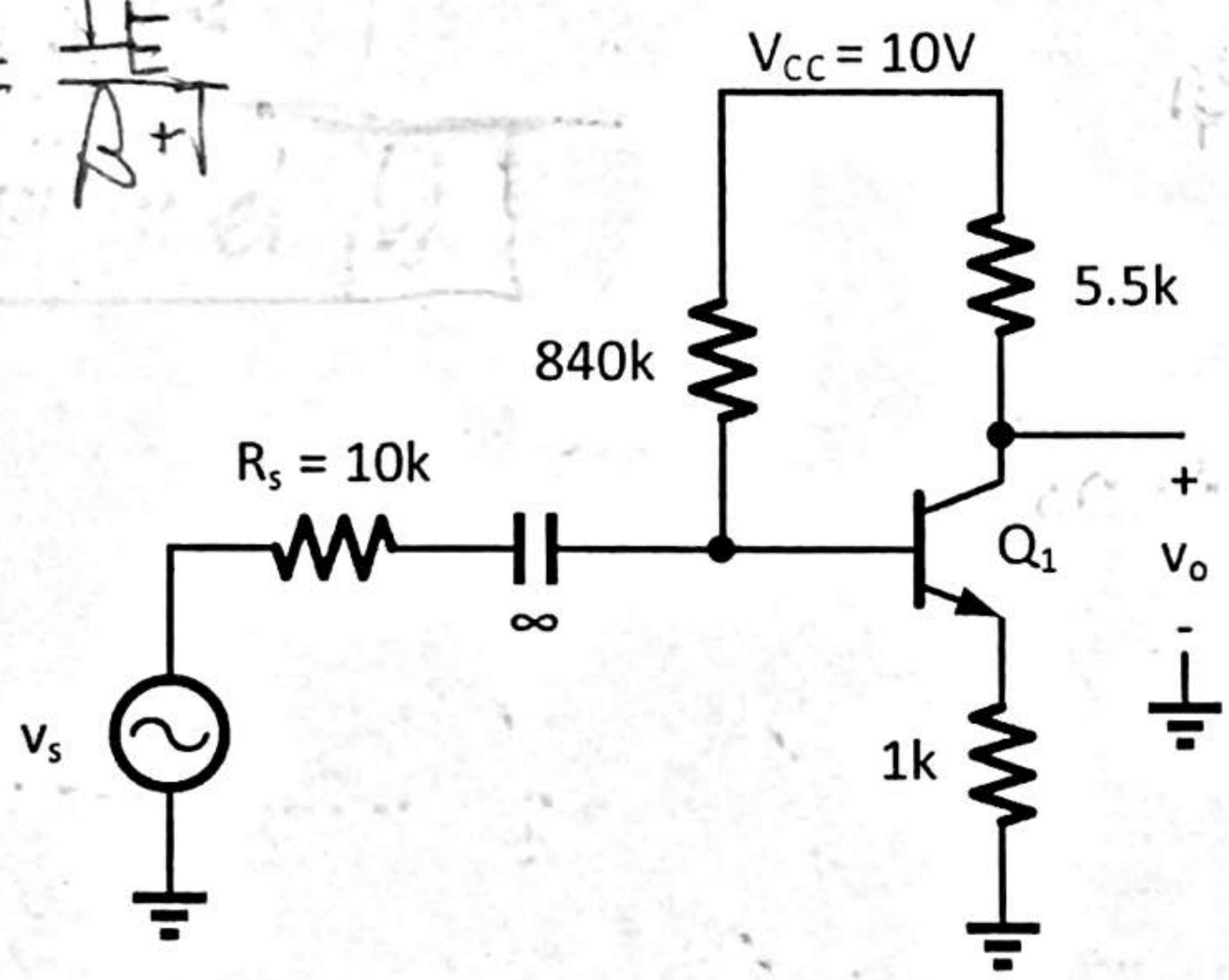
$$\frac{20^V - V_{CE,sat} - V_{BE,on}}{I_{C2}} = \boxed{R_{C,max} = 1940^\Omega}$$

3. In the common-emitter amplifier below, $V_{BE,ON} = 0.6V$, $V_A = \infty$, $\beta = 100$, $V_{CE,SAT} = 0.2V$.

a. Find the DC operating point and the transistor region of operation.

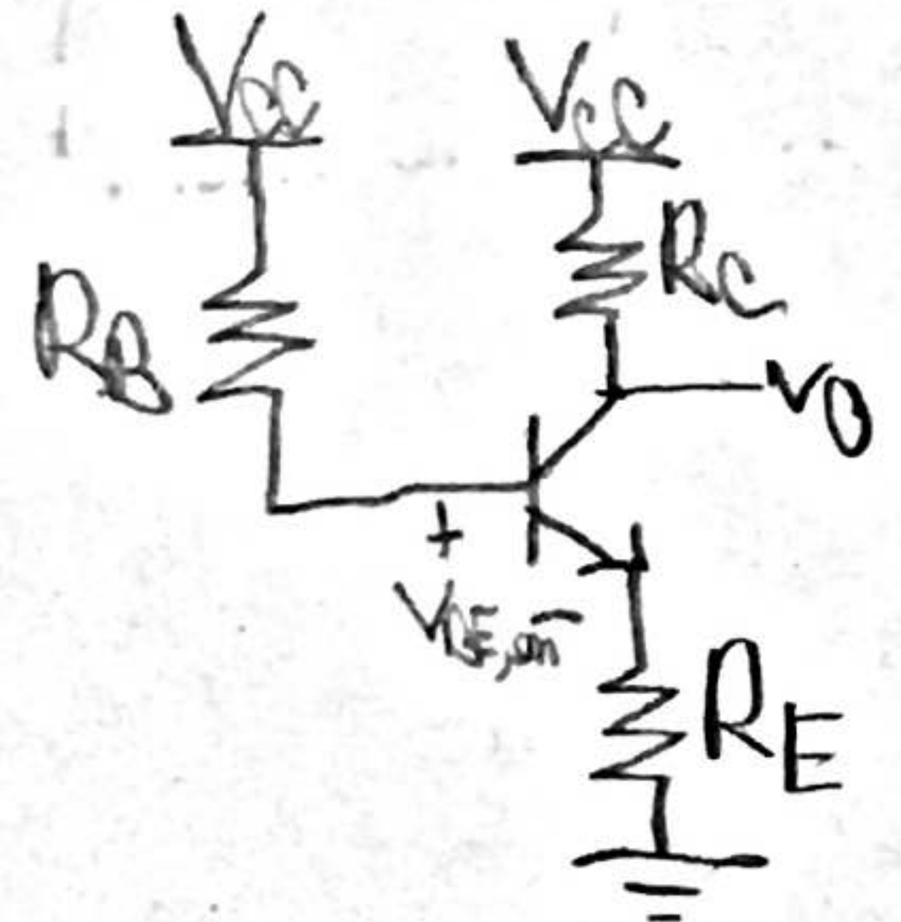
b. Calculate the amplifier small signal voltage gain ($\frac{v_o}{v_s}$).

$I_B = \frac{I_C}{\beta} = \frac{I_E}{\beta+1}$



$I_B(\beta+1) = I_E$

a)



$V_{CC} - I_B R_B - V_{BE,on} - I_E R_E = 0$

$V_{CC} - V_{BE,on} = I_B(R_B + (\beta+1)R_E)$

$I_B = \frac{V_{CC} - V_{BE,on}}{R_B + R_E(\beta+1)} = 9.99\ \mu A$

$\boxed{I_C = \beta I_B = 0.999\ mA}$

$I_E = (\beta+1)I_B = 1.01\ mA$

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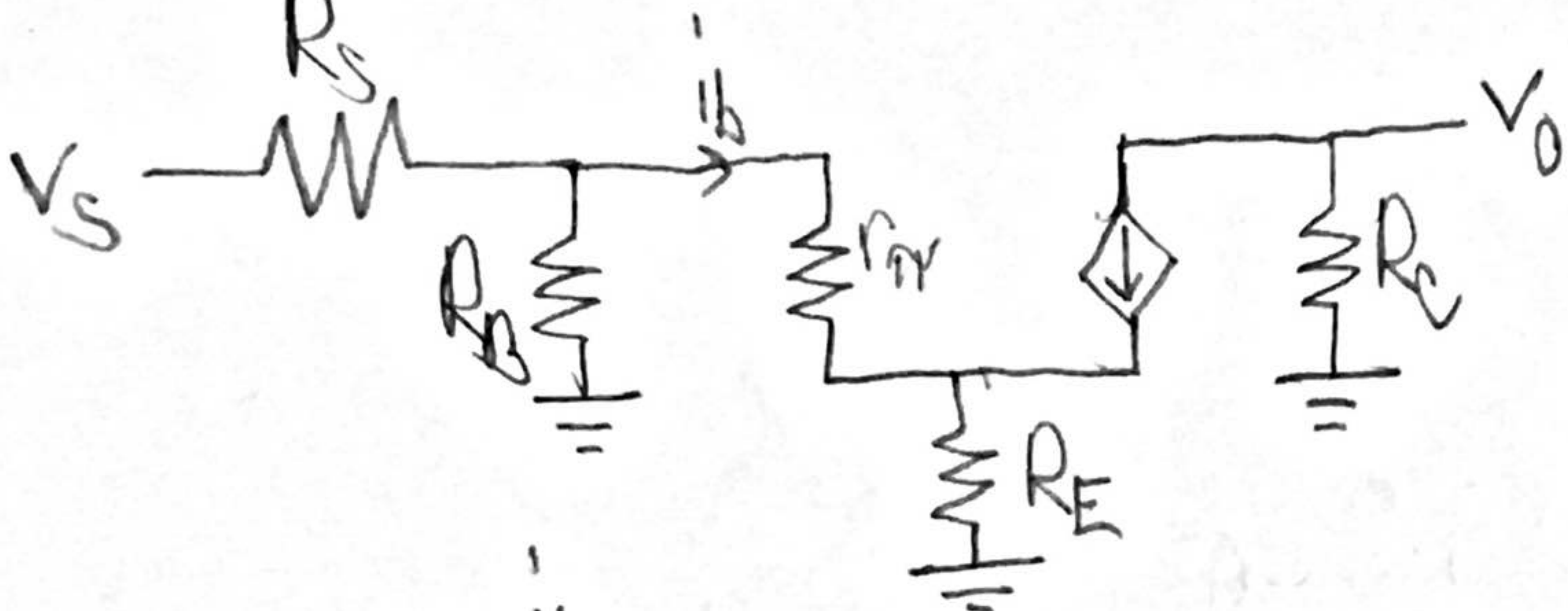
$10^V - I_C R_C - V_{CE} - I_E R_E = 0$

$V_{CE} = 10^V - I_C R_C - I_E R_E = 3.50^V > V_{CE,sat}$

$\boxed{Q_1 \text{ is in forward active}}$

b) 

$r_\pi = \frac{\beta}{g_m} = \frac{V_T}{I_B} = 2593\,\Omega$

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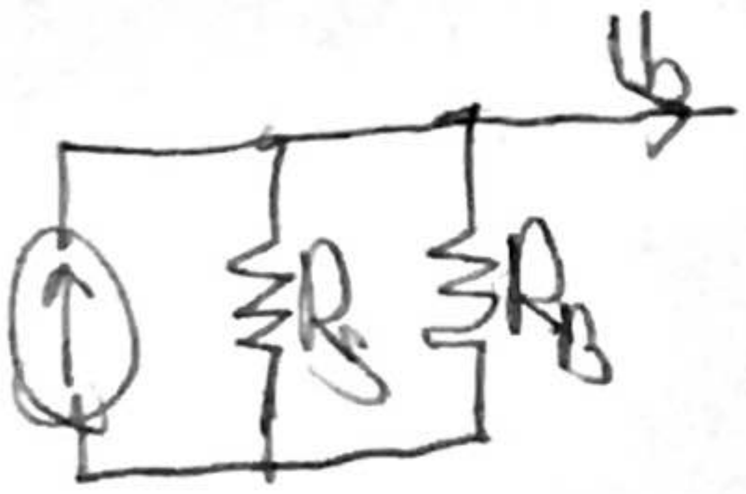
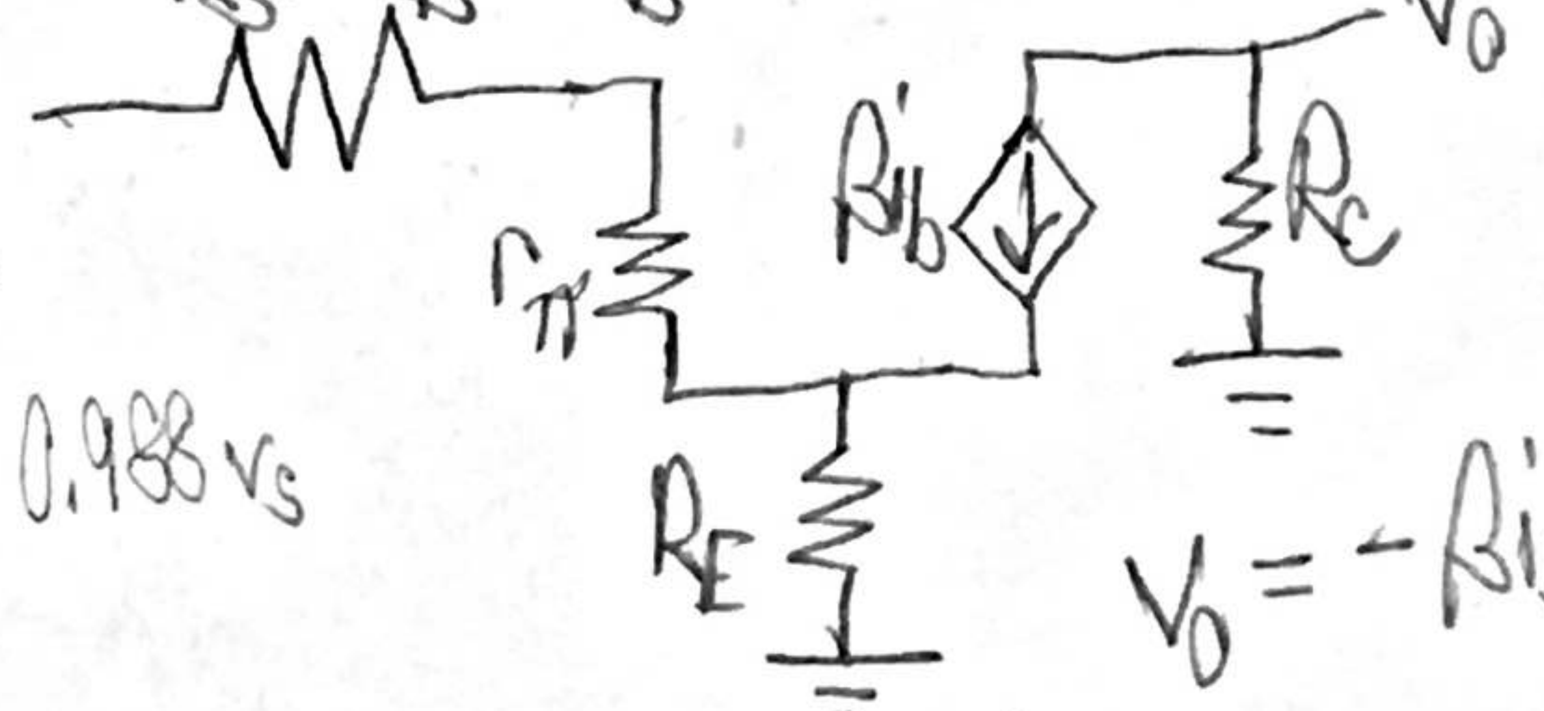
$\frac{V_S}{R_S}$ 

$\frac{V_S}{R_S} \cdot \frac{R_S R_B}{R_S+R_B} = \frac{R_B}{R_S+R_B} V_S$

$R_S \| R_B = R_B' \approx 9882\,\Omega$

$V_S \frac{R_B}{R_S+R_B} = V_S' \approx 0.988\, V_S$



$V_0 = -\beta i_b R_C$

$V_S' - i_b R_B' - i_b r_\pi - (\beta+1) i_b R_E = 0$

$\frac{V_S'}{R_B' + r_\pi + (\beta+1)R_E} = i_b = V_S \frac{\frac{R_B}{R_S+R_B}}{(R_S \| R_B) + r_\pi + (\beta+1)R_E} \approx \frac{V_S}{(\beta+1)R_E}$

$$\boxed{\frac{V_0}{V_S} = -\frac{\beta \frac{R_B R_C}{R_S+R_B}}{(R_S \| R_B) + r_\pi + (\beta+1)R_E} = -4.79}$$

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$R_B = 840\,k\Omega$, $R_C = 5.5\,k\Omega$, $r_\pi = 2593\,\Omega$, $R_E = 1\,k\Omega$, $R_S = 10\,k\Omega$

sanity check: using i_b approximation, $\frac{V_0}{V_S} = \frac{-\beta R_C}{(\beta+1)R_E} = -\alpha \frac{R_C}{R_E} = -5.55$

gain approximation for CE w/degeneration

Good.