EE114 Midterm Exam

Notes: - For all problems please circle or otherwise clearly indicate your answers!

The use of calculators or other electronic devices with calculator-like functionality is not permitted on this test.

1. Signal processing basics (x points)

1a) Sampling and Reconstruction (x points)

Suppose that a signal consisting of a pure cosine at 4000 Hz is subject to ideal sampling at a sampling frequency of 5000 Hz, and then subject to reconstruction using an ideal low pass filter with a cutoff of 2500 Hz. What is the frequency of the resulting cosine?

Answer:

1000 Hz. The spectrum of the original cosine is a pair of delta functions at plus and minus 4 KHz. After sampling, these delta functions will be replicated at intervals of 5 KHz. The delta function locations in KHz will then be

. . . .-11,-9,-6, -4, -1,1, 4, 6, 9, 11, 14, 16 . . .

After application of the low pass filter, only those delta functions at plus and minus 1KHz will remain.

1b) 1D Convolution (x points)

Figure 1 below shows a triangle function that starts on the horizontal axis at position 4, rises to a maximum height of 1 at 5, and then declines again until it reaches a value of zero again horizontal axis position 6.

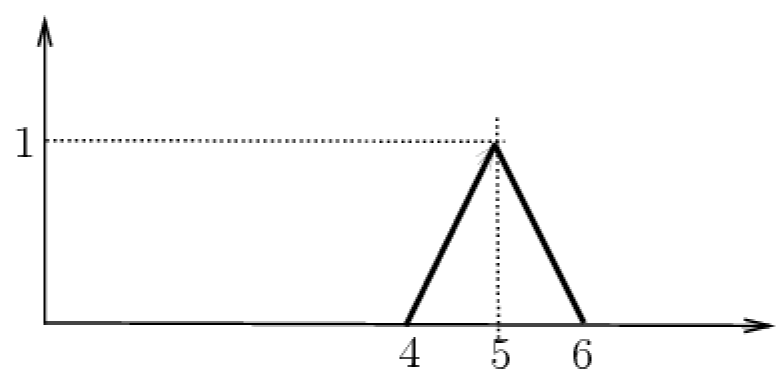

Figure 1

Determine 1) the location of the maximum of the convolution of this function with itself, and 2) the value of the maximum.

Answer:

It is easiest to consider the two questions asked in the problem separately. With respect to the location of the maximum, this can be solved in the time domain by noting that the maximum will occur when the two triangles being convolved overlap exactly, which happens at location 10.

For finding the maximum we can treat the function as if it is centered on the x axis, since the shift will only change the location, not the shape, of the output. At the point of maximum overlap, the product of the functions will have an area given by:

$$
\begin{aligned}
&2\int_{-1}^{0}(x+1)^2\,dx \\
&= 2\left[\frac{x^3}{3} + x^2 + x\Big|_{-1}^{0}\right] \\
&= 2/3
\end{aligned}
$$

1c) Low pass filter (x points)

Suppose that a speech waveform is passed through an analog low pass filter that removes all frequencies above 4KHz. Will speech emerging from this filter generally be intelligible?

Answer: Yes. As discussed in lecture, this is actually more bandwidth that is often present in mobile phone voice calls. The speech will generally be intelligible, though the quality will not be great.

2. All-pole vocal tract system model (x points)

For parts 2a, 2b, and 2c of this problem consider the all-pole plot shown in Figure 1 below. The figure shows a system with two pairs of complex conjugate poles. The first pair is at a distance of a from the origin at radial angles $\pm p / 4$, and the second pair is at a distance of b from the origin at radial angles $\pm 3p / 4$. The unit circle is indicated by the dotted line, and, as is customary, the horizontal and vertical axes are real and imaginary respectively.

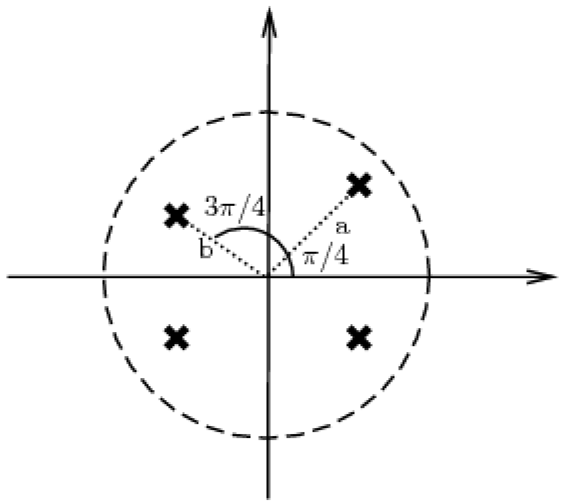

Figure 2

2a) (x points) Provide an expression for $H(z)$ for the system shown in figure 2.

2b) (x points) Provide a plot of the magnitude response $|H(w)|$ of the system in figure 2 over the interval from 0 to p. Your plot must have horizontal axis labels. Your plot does not need to have vertical axis labels, but it should correctly indicate the approximate relative heights of the peaks.

2c) (x points) If the sampling rate is 8 KHz, what are the formant frequencies for the system shown in figure 2?

2d) (x points) Now, consider the all-pole plot shown in figure 3 below. What is the relationship between a and b that must hold in figure 3 such that $\frac{|H(p)|}{|H(0)|} = 2$.

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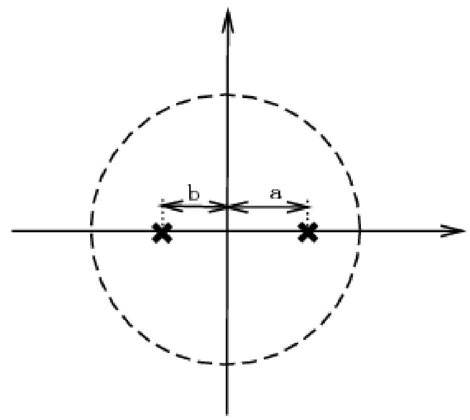


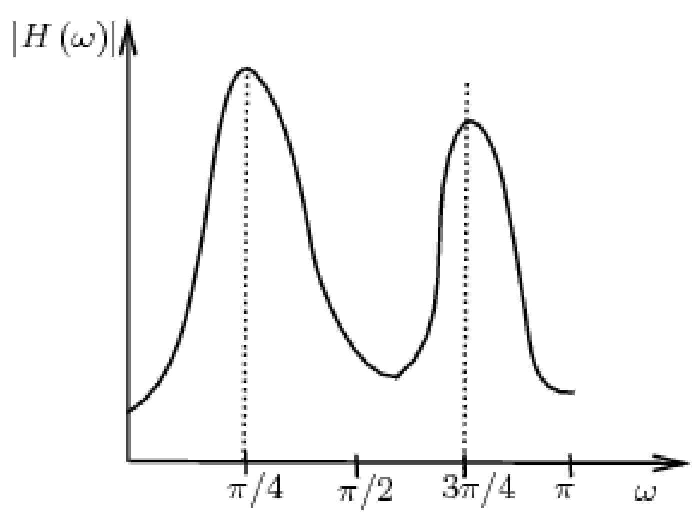
Figure 3

Answer:

2a)

$$H(z) = \frac{1}{\left(1 - ae^{-j\frac{p}{4}}z^{-1}\right)\left(1 - ae^{j\frac{p}{4}}z^{-1}\right)\left(1 - be^{-j\frac{3p}{4}}z^{-1}\right)\left(1 - be^{j\frac{3p}{4}}z^{-1}\right)}$$

2b)



2c) There are two formant frequencies (one conjugate pair of poles for each formant).

$$F_i = \frac{w_i F_s}{2p}, i = 1, 2$$

$$\left|H(w)\right| \; F_1 = \frac{w_1 F_s}{2p} = \frac{p/4(8000)}{2p} = 1000\text{Hz}$$

$$F_2 = \frac{w_2 F_s}{2p} = \frac{3p/4(8000)}{2p} = 3000\text{Hz}$$

2d) Poles are located at $w = 0$ and $w = p$

$$H(z) = \frac{1}{\left(1 - ae^{-j0} z^{-1}\right)\left(1 - be^{jp} z^{-1}\right)}$$

$$= \frac{1}{\left(1 - az^{-1}\right)\left(1 + bz^{-1}\right)}$$

$$H(w) = H(z)\Big|_{z=e^{jw}}$$

$$= \frac{1}{\left(1 - ae^{-jw}\right)\left(1 + be^{-jw}\right)}$$

For $w = 0$,

$$\left|H(0)\right| = \frac{1}{(1-a)(1+b)}$$

For $w = p$,

$$\left|H(p)\right| = \frac{1}{(1+a)(1-b)}$$

Therefore,

$$\frac{|H(p)|}{|H(0)|} = 2$$

$$\frac{(1-a)(1+b)}{(1+a)(1-b)} = 2$$

$$1+b-a-ab = 2(1-b+a-ab)$$

$$b = \frac{1+3a}{3+3a} \text{ OR } a = \frac{3b-1}{3-3b}$$

3. 1D DFT (x points)

For this problem, assume that the 1D DFT is defined as follows

$$v(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}x(n)e^{-j2pkn/N}$$

Given the following $N=8$ transform pair:
$[a \quad b \quad c \quad d \quad e \quad f \quad g \quad h] \leftrightarrow [A \quad B \quad C \quad D \quad E \quad F \quad G \quad H]$.
What is the $N=16$ transform of $[a \quad 0 \quad b \quad 0 \quad c \quad 0 \quad d \quad 0 \quad e \quad 0 \quad f \quad 0 \quad g \quad 0 \quad h \quad 0]$ in terms of A, B, C, D, E, F, G and H ?

Answer:
Denote $x_1(n)=[a \quad b \quad c \quad d \quad e \quad f \quad g \quad h]$ and
$x_2(n)=[a \quad 0 \quad b \quad 0 \quad c \quad 0 \quad d \quad 0 \quad e \quad 0 \quad f \quad 0 \quad g \quad 0 \quad h \quad 0]$

From the problem statement, we know that
$v_1(k)=\frac{1}{\sqrt{8}}\sum_{n=0}^{7}x_1(n)e^{-j2pkn/8}$ gives $[A \quad B \quad C \quad D \quad E \quad F \quad G \quad H]$.
The transform of $x_2(n)$ is

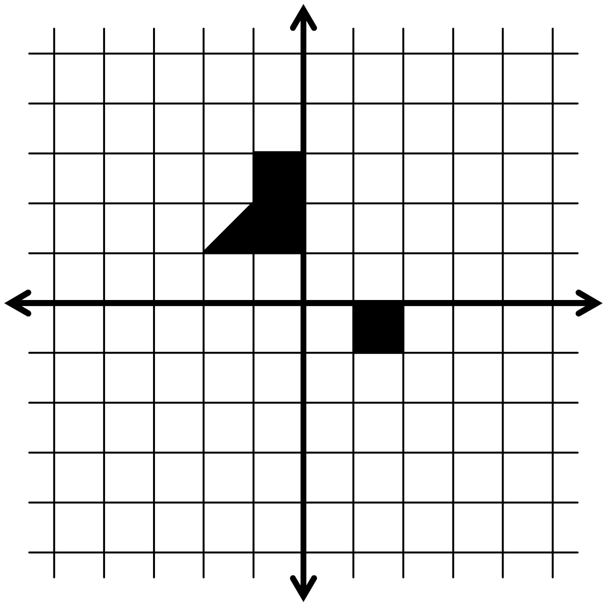
$$v_2(k)=\frac{1}{\sqrt{16}}\sum_{n=0}^{15}x_2(n)e^{-j2pkn/16}$$
$$=\frac{1}{\sqrt{16}}\sum_{n=0}^{7}x_2(2n)e^{-j2pk2n/16}$$
$$=\frac{1}{\sqrt{16}}\sum_{n=0}^{7}x_1(n)e^{-j2pkn/8}$$
$$=\frac{1}{\sqrt{2}}v_1(k)$$

where k is evaluated over the range from 0 to 7. Note that evaluating $v_1(k)$ outside the range [0, 7] involves using the periodic properties of DFTs – in other words, $v_1(8+k)=v_1(k)$. Thus,

$$[a \quad 0 \quad b \quad 0 \quad c \quad 0 \quad d \quad 0 \quad e \quad 0 \quad f \quad 0 \quad g \quad 0 \quad h \quad 0] \leftrightarrow$$
$$\frac{1}{\sqrt{2}}[A \quad B \quad C \quad D \quad E \quad F \quad G \quad H \quad A \quad B \quad C \quad D \quad E \quad F \quad G \quad H]$$

4. 2D Convolution (x points)

Consider the continuous 2D function $f(x, y)$ given below, which has value 1 where the dark regions are located and 0 elsewhere.



Make a sketch showing the boundaries of non-zero regions of the self-convolution of $f(x, y)$. Be sure to place your sketch on a set of marked axis such that relevant dimensions and positions of features are clearly indicated.

Answer: