EE113: Digital Signal Processing
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MIDTERM EXAMINATION

(Closed Book)

1. (40 PTS) A relaxed system is described by the difference equation

$$y(n) = \frac{1}{2}y(n-1) + x(2n), \quad n \geq 0$$

where $x(n)$ denotes the input sequence and $y(n)$ denotes the output sequence. Prove or give counter-examples:

   (a) (8 PTS) Is the system linear?
   (b) (8 PTS) Is the system time-invariant?
   (c) (8 PTS) Is the system causal?
   (d) (8 PTS) Is the system BIBO stable?
   (e) (8 PTS) Express $y(n)$ in terms of the samples of $x(n)$ only. Assume $x(n) = 0$ for $n < 0$.

2. (30 PTS) The following is known about a discrete-time linear time-invariant system with input $x(n)$ and output $y(n)$:

   – if $x(n) = (-1)^n$ for all n, then $y(n) = 0$ for all n.
   – if $x(n) = \left(\frac{1}{2}\right)^n u(n)$ for all n, then $y(n)$ has the form

$$y(n) = \delta(n) + a\left(\frac{1}{4}\right)^n u(n)$$

   for all n, where a is a constant.

   For this system:

   (a) (15 PTS) Determine the value of the constant a and find the system function $H(z)$.
   (b) (15 PTS) Find the step response of the system, i.e. find the output $y(n)$ corresponding to the input $x(n) = u(n)$.

3. (30 PTS) A system's output is defined by the relationship

$$y(n) = x(n) - 0.1(x(n-1) + y(n-1)) + 0.06y(n-2)$$

   (a) (10 PTS) What is the closed-form impulse response of this system?
   (b) (10 PTS) Given that $y(-1) = 1$ and $y(-2) = 1$, what is the closed-form system output $y(n)$ for $n \geq 0$ given that $x(n) = 0$ for all n?
   (c) (10 PTS) With the same initial conditions as part (b), what is the output for the input $x(n) = \delta(n)$?