EE113: Digital Signal Processing

Prof. Mihaela van der Schaar — Spring 2012

Name: **Student ID:**

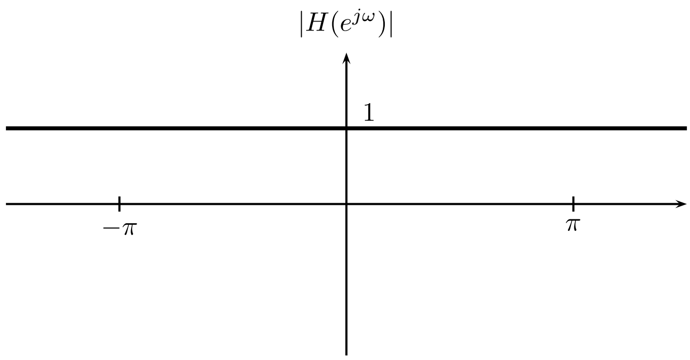
FINAL EXAMINATION SOLUTIONS (Version A)

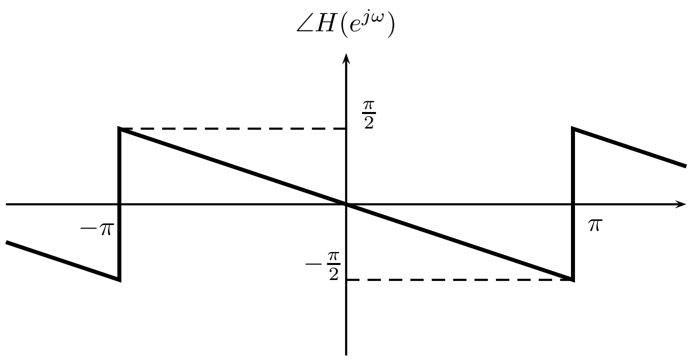
(Closed Book, One page of notes allowed, No electronic devices)

1. The frequency response of a discrete-time LTI system is $H(e^{j\omega})$. The input sequence is

$$x(n) = \cos\left(\frac{5\pi}{2}n - \frac{\pi}{4}\right)$$

Determine and sketch the output $y(n)$ if the magnitude and the phase of $H(e^{j\omega})$ are given below:





The input sequence is

$$x(n) = \cos(\frac{5\pi}{2}n - \frac{\pi}{4}) = \frac{1}{2}(e^{j(\frac{5\pi}{2}n-\frac{\pi}{4})} + e^{-j(\frac{5\pi}{2}n-\frac{\pi}{4})}) = \frac{1}{2}e^{-j\frac{\pi}{4}}e^{j\frac{\pi}{2}n} + \frac{1}{2}e^{j\frac{\pi}{4}}e^{-j\frac{\pi}{2}n}$$

Its DTFT is

$$X(e^{j\omega}) = \sum_{l=-\infty}^{+\infty}(\pi e^{-j\frac{\pi}{4}}\delta(\omega - \frac{\pi}{2} + 2\pi l) + \pi e^{j\frac{\pi}{4}}\delta(\omega + \frac{\pi}{2} + 2\pi l)$$

There are only two frequencies $-\pi/2$ and $\pi/2$ in $[-\pi, \pi]$. According to the system frequency response, the output DTFT at each frequency is thus,

$$\begin{aligned}
Y(e^{j(\pi/2+2\pi l)}) &= \pi\delta(\omega - \pi/2 + 2\pi l)e^{-j\pi/4}e^{-j\pi/4}, l = 0, \pm 1, \pm 2, ...\\
Y(e^{j(-\pi/2+2\pi l)}) &= \pi\delta(\omega + \pi/2 + 2\pi l)e^{j\pi/4}e^{j\pi/4}, l = 0, \pm 1, \pm 2, ...\\
Y(e^{j\omega}) &= 0, \text{for all other } \omega
\end{aligned}$$

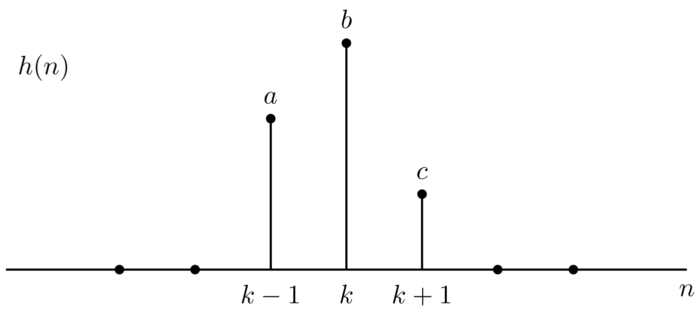
Therefore, the output DTFT is

$$Y(e^{j\omega}) = \sum_{l=-\infty}^{+\infty}(\frac{\pi}{j}\delta(\omega - \pi/2 + 2\pi l) - \frac{\pi}{j}\delta(\omega + \pi/2 + 2\pi l))$$

Therefore, the output sequence is

$$y(n) = \sin\frac{\pi}{2}n$$

2. Consider the discrete-time LTI system whose unit sample response $h(n)$ is shown below



where k is an unknown integer and a, b and c are unknown real numbers. It is known that $h(n)$ satisfies the following conditions:

(a) Let $H(e^{j\omega})$ be the DTFT of $h(n)$. $H(e^{j\omega})e^{j\omega}$ is real and even.

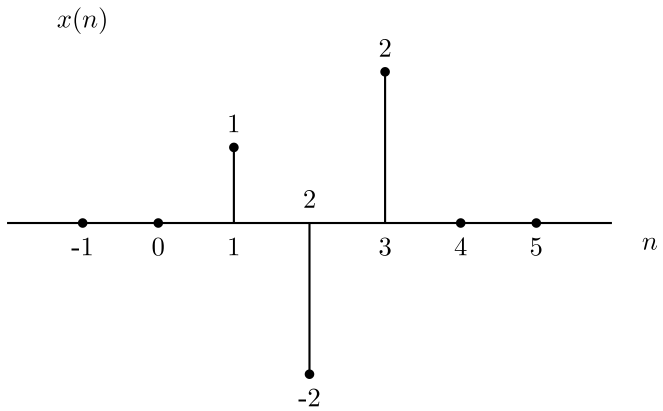
(b) If $x(n) = (-1)^n$ for all n, then $y(n) = 0$.

(c) If $x(n) = \left(\frac{1}{2}\right)^n u(n)$ for all n, then $y(2) = \frac{9}{2}$.

Answer the following questions:

(a) Show that if a sequence $x(n)$ is real and even, its DTFT $X(e^{j\omega})$ is also real and even.

(b) Provide a labeled sketch of the output $y(n)$ when the input $x(n)$ is shown below. Your answer should not include a, b, c, nor k.



(a) $x(n)$ is even, i.e. $x(n) = x(-n)$, therefore $X(e^{j\omega}) = X(e^{-j\omega})$. This means that the DTFT is also even.

$x(n)$ is real, i.e. $x(n) = x^*(n) = x^*(-n)$, therefore $X(e^{j\omega} = X^*(e^{j\omega})$. This means that the DTFT is real.

(b)

(1) Denote the system transfer function by $H(z) = az^{-(k-1)} + bz^{-k} + cz^{-(k+1)}$.

(2) Since $H(e^{j\omega})e^{j\omega}$ is real and even, then $h(n+1)$ is real and even. Therefore $a = c$, and $k = 1$.

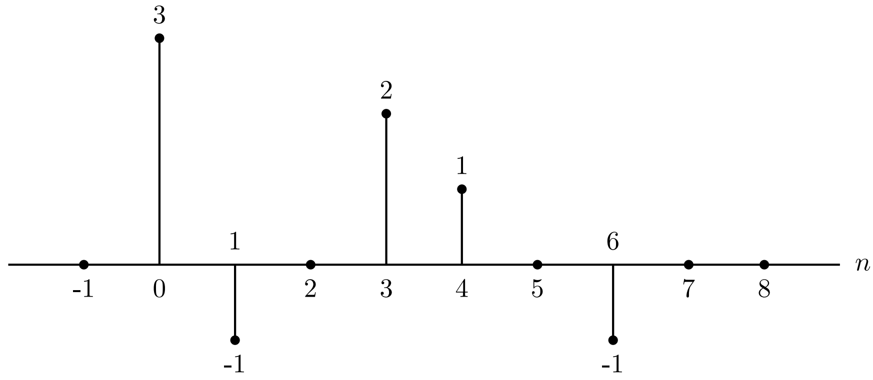
(3) The transfer function becomes $H(z) = a + bz^{-1} + az^{-2}$.

(4) If $x(n) = (-1)^n$, $y(n) = 0$, then $H(-1) = 0$. Therefore $H(-1) = a - b + a = 0$, $2a = b$.

(5) If $x(n) = (1/2)^n u(n)$, $y(2) = 9/2$, then $y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0)$. Therefore, $y(2) = 9/4a$, $a = c = 2, b = 4$.

(6) The z transform of the output sequence is $Y(z) = H(z)X(z) = 2z^{-1} - 2z^{-3} + 4z^{-4} + 4z^{-5}$. Therefore the output sequence $y(n) = 2\delta(n-1) - 2\delta(n-3) + 4\delta(n-4) + 4\delta(n-5)$.

3. Each part of this problem may be solved independently. All parts use the signal $x(n)$ shown below.

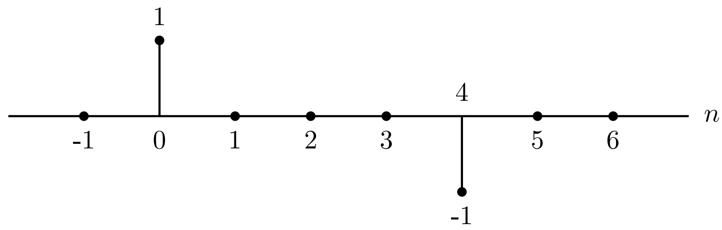


(a) Let $X(e^{j\omega})$ be the DTFT of $x(n)$. Define

$$R(k) = X(e^{j\omega})|_{\omega=\frac{2\pi k}{4}}, 0 \le k \le 3$$

Sketch the signal $r(n)$ which is the four-point inverse DFT of $R(k)$.

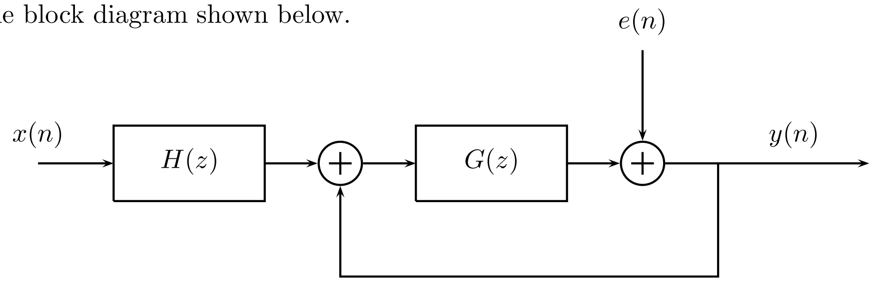
(b) Let $X(k)$ be the eight-point DFT of $x(n)$, and let $H(k)$ be the eight-point DFT of the impulse response $h(n)$ shown below. Define $Y(k) = X(k)H(k)$ for $0 \le k \le 7$. Sketch $y(n)$, the eight-point inverse DFT of $Y(k)$.



(a) Note $R(k)$ is the 4-pt DFT of $x(n)$. Inverting $R(k)$ creates time aliasing. $R(k) = X(k) + X(k+4), \forall k = 0, 1, 2, 3$. Therefore, the sequence is $(\boxed{4}, -1, -1, 2)$.

(b) The product of the DFT corresponds to circular convolution, i.e. linear convolution followed by time aliasing. $(\boxed{2}, -1, 1, 2, -2, 1, -1, -2)$.

4. Consider the block diagram shown below.



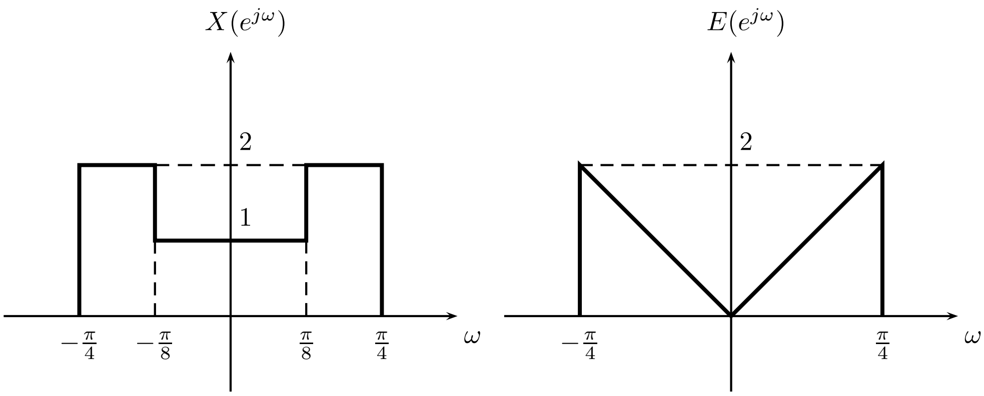
The transfer functions $H(z)$ and $G(z)$ are given by

$$H(z) = \frac{1}{z - \frac{1}{2}}, \qquad G(z) = 1 - \frac{1}{2}z^{-1}$$

These transfer functions denote stable and causal LTI systems. Let $\{Y(e^{j\omega}), X(e^{j\omega}), E(e^{j\omega})\}$ denote the DTFTs of the signals indicated in the figure. Let also $H(e^{j\omega})$ and $G(e^{j\omega})$ denote the frequency responses of the above systems.

(a) The DTFTs of the signal $\{x(n), e(n)\}$ are shown below. Compute the energies of these sequences. Compute also the signal-to-noise energy ratio at the input of the system, which is defined as

$$\text{SNR} = \frac{\text{energy of } x(n)}{\text{energy of } e(n)}$$



(b) Show that

$$Y(e^{j\omega}) = \frac{H(e^{j\omega})G(e^{j\omega})}{1 - G(e^{j\omega})} X(e^{j\omega}) + \frac{1}{1 - G(e^{j\omega})} E(e^{j\omega}) \triangleq X'(e^{j\omega}) + E'(e^{j\omega})$$

where $X'(e^{j\omega})$ refers to the contribution of the input signal $x(n)$ at the output, while $E'(e^{j\omega})$ refers to the contribution of the interfering signal $e(n)$ at the output.

(c) Compute the signal-to-noise energy ratio at the output of the system, which is defined as

$$\text{SNR} = \frac{\text{energy of } x'(n)}{\text{energy of } e'(n)}$$

(d) Assume instead that

$$x(n) = \cos\left(\frac{\pi}{3}n\right), \quad e(n) = \sin\left(\frac{\pi}{4}n + \frac{\pi}{6}\right)$$

Compute the steady-state response $y_{ss}(n)$.

(a) energy of $x(n)$

$$\sum_{n=-\infty}^{+\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega = \frac{1}{2\pi}(\frac{\pi}{2}4 - \frac{\pi}{4}3) = \frac{5}{8}$$

energy of $e(n)$,

$$\sum_{n=-\infty}^{+\infty} |e(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |E(e^{j\omega})|^2 d\omega = \frac{1}{2pi}2\int_{\omega=0}^{\frac{\pi}{4}} \frac{64}{\pi^2}\omega^2 d\omega = \frac{1}{3}$$

Therefore,

$$SNR = \frac{5}{8}3 = \frac{15}{8} \tag{1}$$

(b) According to the system diagram,

$$(X(e^{j\omega})H(e^{j\omega}) + Y(e^{j\omega}))G(e^{j\omega}) + E(e^{j\omega}) = Y(e^{j\omega})$$

rearranging yields the result.
(c)

$$\begin{aligned} H_1(z) &= \frac{H(z)G(z)}{1-G(z)} = 2 \\ H_2(z) &= \frac{1}{1-G(z)} = 2z \end{aligned}$$

Therefore, $|H_1(e^{j\omega})| = |H_2(e^{j\omega})| = 2$, and $SNR = 15/8$.
(d)

$$\begin{aligned} y_{ss}(n) &= 2\cos(\frac{\pi}{3}n) + 2\sin(\frac{\pi}{4}(n+1) + \frac{\pi}{6}) \\ &= 2\cos(\frac{\pi}{3}n) + 2\sin(\frac{\pi}{4}n + \frac{5\pi}{12}) \end{aligned}$$

5. (a) $x(n)$ is a real-valued, causal sequence with DTFT $X(e^{j\omega})$. Show that the DTFT of $x_o(n) = \frac{1}{2}(x(n) - x(-n))$ is $j\mathrm{Im}\{X(e^{j\omega})\}$.

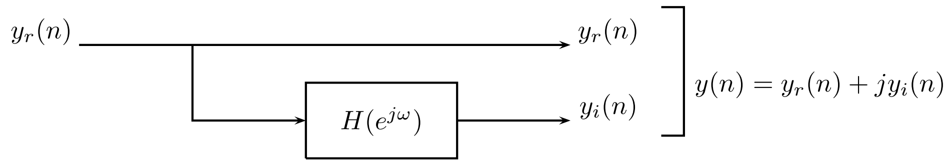
(b) $x(n)$ is a real-valued, causal sequence with DTFT $X(e^{j\omega})$. Determine a choice for $x(n)$ if the imaginary part of $X(e^{j\omega})$ is given by:

$$Im\{X(e^{j\omega})\} = 3\sin(2\omega) - 2\sin(3\omega)$$

and

$$\int_{\omega=-\pi}^{\pi} X(e^{j\omega})d\omega = 3\pi$$

(c) $y_r(n)$ is a real-valued sequence with DTFT $Y_r(e^{j\omega})$. The sequences $y_r(n)$ and $y_i(n)$ in the figure below are interpreted as the real and imaginary parts of a complex sequence $y(n)$, i.e. $y(n) = y_r(n) + jy_i(n)$.



Determine a choice of $H(e^{j\omega})$ so that $Y(e^{j\omega})$ is $Y_r(e^{j\omega})$ for *negative* frequencies and zero for *positive* frequencies between $-\pi$ and π, i.e.

$$Y(e^{j\omega}) = \begin{cases} Y_r(e^{j\omega}), & -\pi < \omega < 0 \\ 0, & 0 < \omega < \pi \end{cases}$$

(a) Because $x_o(n) = \frac{1}{2}(x(n) - x(-n))$,

$$X_o(e^{j\omega}) = \frac{1}{2}(X(e^{j\omega}) - X(e^{-j\omega})) \tag{2}$$

Because $x(n)$ is real, $x(-n) = x^*(-n)$. Therefore $X(e^{-j\omega}) = X^*(e^{j\omega})$. Therefore,

$$X_o(e^{j\omega}) = \frac{1}{2}(X(e^{j\omega}) - X^*(e^{j\omega})) = j\mathrm{Im}\{X(e^{j\omega})\} \tag{3}$$

(b) Because $j\mathrm{Im}\{X(e^{j\omega}\} = 3/2e^{j2\omega} - 3/2e^{-j2\omega} - e^{j3\omega} + e^{-j3\omega}$, $x_o(n) = 3/2\delta(n+2) - 3/2\delta(n-2) - \delta(n+3) + \delta(n-3)$. Also since the sequence is causal, $x(n) = 2\delta(n-3) - 3\delta(n-2) + x(0)\delta(n)$.
$x(0)$ is determined as follows,

$$x(0) = \frac{1}{2\pi} \int_{\omega=-\pi}^{\pi} X(e^{j\omega}) d\omega = 3/2 \tag{4}$$

Therefore, $x(n) = 2\delta(n-3) - 3\delta(n-2) + 3/2\delta(n)$.
(c) $Y(e^{j\omega}) = Y_r(e^{j\omega}) + jY_i(e^{j\omega}) = Y_r(e^{j\omega})(1 + jH(e^{j\omega}))$. To satisfy the constraints,

$$1 + jH(e^{j\omega}) = \begin{cases} 1, -\pi < \omega < 0 \\ 0, 0 < \omega < \pi \end{cases}$$

Therefore,

$$H(e^{j\omega}) = \begin{cases} 0, -\pi < \omega < 0 \\ j, 0 < \omega < \pi \end{cases}$$

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Prof. Mihaela van der Schaar | Spring 2012

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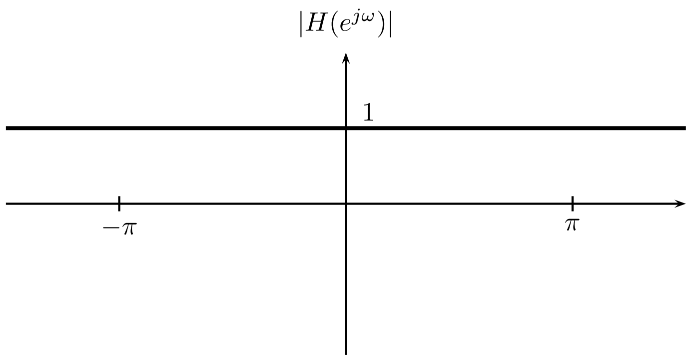
FINAL EXAMINATION SOLUTIONS (Version B)

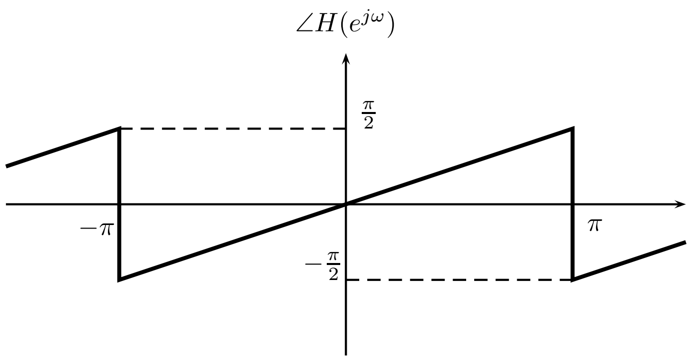
(Closed Book, One page of notes allowed, No electronic devices)

1. The frequency response of a discrete-time LTI system is $H(e^{j\omega})$. The input sequence is

$$x(n) = \cos\left(\frac{5\pi}{2}n - \frac{\pi}{4}\right)$$

Determine and sketch the output $y(n)$ if the magnitude and the phase of $H(e^{j\omega})$ are given below:





The input sequence is

$$x(n) = \cos(\frac{5\pi}{2}n - \frac{\pi}{4}) = \frac{1}{2}(e^{j(\frac{5\pi}{2}n-\frac{\pi}{4})} + e^{-j(\frac{5\pi}{2}n-\frac{\pi}{4})}) = \frac{1}{2}e^{-j\frac{\pi}{4}}e^{j\frac{\pi}{2}n} + \frac{1}{2}e^{j\frac{\pi}{4}}e^{-j\frac{\pi}{2}n}$$

Its DTFT is

$$X(e^{j\omega}) = \sum_{l=-\infty}^{+\infty}(\pi e^{-j\frac{\pi}{4}}\delta(\omega - \frac{\pi}{2} + 2\pi l) + \pi e^{j\frac{\pi}{4}}\delta(\omega + \frac{\pi}{2} + 2\pi l)$$

There are only two frequencies $-\pi/2$ and $\pi/2$ in $[-\pi, \pi]$. According to the system frequency response, the output DTFT at each frequency is thus,

$$\begin{aligned}
Y(e^{j(\pi/2+2\pi l)}) &= \pi\delta(\omega - \pi/2 + 2\pi l)e^{-j\pi/4}e^{j\pi/4}, l = 0, \pm1, \pm2, ... \\
Y(e^{j(-\pi/2+2\pi l)}) &= \pi\delta(\omega + \pi/2 + 2\pi l)e^{j\pi/4}e^{-j\pi/4}, l = 0, \pm1, \pm2, ... \\
Y(e^{j\omega}) &= 0, \text{for all other } \omega
\end{aligned}$$

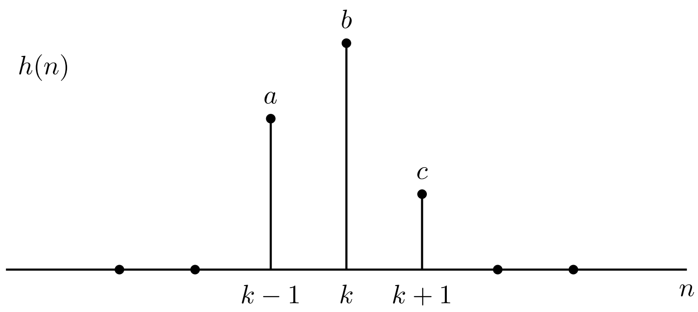
Therefore, the output DTFT is

$$Y(e^{j\omega}) = \sum_{l=-\infty}^{+\infty}(\pi\delta(\omega - \pi/2 + 2\pi l) + \pi\delta(\omega + \pi/2 + 2\pi l))$$

Therefore, the output sequence is

$$y(n) = \cos\frac{\pi}{2}n$$

2. Consider the discrete-time LTI system whose unit sample response $h(n)$ is shown below



where k is an unknown integer and a, b and c are unknown real numbers. It is known that $h(n)$ satisfies the following conditions:

(a) Let $H(e^{j\omega})$ be the DTFT of $h(n)$. $H(e^{j\omega})e^{j\omega}$ is real and even.

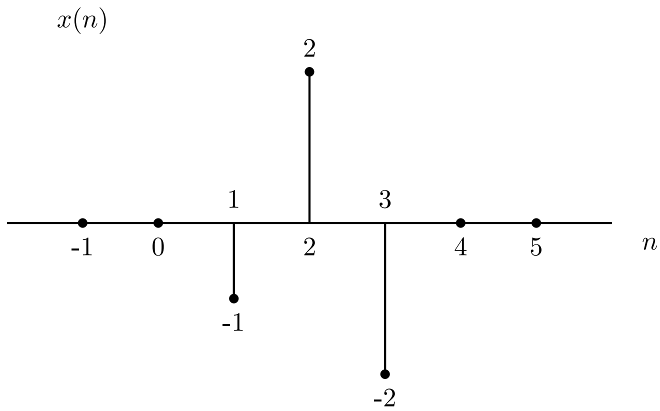
(b) If $x(n) = (-1)^n$ for all n, then $y(n) = 0$.

(c) If $x(n) = \left(-\frac{1}{2}\right)^n u(n)$ for all n, then $y(2) = \frac{3}{4}$.

Answer the following questions:

(a) Show that if a sequence $x(n)$ is real and even, its DTFT $X(e^{j\omega})$ is also real and even.

(b) Provide a labeled sketch of the output $y(n)$ when the input $x(n)$ is shown below. Your answer should not include a, b, c, nor k.



(a) $x(n)$ is even, i.e. $x(n) = x(-n)$, therefore $X(e^{j\omega}) = X(e^{-j\omega})$. This means that the DTFT is also even.

$x(n)$ is real, i.e. $x(n) = x^*(n) = x^*(-n)$, therefore $X(e^{j\omega} = X^*(e^{j\omega})$. This means that the DTFT is real.

(b)

(1) Denote the system transfer function by $H(z) = az^{-(k-1)} + bz^{-k} + cz^{-(k+1)}$.

(2) Since $H(e^{j\omega})e^{j\omega}$ is real and even, then $h(n+1)$ is real and even. Therefore $a = c$, and $k = 1$.

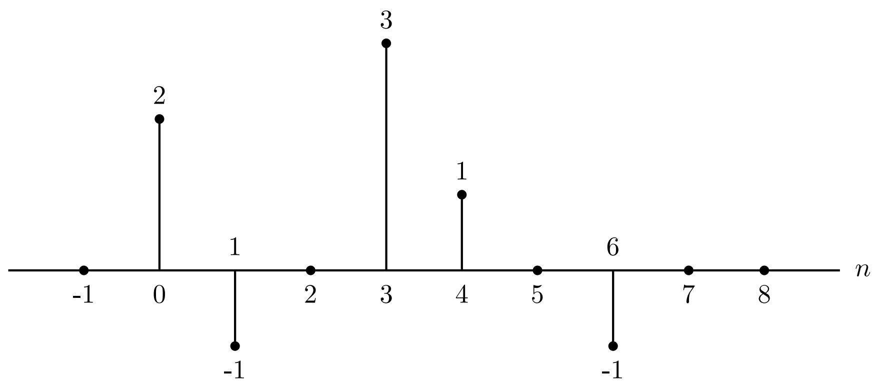
(3) The transfer function becomes $H(z) = a + bz^{-1} + az^{-2}$.

(4) If $x(n) = (-1)^n$, $y(n) = 0$, then $H(-1) = 0$. Therefore $H(-1) = a - b + a = 0$, $2a = b$.

(5) If $x(n) = (-1/2)^n u(n)$, $y(2) = 9/2$, then $y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0)$. Therefore, $y(2) = 1/4a$, $a = c = 3, b = 6$.

(6) The z transform of the output sequence is $Y(z) = H(z)X(z) = -3z^{-1} + 3z^{-3} - 6z^{-4} - 6z^{-5}$. Therefore the output sequence $y(n) = -3\delta(n-1) + 3\delta(n-3) - 6\delta(n-4) - 6\delta(n-5)$.

3. Each part of this problem may be solved independently. All parts use the signal $x(n)$ shown below.

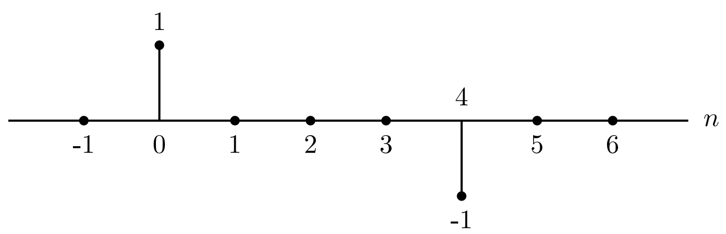


(a) Let $X(e^{j\omega})$ be the DTFT of $x(n)$. Define

$$R(k) = X(e^{j\omega})|_{\omega=\frac{2\pi k}{4}}, 0 \leq k \leq 3$$

Sketch the signal $r(n)$ which is the four-point inverse DFT of $R(k)$.

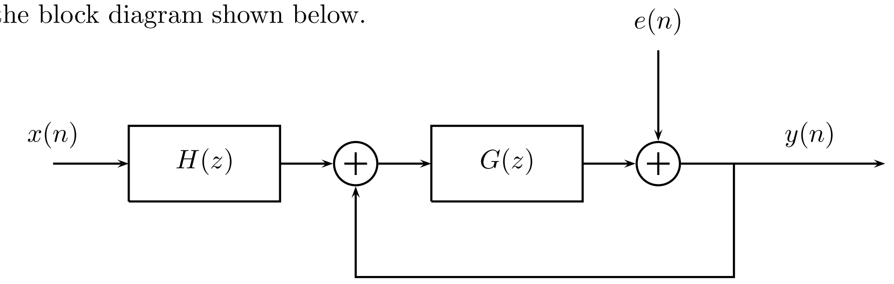
(b) Let $X(k)$ be the eight-point DFT of $x(n)$, and let $H(k)$ be the eight-point DFT of the impulse response $h(n)$ shown below. Define $Y(k) = X(k)H(k)$ for $0 \leq k \leq 7$. Sketch $y(n)$, the eight-point inverse DFT of $Y(k)$.



(a) Note $R(k)$ is the 4-pt DFT of $x(n)$. Inverting $R(k)$ creates time aliasing. $R(k) = X(k) + X(k + 4), \forall k = 0, 1, 2, 3$. Therefore, the sequence is $(\boxed{3}, -1, -1, 3)$

(b) The product of the DFT corresponds to circular convolution, i.e. linear convolution followed by time aliasing. $(\boxed{1}, -1, 1, 3, -1, 1, -1, -3)$.

4. Consider the block diagram shown below.



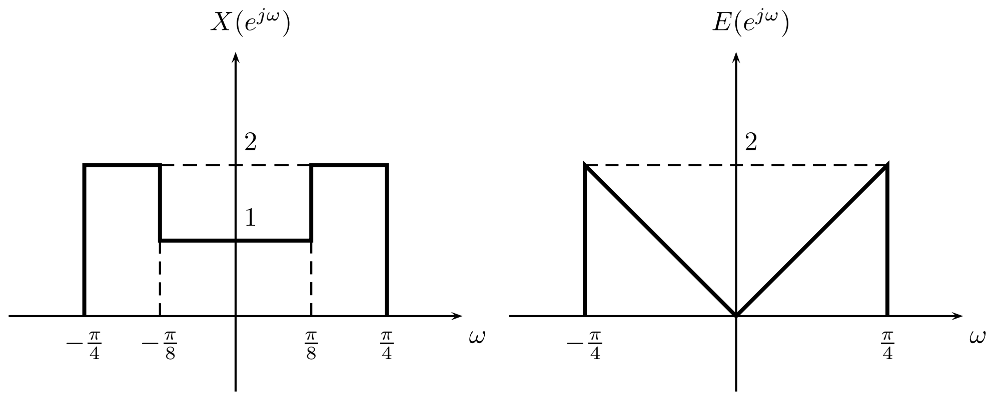
The transfer functions $H(z)$ and $G(z)$ are given by

$$H(z) = \frac{1}{z - \frac{1}{3}}, \qquad G(z) = 1 - \frac{1}{3}z^{-1}$$

These transfer functions denote stable and causal LTI systems. Let $\{Y(e^{j\omega}), X(e^{j\omega}), E(e^{j\omega})\}$ denote the DTFTs of the signals indicated in the figure. Let also $H(e^{j\omega})$ and $G(e^{j\omega})$ denote the frequency responses of the above systems.

(a) The DTFTs of the signal $\{x(n), e(n)\}$ are shown below. Compute the energies of these sequences. Compute also the signal-to-noise energy ratio at the input of the system, which is defined as

$$\text{SNR} = \frac{\text{energy of } x(n)}{\text{energy of } e(n)}$$



(b) Show that

$$Y(e^{j\omega}) = \frac{H(e^{j\omega})G(e^{j\omega})}{1 - G(e^{j\omega})}X(e^{j\omega}) + \frac{1}{1 - G(e^{j\omega})}E(e^{j\omega}) \triangleq X'(e^{j\omega}) + E'(e^{j\omega})$$

where $X'(e^{j\omega})$ refers to the contribution of the input signal $x(n)$ at the output, while $E'(e^{j\omega})$ refers to the contribution of the interfering signal $e(n)$ at the output.

(c) Compute the signal-to-noise energy ratio at the output of the system, which is defined as

$$\text{SNR} = \frac{\text{energy of } x'(n)}{\text{energy of } e'(n)}$$

(d) Assume instead that

$$x(n) = \cos\left(\frac{\pi}{4}n\right), \quad e(n) = \sin\left(\frac{\pi}{3}n + \frac{\pi}{9}\right)$$

Compute the steady-state response $y_{ss}(n)$.

(a) energy of $x(n)$

$$\sum_{n=-\infty}^{+\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{j\omega})|^2 d\omega = \frac{1}{2\pi} (\frac{\pi}{2} 4 - \frac{\pi}{4} 3) = \frac{5}{8}$$

energy of $e(n)$,

$$\sum_{n=-\infty}^{+\infty} |e(n)|^2 = \frac{1}{2\pi} \int_{2\pi} |E(e^{j\omega})|^2 d\omega = \frac{1}{2pi} 2 \int_{\omega=0}^{\frac{\pi}{4}} \frac{64}{\pi^2} \omega^2 d\omega = \frac{1}{3}$$

Therefore,

$$SNR = \frac{5}{8} 3 = \frac{15}{8} \tag{1}$$

(b) According to the system diagram,

$$(X(e^{j\omega})H(e^{j\omega}) + Y(e^{j\omega}))G(e^{j\omega}) + E(e^{j\omega}) = Y(e^{j\omega})$$

rearranging yields the result.
(c)

$$\begin{aligned} H_1(z) &= \frac{H(z)G(z)}{1 - G(z)} = 3 \\ H_2(z) &= \frac{1}{1 - G(z)} = 3z \end{aligned}$$

Therefore, $|H_1(e^{j\omega})| = |H_2(e^{j\omega})| = 3$, and $SNR = 15/8$.
(d)

$$\begin{aligned} y_{ss}(n) &= 3\cos(\frac{\pi}{4} n) + 3\sin(\frac{\pi}{3}(n+1) + \frac{\pi}{9}) \\ &= 3\cos(\frac{\pi}{4} n) + 3\sin(\frac{\pi}{3} n + \frac{4\pi}{9}) \end{aligned}$$

5. (a) $x(n)$ is a real-valued, causal sequence with DTFT $X(e^{j\omega})$. Show that the DTFT of $x_o(n) = \frac{1}{2}(x(n) - x(-n))$ is $j\mathrm{Im}\{X(e^{j\omega})\}$.

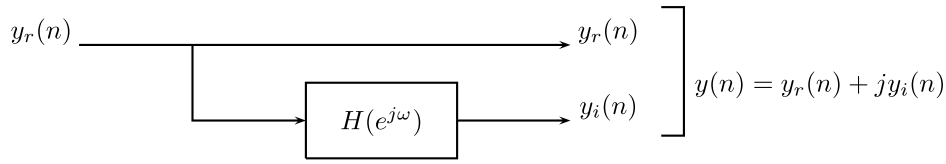
(b) $x(n)$ is a real-valued, causal sequence with DTFT $X(e^{j\omega})$. Determine a choice for $x(n)$ if the imaginary part of $X(e^{j\omega})$ is given by:

$$Im\{X(e^{j\omega})\} = 2\sin(2\omega) - 3\sin(3\omega)$$

and

$$\int_{\omega=-\pi}^{\pi} X(e^{j\omega})d\omega = 5\pi$$

(c) $y_r(n)$ is a real-valued sequence with DTFT $Y_r(e^{j\omega})$. The sequences $y_r(n)$ and $y_i(n)$ in the figure below are interpreted as the real and imaginary parts of a complex sequence $y(n)$, i.e. $y(n) = y_r(n) + jy_i(n)$.



Determine a choice of $H(e^{j\omega})$ so that $Y(e^{j\omega})$ is $Y_r(e^{j\omega})$ for *negative* frequencies and zero for *positive* frequencies between $-\pi$ and π, i.e.

$$Y(e^{j\omega}) = \begin{cases} Y_r(e^{j\omega}), & -\pi < \omega < 0 \\ 0, & 0 < \omega < \pi \end{cases}$$

(a) Because $x_o(n) = \frac{1}{2}(x(n) - x(-n))$,

$$X_o(e^{j\omega}) = \frac{1}{2}(X(e^{j\omega}) - X(e^{-j\omega})) \tag{2}$$

Because $x(n)$ is real, $x(-n) = x^*(-n)$. Therefore $X(e^{-j\omega}) = X^*(e^{j\omega})$. Therefore,

$$X_o(e^{j\omega}) = \frac{1}{2}(X(e^{j\omega}) - X^*(e^{j\omega})) = j\mathrm{Im}\{X(e^{j\omega})\} \tag{3}$$

(b) Because, $j\mathrm{Im}\{X(e^{j\omega}\} = e^{j2\omega} - e^{-j2\omega} - 3/2e^{j3\omega} + 3/2e^{-j3\omega}$, $x_o(n) = \delta(n+2) - \delta(n-2) - 3/2\delta(n+3) + 3/2\delta(n-3)$. Therefore, Also since the sequence is causal, $x(n) = 3\delta(n-3) - 2\delta(n-2) + x(0)\delta(n)$. $x(0)$ is determined as follows,

$$x(0) = \frac{1}{2\pi} \int_{\omega=-\pi}^{\pi} X(e^{j\omega}) d\omega = 5/2 \tag{4}$$

Therefore, $x(n) = 3\delta(n-3) - 2\delta(n-2) + 5/2\delta(n)$.

(c) $Y(e^{j\omega}) = Y_r(e^{j\omega}) + jY_i(e^{j\omega}) = Y_r(e^{j\omega})(1 + jH(e^{j\omega}))$. To satisfy the constraints,

$$1 + jH(e^{j\omega}) = \begin{cases} 1, -\pi < \omega < 0 \\ 0, 0 < \omega < \pi \end{cases}$$

Therefore,

$$H(e^{j\omega}) = \begin{cases} 0, -\pi < \omega < 0 \\ j, 0 < \omega < \pi \end{cases}$$