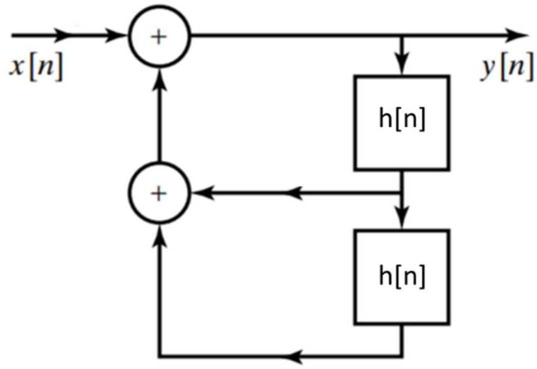
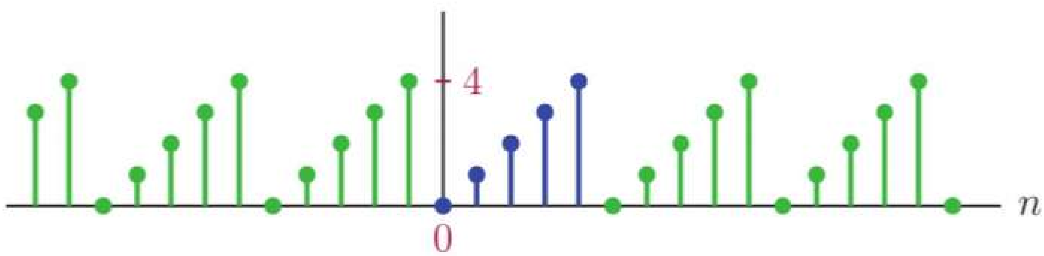
Consider the system below:



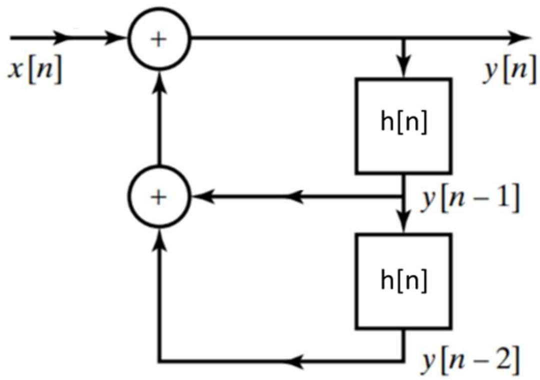
$$where\ h[n] = \delta[n-1]$$

You observe that the output of the system y[n] is as given below:



a. Plot one period of x[n]. Show all work
b. Give the Fourier Series Coefficients ck for x[n].
c. Using your answer from b give the Fourier Series Coefficients, dk, for x[6+n] in terms of ck. Put your answer in simplest form.

Solution



$$where\ h[n] = \delta[n-1]$$

$$y[n] = x[n] + y[n-1] + y[n-2]$$

Over the points n=0 to n=4, we have

$$[0, 1, 2, 3, 4\ ] = \big[x[0], x[1], x[2], x[3], x[4]\big] + [4, 0, 1, 2, 3] + [3, 4, 0, 1, 2]$$

$$so\ x[n] = [0-3-4, 1-0-4, 2-1-0, 3-2-1, 4-3-2] = [-7, -3, 1, 0, -1]$$

We see that x[n] has a period of N=5, so the Fourier series coefficients become:

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{\frac{-2j\pi kn}{N}}$$

$$c_0 = \frac{1}{5}\left((-7)e^{\frac{-2j\pi(0)(0)}{N}} + (-3)e^{\frac{-2j\pi(0)(1)}{N}} + (1)e^{\frac{-2j\pi(0)(2)}{N}} + (0)e^{\frac{-2j\pi(0)(3)}{N}} + (-1)e^{\frac{-2j\pi(0)(4)}{N}}\right)$$

$$c_1 = \frac{1}{5}\left((-7)e^{\frac{-2j\pi(1)(0)}{N}} + (-3)e^{\frac{-2j\pi(1)(1)}{N}} + (1)e^{\frac{-2j\pi(1)(2)}{N}} + (0)e^{\frac{-2j\pi(1)(3)}{N}} + (-1)e^{\frac{-2j\pi(1)(4)}{N}}\right)$$

$$c_2 = \frac{1}{5}\left((-7)e^{\frac{-2j\pi(2)(0)}{N}} + (-3)e^{\frac{-2j\pi(2)(1)}{N}} + (1)e^{\frac{-2j\pi(2)(2)}{N}} + (0)e^{\frac{-2j\pi(2)(3)}{N}} + (-1)e^{\frac{-2j\pi(2)(4)}{N}}\right)$$

$$c_3 = \frac{1}{5}\left((-7)e^{\frac{-2j\pi(3)(0)}{N}} + (-3)e^{\frac{-2j\pi(3)(1)}{N}} + (1)e^{\frac{-2j\pi(3)(2)}{N}} + (0)e^{\frac{-2j\pi(3)(3)}{N}} + (-1)e^{\frac{-2j\pi(3)(4)}{N}}\right)$$

$$c_4 = \frac{1}{5}\left((-7)e^{\frac{-2j\pi(4)(0)}{N}} + (-3)e^{\frac{-2j\pi(4)(1)}{N}} + (1)e^{\frac{-2j\pi(4)(2)}{N}} + (0)e^{\frac{-2j\pi(4)(3)}{N}} + (-1)e^{\frac{-2j\pi(4)(4)}{N}}\right)$$

Apply time shift property:

$$d_k = e^{\frac{-j2\pi k(-6)}{5}} c_k = e^{\frac{-j2\pi k(-1)}{5}} c_k = e^{\frac{-j2\pi k(4)}{5}} c_k$$

The important part here is that $e^{-j\omega}$ is periodic with a period of 2π, so we can always express it in simplest form as $e^{\frac{-j2\pi kn}{N}}$ where n is no larger than N. Another way of stating this is that a shift of n where n > N is can more simply be written as a shift of n mod N.