EE 113 Digital Signal Processing

Spring 2014

Final Exam

Closed Book, 2 sheet of notes allowed

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9) ________ / 10

TOTAL ________ / 125

	Sequence	z-transform	ROC	Property
1.	$x(n)$	$X(z)$	$R_x = \{r_1 < \|z\| < r_2\}$	
2.	$y(n)$	$Y(z)$	$R_y = \{r' < \|z\| < r''\}$	
3.	$ax(n) + by(n)$	$aX(z) + bY(z)$	$\{R_x \cap R_y\}$ including possibly $z = 0$ or $z = \pm\infty$	linearity

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4.	$x(n - n_0)$	$z^{-n_0} X(z)$	R_x excluding possibly $z = 0$ or $z = \pm\infty$	time-shifts
5.	$a^n x(n)$	$X(z/a)$	$\|a\|r_1 < \|z\| < \|a\|r_2$	exponential modulation
6.	$(-1)^n x(n)$	$X(-z)$	R_x	alternating sign
7.	$x(-n)$	$X(1/z)$	$1/r_2 < \|z\| < 1/r_1$	time reversal
8.	$nx(n)$	$-z\,\frac{dX(z)}{dz}$	R_x excluding possibly $z = 0$ or $z = \pm\infty$	linear modulation
9.	$x^*(n)$	$[X(z^*)]^*$	R_x	conjugation
10.	Re $[x(n)]$	$\frac{1}{2}\left[X(z) + (X(z^*))^*\right]$	R_x excluding possibly $z = 0$ or $z = \pm\infty$	real part
11.	Im $[x(n)]$	$\frac{1}{2j}\left[X(z) - (X(z^*))^*\right]$	R_x excluding possibly $z = 0$ or $z = \pm\infty$	imaginary part
12.	$x(n) \star y(n)$	$X(z)Y(z)$	$\{R_x \cap R_y\}$ including possibly $z = 0$ or $z = \pm\infty$	convolution
13.	$y(n) = \begin{cases} x\left(\frac{n}{L}\right), & \frac{n}{L} \text{ integer} \\ 0, & \text{otherwise} \end{cases}$	$X(z^L)$	$r_1^{1/L} < \|z\| < r_2^{1/L}$	upsampling
14.	$y(n) = x(2n)$	$\frac{1}{2}\left[X(z^{1/2}) + X(-z^{1/2}\right]$	$r_1^2 < \|z\| < r_2^2$	2-fold downsampling
15.	$y(n) = x(Mn)$	$\frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{-\frac{j2\pi k}{M}} z^{1/M}\right)$	$r_1^M < \|z\| < r_2^M$	M-fold downsampling

Sequence	**z-Transform**	**ROC**
$\delta(n)$	1	$\mathbb{C}$
$u(n)$	$\frac{z}{z-1}$	$\|z\| > 1$
$\alpha^n u(n)$	$\frac{z}{z-\alpha}$	$\|z\| > \|\alpha\|$
$-\alpha^n u(-n-1)$	$\frac{z}{z-\alpha}$	$\|z\| < \|\alpha\|$
$nu(n)$	$\frac{z}{(z-1)^2}$	$\|z\| > 1$
$-nu(-n-1)$	$\frac{z}{(z-1)^2}$	$\|z\| < 1$
$n\alpha^n u(n)$	$\frac{\alpha z}{(z-\alpha)^2}$	$\|z\| > \|\alpha\|$
$-n\alpha^n u(-n-1)$	$\frac{\alpha z}{(z-\alpha)^2}$	$\|z\| < \|\alpha\|$
$\cos(\omega_o n)u(n)$	$\frac{z^2 - z\cos\omega_o}{z^2 - 2z\cos\omega_o + 1}$	$\|z\| > 1$

$\cos(\omega_o n)u(n)$	$\frac{z^2 - z\cos\omega_o}{z^2 - 2z\cos\omega_o + 1}$	$\lvert z\rvert > 1$
$\sin(\omega_o n)u(n)$	$\frac{z\sin\omega_o}{z^2 - 2z\cos\omega_o + 1}$	$\lvert z\rvert > 1$

$\alpha^n\cos(\omega_o n)u(n)$	$\frac{z^2 - \alpha z\cos\omega_o}{z^2 - 2\alpha z\cos\omega_o + \alpha^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$\alpha^n\sin(\omega_o n)u(n)$	$\frac{\alpha z\sin\omega_o}{z^2 - 2\alpha z\cos\omega_o + \alpha^2}$	$\lvert z\rvert > \lvert\alpha\rvert$

10 pts **Problem 1**

Consider the following LTI system operating with a sampling frequency equal to F_s KHz:

5 pts

$$y(n) = \frac{3}{4}y(n-1) + \frac{1}{2}x(n-1)$$

(a) By how much will a tone at $F_s/4$ KHz be attenuated when filtered by this system? Is the attenuation dependent on the value of F_s?

5 pts

(b) If a tone of 4 KHz is attenuated by $\frac{2}{\sqrt{13}}$, can you tell what the sampling frequency F_s is?

15 pts **Problem 2**

A causal system is composed of the series cascade of two LTI systems with impulse response sequences given by the expressions:

$$h_1(n) = \left(\frac{1}{2}\right)^{2n-1} u(2n-3) \qquad \text{and} \qquad h_2(n) = \left(\frac{1}{2}\right)^{n} u(n-1)$$

5 pts

(a) Determine the transfer function of the system

5 pts

(b) Determine the impulse response sequence of the system

5 pts

(c) Is the system stable? What are its zeros? Poles?

20 pts **Problem 3**

A causal LTI system has the transfer function: $H(z) = \frac{1 + 2z^{-1} + z^{-2}}{(1 + 0.5z^{-1})(1 - 0.75z^{-1})}$

5 pts (a) Find the impulse response of this system

A causal LTI system has the transfer function: $(1+0.5z^{-1})(1-0.75z^{-1})$

5 pts (a) Find the impulse response of this system

5 pts (b) Find the output of the system, y(n) for an input $x(n) = e^{j\frac{\pi}{2}n}$

5 pts (c) Find the LCCDE that relates the input and output of the system

5 pts (d) Sketch the block diagram of the system using only 2 delay elements

10 pts **Problem 4**

The figure below shows 2 finite-length sequences:

5 pts (a) Find their 6-Point circular convolution of the two sequences

5 pts (b) Without computing the linear convolution of the two signals determine whether or not the answer for part (a) equals the linear convolution of the 2 signals.

20 pts **Problem 5**

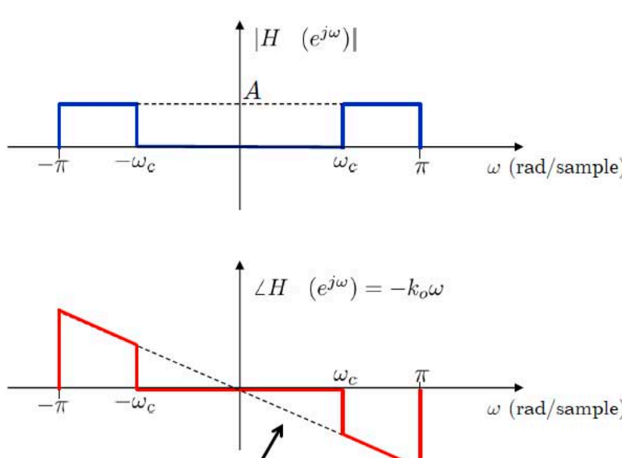
The figure below shows the frequency response of a LTI system with $W_c = \frac{\pi}{4}$

3 pts (a) Is the system BIBO stable? Why?

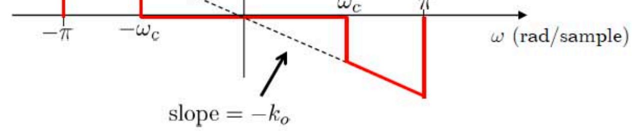
3 pts (b) Is the system realizable? Why?

3 pts (c) Find the energy of the impulse response sequence.

5 pts (d) Find the impulse response sequence of the system



5 pts (d) Find the impulse response sequence of the system.

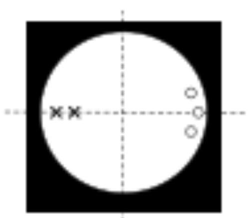


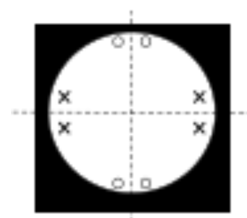
6 pts (e) What is the response of the system to:

$$x(n) = \sin^2\left(\frac{\pi}{6}n - \frac{\pi}{4}\right)$$

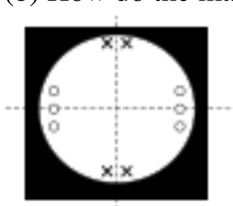
10 pts **<u>Problem 6:</u>**

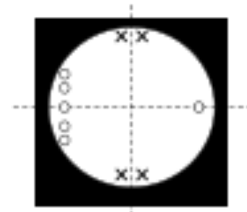
5 pts (a) Sketch the magnitude of the frequency response for each of the following pole-zero plots. Please provide the reasoning behind your sketches. Be sure to indicate the angular frequency for "points of interest" (e.g. places wehre the response peaks or reaches a local minima, etc.)





5 pts (b) How do the magnitude responses of the following two pole-zero plots differ?





10 pts **<u>Problem 7:</u>**

A causal system is described by the difference equation:

$$y(n) + 2y(n-1) + 2y(n-2) = x(n)$$ *y(-1)=0* *y(-2)=1.*

$$x(n) = \left(\frac{1}{3}\right)^n u(n-1)$$

Use the unilateral z-transform to find its complete response to

Use the unilateral z-transform to find its complete response to $x(n) = \left(\frac{1}{3}\right) u(n-1)$

20 pts **Problem 8:**

A relaxed system is described by the equation $y(n) - \frac{1}{2}y(n-2) = x(n^2)$

5 pts (a) is the system linear?

5 pts (b) is the system time invariant?

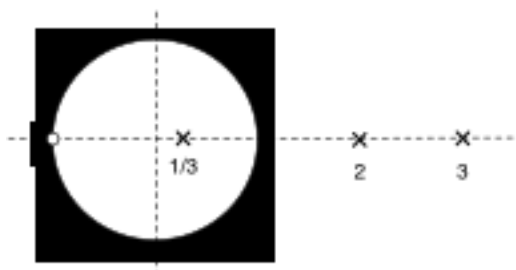
5 pts (c) is the system causal?

5 pts (d) is the system BIBO stable?

Please be sure to justify your answer to the above via either a proof or a counter example.

10 pts **Problem 9:**

The figure shows the pole-zero plot for the transfer function of an LTI system.



5 pts (a) How many possible two sided sequences are possible for the pole-zero plot shown in the figure?

5 pts (b) If the system is causal, can it also be stable? Explain.