10 pts

Problem 1:

Given $X(k) = \{1, -1, 1, -1\}$ find the 4 point IDFT of $Y(k) = \cos(\pi k/2)\, X(k)$

$$x(n) = \frac{1}{4}\sum_{k=0}^{3} X(k)\, e^{j\frac{2\pi}{4}kn} \quad \frac{\pi}{2}$$

$$= \frac{1}{4}\left[1 - e^{j\frac{\pi}{2}n} + e^{j\pi n} - e^{j\frac{3\pi}{2}n}\right]$$

$$= \frac{1}{4}\left[1 - j^{n} + (-1)^{n} - (-j)^{n}\right]$$

$\underline{n}$

$x(0) = \frac{1}{4}[1-1+1-1] = 0$

$x(1) = \frac{1}{4}[1-j-1+j] = 0$ 		$\{0, 0, 1, 0\}$

$x(2) = \frac{1}{4}[1+1+1+1] = 1$

$x(3) = \frac{1}{4}[1+j-1-j] = 0$

$$\cos(\pi k/2) = \frac{1}{2}\left[e^{j\frac{\pi k}{2}} + e^{-j\frac{\pi k}{2}}\right]$$

$$Y(k) = \frac{1}{2}X(k)\, e^{j\frac{\pi}{2}k} + \frac{1}{2}X(k)\, e^{-j\frac{\pi}{2}k}$$ 		$j\frac{\pi}{2}k \Leftrightarrow j\frac{2\pi n_0 k}{4}$ 	$n_0 = 1$

$\frac{1}{2}X(k)\, e^{j\frac{\pi}{2}k} \rightarrow \frac{1}{2}\{0, 0, 0, 1\}$

$\frac{1}{2}X(k)\, e^{-j\frac{\pi}{2}k} \rightarrow \frac{1}{2}\{0, 1, 0, 0\}$

$$y(n) = \{0, \tfrac{1}{2}, 0, \tfrac{1}{2}\}$$

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10 pts

Problem 2:
Find the 4 point DFT of *x[((n+2)mod 4)]*, where *x(n) = {1, 0, 0.5, -2}*

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi}{4}kn} \rightarrow \frac{\pi}{2}$$

$$= 1 + 0 + \frac{1}{2}e^{-j\pi k} - 2e^{-j\frac{3\pi}{2}k} \quad \rightarrow (-1)^n \quad \rightarrow (j)^n$$

$$X(0) = 1 + \frac{1}{2} - 2 = -\frac{1}{2}$$

$$X(1) = 1 + \frac{1}{2}(-1) - 2(j) = \frac{1}{2} - j2$$

$$X(2) = 1 + \frac{1}{2}(1) + 2(1) = 7/2$$

$$X(3) = 1 + \frac{1}{2}(-1) + 2(j) = \frac{1}{2} + j2$$

$$\left\{-\frac{1}{2}, \frac{1}{2} - j2, 7/2, \frac{1}{2} + j2\right\}$$

$$x[((n+2)\bmod 4)] \Rightarrow X(k)\, e^{-j\frac{2\pi}{4}(-2)k} = X(k)\, e^{j\pi k}$$

$$X(k) = \left\{-\frac{1}{2}, -\frac{1}{2} + j2, \frac{7}{2}, -\frac{1}{2} - j2\right\}$$

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15 pts

Problem 3:

$-3 \rightarrow 8$

Given $x(n) = \sin\left(\frac{\pi}{3}(n-3)\right)$ and $y(n) = u(n+3) - u(n-9)$ find the DTFT of $z(n)=x(n)y(n)$. you need not simplify the final answer.

$$\sin\left(\frac{\pi}{3}n\right) = \frac{1}{j2}\left[e^{j\frac{\pi}{3}n} - e^{-j\frac{\pi}{3}n}\right]$$

$$z(n) = x(n)\,y(n)$$

$$Z(e^{j\omega}) = X(e^{j\omega}) * Y(e^{j\omega})$$

$$Z(e^{j\omega}) = \sum_{n=-\infty}^{\infty} z(n)\, e^{-j\omega n}$$

$z(-3) = 0$
$z(-2) = \sqrt{3}/2$
$z(-1) = \sqrt{3}/2$
$z(0) = 0$
$z(1) = \sqrt{3}/2$
$z(2) = -\sqrt{3}/2$
$z(3) = 0$
$z(4) = \sqrt{3}/2$
$z(5) = \sqrt{3}/2$
$z(6) = 0$
$z(7) = -\sqrt{3}/2$
$z(8) = -\sqrt{3}/2$

$$Z(e^{j\omega}) = \frac{\sqrt{3}}{2}\left(e^{j2\omega} + e^{j\omega} - e^{-j\omega} - e^{-j2\omega} + e^{-j4\omega} + e^{-j5\omega} - e^{-j7\omega} - e^{-j8\omega}\right)$$

You should use the properties. You were on right track.

-1

Problem 4:

20 pts Consider a 50 Hz sinusoid $x(t) = -\sin(2\pi \cdot 50 \cdot t)$ sampled at a 90 Hz sampling rate.

4 pts (a) Find the sampled sequence $x(n)$ and its angular frequency ω_x, where $-\pi \le \omega_x < \pi$.

4 pts (b) What is the period of the sampled sequence? Please show your work and explain your answer

4 pts (c) If the signal is reconstructed by passing $x(n)$ through an ideal LPF filter, what is the frequency in (Hz) of the resulting signal?

8 pts (d) Let $x_{18}(n) = x(n), n = 0, 1, 2, \ldots 17$. Its 18-point DFT is $X_{18}(k), k = 0, 1, 2, \ldots 17$. For what values of k, $0 \le k < 17$, is $X_{18}(k)$ nonzero? Please provide an explanation.

a) $x(n) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t - \frac{n}{90})$

$= -\sin(2\pi\, 50 \cdot \frac{n}{90})$

$= -\sin(2\pi \frac{5}{9} n)$

$\omega_x = \frac{10\pi}{9} - \frac{18\pi}{9} = -\frac{8\pi}{9}$

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b) $2\pi \frac{5}{9} n + 2\pi k = 2\pi \frac{5}{9}(n + N)$

$2\pi k = 2\pi \frac{5}{9} N$

$N = \frac{9}{5} k$

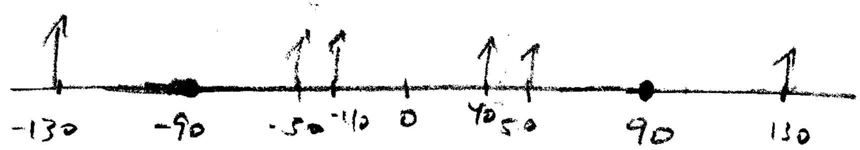
Period of length 9

4

$\rightarrow -\sin(2\pi \frac{5}{9}(n)) = -\sin(2\pi \frac{5}{9}(n + 9x)) = -\sin(2\pi \frac{5}{9} n + 2\pi (5) x)$

$10\pi x$ multiple of 2π

c)



resulting frequency of 40 Hz

4

d) There will be nonzero values at k = 8 and k = 10 because there are two frequencies present, 40Hz and 50 Hz, due to aliasing. These two frequencies correspond to k = 8 and 10 because the was over 2 periods of the signal

8

10 pts **Problem 5:**

Express the DFT of the 9-point sequence {x(0), x(1), …, x(8)} in terms of the DFTs of the following 3-point sequences:
{x(0), x(3), x(6)} $\longrightarrow X_0$
{x(1), x(4), x(7)} $\longrightarrow X_1$
{x(2), x(5), x(8)} $\longrightarrow X_2$

$$X(k) = \sum_{n=0}^{8} x(n) e^{-j\frac{2\pi}{9}kn}$$

$$= \sum_{a=0}^{2} x(3a) e^{-j\frac{2\pi}{9}k(3a)} + \sum_{a=0}^{2} x(3a+1) e^{-j\frac{2\pi}{9}k(3a+1)} + \sum_{a=0}^{2} x(3a+2) e^{-j\frac{2\pi}{9}k(3a+2)}$$

$$= \sum_{a=0}^{3} x(3a) e^{-j\frac{2\pi}{3}ka} + e^{-j\frac{2\pi}{9}k} \sum_{a=0}^{2} x(3a+1) e^{-j\frac{2\pi}{3}ka} + e^{-j\frac{4\pi}{9}k} \sum_{a=0}^{2} x(3a+2) e^{-j\frac{2\pi}{3}ka}$$

$$= X_0(k) + e^{-j\frac{2\pi}{9}k} X_1(k) + e^{-j\frac{4\pi}{9}k} X_2(k)$$

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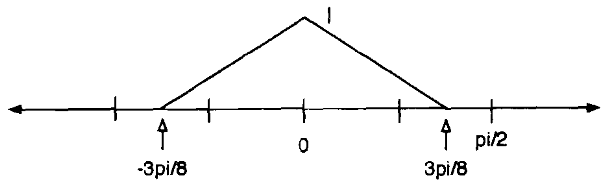
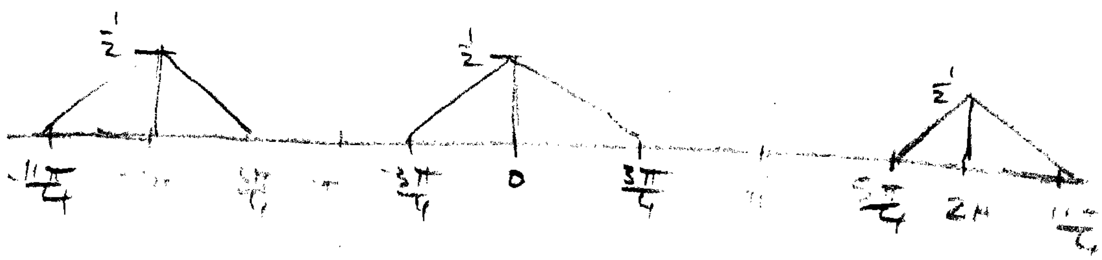
10 pts

Problem 6:

A sequence x(n) has the DTFT shown in the figure below. Sketch the DTFT of y(n) obtained by first downsampling x(n) by 2 and the upsampling it by 3. Be sure to show your work and provide justifications for your answer

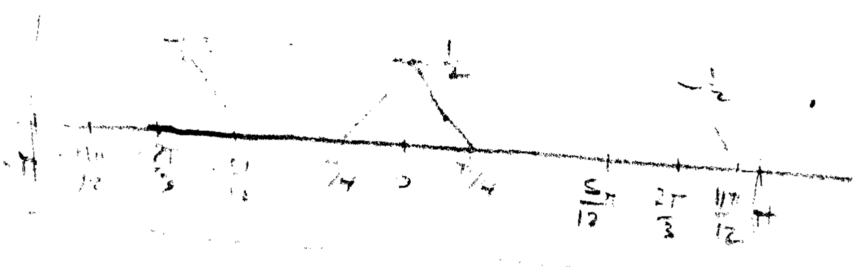


down sample by 2



no aliasing, does not expand past π or -π

up sample by 3



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Because the DTFT is periodic, it introduces additional components from neighboring repetitions as their centers are also time scaled.