UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE113: DIGITAL SIGNAL PROCESSING

Midterm 1 Exam
Date: November 2, 2020, Duration: 1 hour 50 minutes

INSTRUCTIONS:

- The exam has 6 problems
- The exam is open-book and open notes.
- Calculator is allowed.
- Please submit all your work as a single PDF file on CCLE.

Your name:

Student ID:

Table 1: Score Table

Problem	a	b	c	d	Total	Score
1	5	5	5	5	20	
2	10	10			20	
3	10				10	
4	10	10			20	
5	10	10			20	
6	10				10	
Sum					100	

Problem 1 (20 pts)

An LTI discrete-time system has an impulse response $h[n] = u[n+1] - u[n-4]$, and as input the signal $x[n] = u[n] - u[n-(N+1)]$ for a positive integer N. The output of the system is denoted as $y[n]$.

a) (5 pts) Derive input output relationship in the form of difference equation.

b) (5 pts) If $N = 4$, without calculating $y[n]$, what is the length of the output $y[n]$? Explain your answer.

c) (5 pts) Is the system stable?

d) (5 pts) Is the system causal?

a) $y[n] = x[n] * h[n]$

$= [u[n] - u[n] * \delta(n-(N+1))] * [u[n] * \delta[n+1] - u[n] * \delta[n-4]]$

$= u[n] * u[n] * \delta[n+1] - u[n] * u[n] * \delta[n-4]$

$- u[n] * \delta(n-(N+1)) * u[n] * \delta[n+1] + u[n] * \delta[n-(N+1)] * u[n] * \delta[n-4]$

$= r[n+1] * \delta(n+1] - r[n+1] * \delta[n-4] - r[n+1] * \delta[n+1-N-1]$

$+ r[n+1] * \delta[n-4-N-1]$

$= r[n+2] - r[n-3] - r[n-N+1] + r[n-N-5+1]$

$= r[n+2] - r[n-3] - r[n-N+1] + r[n-N-4]$

$y[n] = \underbrace{r[n+2] - r[n-3]} - \underbrace{r[n-(N-1)] + r[n-(N+4)]}$

b) $u[n] - u[n-5]$

length 4

$h[n] = u[n+1] - u[n-4]$

length 5

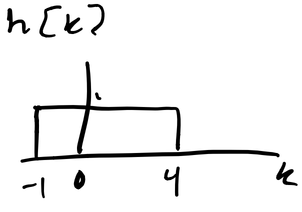
$y[n]$ has length 8. Convolution slides and multiplies one function over the other. The length of this result is $4+5-1 = 8$

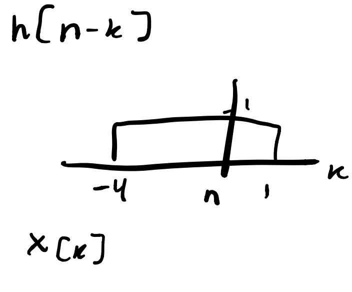
Problem 1 extra page

c) $\lim_{n\to\infty} y[n] =$ constant + constant ∴ y[n] is BIBO

also y[n] has fixed length of non zero values so stable

d) Not causal when $N < 1$





Problem 2 (20 pts)

A discrete-time system is represented by a difference equation

$$y[n] = \frac{1}{3}\left(x[2n+1] + x[n/3] + x[n-1]\right)$$

a) (10 pts) The input of the system is generated by sampling an analog signal $x(t) = 2cos(t)$ using two different sampling periods $T_1 = 1/6$ seconds and $T_2 = \pi/6$ seconds. If we want the discrete-time signal $x[n]$ to be periodic, which of the two sampling periods would you use? For the chosen sampling period what would be the fundamental period of $x[n]$?

b) (10 pts) If $x[n]$ is periodic, would the output of the system also be periodic. What would be the fundamental period?

a) $x(t) = 2\cos(t)$ $\quad w_0 = 2\pi f_0 = 1 \quad f_0 = \frac{1}{2\pi}$

$T_1 = 6\,Hz \quad T_2 = \frac{6}{\pi}\,hz$

for T_2 $\quad F_0 = \frac{f_0}{f_s} = \frac{1}{2\pi} \div \frac{6}{\pi} = \frac{1}{2\cancel{\pi}} \cdot \frac{\cancel{\pi}}{6} = \frac{1}{12}$

$$\boxed{\begin{array}{l}\text{Use } \quad T_2 = \pi/6 \text{ seconds} \\ \text{Fundamental Period} \quad N = 12\end{array}}$$

b) $y[n] = \frac{1}{3}\left(x[2n+1] + x[n/3] + x[n-1]\right)$

$y[n+N_1] = \frac{1}{3}\left(x[2(n+N_1)+1] + x\left[\frac{n}{3} + \frac{N_1}{3}\right] + x[n+N_1-1]\right)$

$= \frac{1}{3}\left(x[2n + \underbrace{2N_1+1}_{12}] + x\left[\frac{n}{3} + \underbrace{\frac{N_1}{3}}_{12}\right] + x[n + \underbrace{N_1-1}_{12}]\right)$

$2N_1 + 1 = 12$

$\frac{2N_1 = 11}{2}$

$N_1 = 5.5$

not a whole number

$\frac{N_1}{3} = 12$

$N_1 = 36$

$N_1 - 1 = 12$

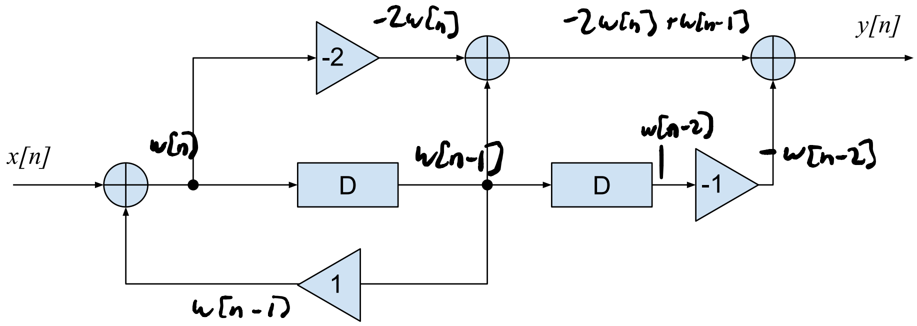
$N_1 = 13$

Problem 2 extra page

The output of the system is not periodic

Problem 3 (10 pts)

Consider the following block diagram representation of an LTI system. Derive the input-output equation.



$$y[n] = -2w[n] + w[n-1] - w[n-2]$$

$$w[n] = x[n] + w[n-1]$$

$$x[n] = w[n] - w[n-1]$$

$$\boxed{y[n] - y[n-1] = -2x[n] + x[n-1] - x[n-2]}$$

Problem 3 extra page

Problem 4 (20 pts)

Assume x[n] has nonzero samples only in the interval $-N_1 \leq n \leq N_2$. Generally, over what interval of time will the following sequences have non-zero samples:

a) (10 pts) $r[n] = x[n] * x[-2n]$

b) (10 pts) $y[n] = x[n] * x[n+1]$

a) $r[n] = x[n] * x[-2n]$

$x[-2n]$ has nonzero samples for $-N_1 \leq -2n \leq N_2$

$$\frac{N_1}{2} \geq n \geq \frac{N_2}{-2}$$

$$\frac{N_2}{-2} \leq n \leq \frac{N_1}{2}$$

$r[n]$ has nonzero samples for

$$-N_1 - \frac{N_2}{2} \leq n \leq N_2 + \frac{N_1}{2}$$

or

$$\boxed{-\left(N_1 + \frac{N_2}{2}\right) \leq n \leq N_2 + \frac{N_1}{2}}$$

b) $y[n] = x[n] * x[n+1]$

$x[n+1]$ has nonzero samples for $-N_1 \leq n+1 \leq N_2$

$$-N_1 - 1 \leq n \leq N_2 - 1$$

$y[n]$ has nonzero samples for

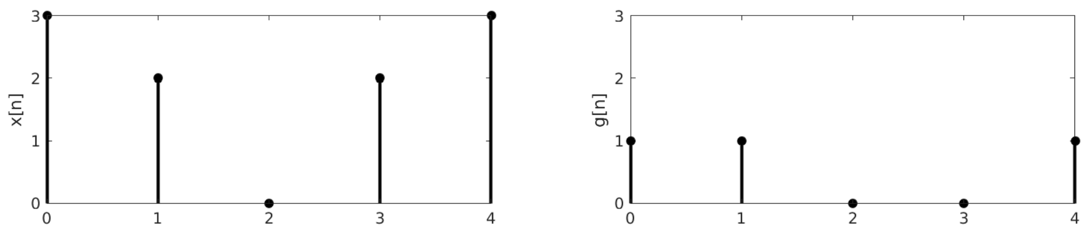
$$-N_1 - N_1 - 1 \leq n \leq N_2 + N_2 - 1 \quad \boxed{-(2N_1+1) \leq n \leq 2N_2 - 1}$$

Problem 4 extra page

Problem 5 (20 pts)

In this problem, you will use the properties of periodic convolution to calculate the DTFS coefficients of the signal.

a) (10 pts) Find the DTFS coefficients of the periodic signals $x[n]$ and $g[n]$ signals shown below. Only one period is shown for each signal.



b) (10 pts) Let $h[n]$ be a signal defined as $h[n] = x[n] \otimes g[n] \otimes x[n]$. Find the DTFS coefficients of $h[n]$.

a) $x[n] = \{ \underset{n=0}{\uparrow 3}, \underset{1}{2}, \underset{2}{0}, \underset{3}{2}, \underset{4}{3} \}$ $N=5$

$$\tilde{c}_k = \frac{1}{5} \sum_{n=0}^{4} x[n] e^{-j \frac{2\pi}{5} kn}$$

$$= \frac{1}{5} \left(3 + 2e^{-j\frac{2\pi}{5}k} + 2e^{-j\frac{6\pi}{5}k} + 3e^{-j\frac{8\pi}{5}k} \right)$$

$$\boxed{\begin{array}{ll} \tilde{c}_0 = 2 & \tilde{c}_1 = 0.5854 + j0.4253 \\ \tilde{c}_2 = -0.0854 - j0.2629 & \tilde{c}_3 = -0.0854 + j0.2629 \\ \tilde{c}_4 = 0.5854 - j0.4253 & \end{array}}$$

Problem 5 extra page

$g[n] = \{ \underset{n=0}{\overset{\uparrow}{1}}, \underset{1}{1}, \underset{2}{0}, \underset{3}{0}, \underset{4}{1} \}$ $\quad N=5$

$\tilde{d}_k = \frac{1}{5} \sum_{k=0}^{4} g[n] e^{-j\frac{2\pi}{5} k \cdot n}$

$\tilde{d}_k = \frac{1}{5} \left(1 + e^{-j\frac{2\pi}{5}k} + e^{-j\frac{8\pi}{5}k} \right)$

$$\boxed{\begin{array}{l} \tilde{d}_0 = 0.6 \quad \tilde{d}_1 = 0.3236 \quad \tilde{d}_2 = -0.1236 \\ \tilde{d}_3 = -0.1236 \quad \tilde{d}_4 = 0.3236 \end{array}}$$

b) $h[n] = x[n] \circledast g[n] \circledast x[n]$

let $r[n] = x[n] \circledast g[n]$

$\downarrow$ DTFS $\quad \downarrow \quad \downarrow$

$\tilde{e}_k \quad \tilde{c}_k \quad \tilde{d}_k$

$\tilde{e}_k = N \cdot \tilde{c}_k \cdot \tilde{d}_k = 5 \cdot \tilde{c}_k \cdot \tilde{d}_k$

$h[n] = r[n] \circledast x[n]$

$\downarrow \quad \downarrow$

$5 \cdot \tilde{c}_k \cdot \tilde{d}_k \quad \tilde{c}_k$

$h[n] \xrightarrow{DTFS} 5(5 \cdot \tilde{c}_k \tilde{d}_k \cdot \tilde{c}_k) = 25(\tilde{c}_k)^2 \tilde{d}_k = \tilde{f}_k$

$$\boxed{\begin{array}{l} \tilde{f}_0 = 60 \quad \tilde{f}_1 = 1.3090 + j\,4.0287 \quad \tilde{f}_2 = 0.1910 - j\,0.1388 \\ \tilde{f}_3 = 0.1910 + j\,0.1388 \quad \tilde{f}_4 = 1.3090 - j\,4.0287 \end{array}}$$

Problem 6 (10 pts)

Let $x[n] = 1 + e^{j\omega_0 n}$ and $y[n] = 1 + \frac{1}{2}e^{j4\omega_0 n} + \frac{1}{2}e^{j3\omega_0 n}$ be two signals with a fundamental period N, such that $\omega_0 = 2\pi/N$.

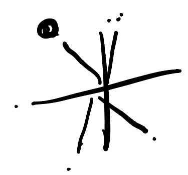
Find the DTFS coefficients of their product $z[n] = x[n]y[n]$, assuming $N = 3$.

$$x[n] = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\omega_0 n \cdot k} = \sum_{k=0}^{2} \tilde{c}_k e^{j\omega_0 n \cdot k}$$

$\tilde{c}_0 = 1 \quad \tilde{c}_1 = 1 \quad \tilde{c}_2 = 0$

$$y[n] = \sum_{k=0}^{2} \tilde{d}_k e^{j\omega_0 n \cdot k}$$

$4 \cdot \frac{2\pi}{3}$ $\overset{\shortparallel}{}$ $1 \cdot \frac{2\pi}{3}$



$\tilde{d}_0 = 1 \quad \tilde{d}_1 = \frac{1}{2} \quad \tilde{d}_2 = \frac{1}{2}$

let $z[n] \xrightarrow{DTFS} \tilde{e}_k$

$z[n]$ has period $N=3$

$z[n+N] = x[n+N]\,y[n+N]$

$= x[n]\,y[n] = z[n]$

$$\tilde{e}_k = \frac{1}{3}\sum_{n=0}^{2} z[n]\, e^{j\omega_0 n k}$$

$$= \frac{1}{3}\left(z[0] + z[1]\, e^{j\omega_0 k} + z[2]\, e^{j\omega_0 2k}\right)$$

$\omega_0 = \frac{2\pi}{3}$

$$= \frac{1}{3}\left(x[0]y[0] + x[1]y[1]\, e^{j\omega_0 k} + x[2]y[2]\, e^{j\omega_0 2k}\right)$$

$x[0]y[0] = (1+1)\cdot(1+\frac{1}{2}+\frac{1}{2})$

$2 \cdot 2 = 4$

$$\boxed{\tilde{e}_0 = 1.5 \qquad \tilde{e}_1 = 0.5 \qquad \tilde{e}_2 = 2}$$

using matlab

Problem 6 extra page