UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE113: DIGITAL SIGNAL PROCESSING

Midterm 1 Exam

Date: November 2, 2020, Duration: 1 hour 50 minutes

INSTRUCTIONS:

- The exam has 6 problems
- The exam is open-book and open notes.
- Calculator is allowed.
- Please submit all your work as a single PDF file on CCLE.

Your name:____

Student ID:____

Table 1: Score Table

Problem	a	b	c	d	Total	Score
1	5	5	5	5	20	
2	10	10			20	
3	10				10	
4	10	10			20	
5	10	10			20	
6	10				10	
Sum					100	

Problem 1 (20 pts)

An LTI discrete-time system has an impulse response $h[n] = u[n+1] - u[n-4]$, and as input the signal $x[n] = u[n] - u[n-(N+1)]$ for a positive integer N. The output of the system is denoted as $y[n]$.

a) (5 pts) Derive input output relationship in the form of difference equation.

b) (5 pts) If $N = 4$, without calculating $y[n]$, what is the length of the output $y[n]$? Explain your answer.

c) (5 pts) Is the system stable?

d) (5 pts) Is the system causal?

Solution:

a) The output of the system is defined as

$$y[n] = h[n] * x[n]$$

$$y[n] = (\delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]) * x[n]$$
$$y[n] = x[n+1] + x[n] + x[n-1] + x[n-2] + x[n-3]$$

b) If two signals $x[n]$ and $h[n]$ are such that $x[n]$ only has non-zero samples in the range $N_{x1} \leq n \leq N_{x2}$ and $h[n]$ has non-zero samples only in the range $N_{h1} \leq n \leq N_{h2}$, then their convolution $y[n] = x[n] * h[n]$ can only have non-zero samples in the range $N_{h1} + N_{x1} \leq n \leq N_{h2} + N_{x2}$.

$$N_{h1} = -139;\ N_{h2} = 3;\ N_{x1} = 0;\ N_{x2} = 4$$

c) The system is stable since

$$\sum_{n=-\infty}^{\infty} |h[n]| = 5 \leq \infty$$

d) The system is not causal since $h[n]$ is not zero everywhere for $n < 0$.

Problem 2 (20 pts)

A discrete-time system is represented by a difference equation

$$y[n] = \frac{1}{3}\left(x[2n+1] + x[n/3] + x[n-1]\right)$$

a) (10 pts) The input of the system is generated by sampling an analog signal $x(t) = 2cos(t)$ using two different sampling periods $T_1 = 1/6$ seconds and $T_2 = \pi/6$ seconds. If we want the discrete-time signal $x[n]$ to be periodic, which of the two sampling periods would you use? For the chosen sampling period what would be the fundamental period of $x[n]$?

b) (10 pts) If $x[n]$ is periodic, would the output of the system also be periodic. What would be the fundamental period?

Solution:

a) The period of the original signal is $\bar{T} = 2\pi$. The sampling period T_s needs to be such that $\frac{\bar{T}}{T_s}$ is a rational number. That will be satisfied if $T_s = T_2$.

   We know that N_1 must be an integer and that $N_1 = k\frac{\bar{T}}{T_s}$, and k is the lowest integer factor such that N_1 is an integer.

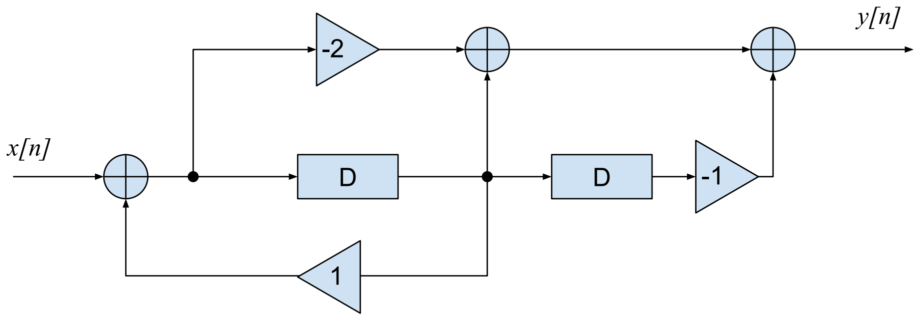
$$N_1 = k\frac{\bar{T}}{T_s} = k\frac{2\pi}{\pi/6} = k\frac{12}{1}$$

   We notice that k is 1, and therefore $N_1 = 12$.

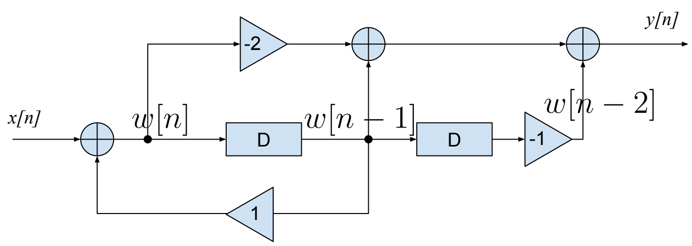
b) The period is the LCM of $x[2n+1]$, $x[n/3]$ and $x[n-1]$. The period of $x[2n+1]$ is 6. The period of $x[n/3]$ is 36. The period of $x[n-1]$ is 12. Therefore, the period of $y[n]$ is $LCM(6, 12, 36) = 36$.

Problem 3 (10 pts)

Consider the following block diagram representation of an LTI system. Derive the input-output equation.



Solution:



From the figure above

$$w[n] = x[n] + w[n-1]$$

Therefore,

$$x[n] = w[n] - w[n-1] \tag{1}$$

Furthermore,

$$y[n] = -2w[n] + w[n-1] - w[n-2] \tag{2}$$

If we can write $y[n]$ as $y[n] = b_0 w[n] + b_1 w[n-1] + b_2 w[n-2]$ and $x[n]$ as $x[n] = a_0 w[n] + a_1 w[n-1]$, then the difference equation of the system can be written as:

$$a_0 y[n] + a_1 y[n-1] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

For this system using equations (1) and (2)

$$a_0 = 1; \; a_1 = -1$$

$$b_0 = -2; \; b_1 = 1; \; b_2 = -1$$

Therefore, the equation of this system is:

$$y[n] - y[n-1] = -2x[n] + x[n-1] - x[n-2].$$

Problem 4 (20 pts)

Assume x[n] has nonzero samples only in the interval $-N_1 \leq n \leq N_2$. Generally, over what interval of time will the following sequences have non-zero samples:

a) (10 pts) $r[n] = x[n] * x[-2n]$

b) (10 pts) $y[n] = x[n] * x[n+1]$

Solution:

If two signals $x[n]$ and $h[n]$ are such that $x[n]$ only has non-zero samples in the range $N_{x1} \leq n \leq N_{x2}$ and $h[n]$ has non-zero samples only in the range $N_{h1} \leq n \leq N_{h2}$, then their convolution $y[n] = x[n] * h[n]$ can only have non-zero samples in the range $N_{h1} + N_{x1} \leq n \leq N_{h2} + N_{x2}$.

a) $x[-2n]$ will have non-zero samples in the interval $-\lfloor N_2/2 \rfloor \leq n \leq \lfloor N_1/2 \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function.

   Therefore, $r[n]$ has non-zero samples in the interval $-\lfloor N_2/2 \rfloor - N_1 \leq n \leq \lfloor N_1/2 \rfloor + N_2$.

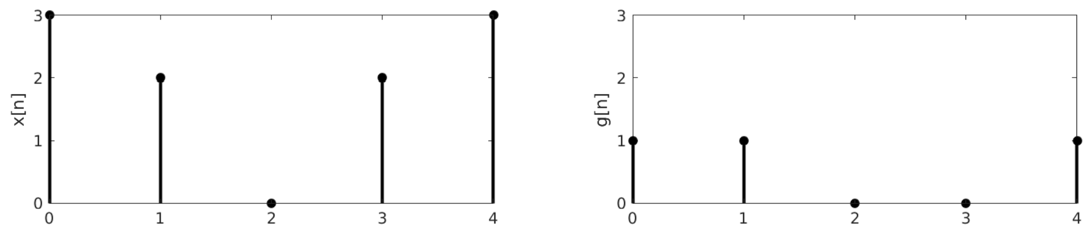
b) $x[n+1]$ has non-zeros samples in the interval $-N_1 - 1 \leq n \leq N_2 - 1$.

   Therefore, $y[n]$ has non-zero samples in the interval $-2N_1 - 1 \leq n \leq 2N_2 - 1$.

Problem 5 (20 pts)

In this problem, you will use the properties of periodic convolution to calculate the DTFS coefficients of the signal.

a) (10 pts) Find the DTFS coefficients of the periodic signals $x[n]$ and $g[n]$ signals shown below. Only one period is shown for each signal.



b) (10 pts) Let $h[n]$ be a signal defined as $h[n] = x[n] \otimes g[n] \otimes x[n]$. Find the DTFS coefficients of $h[n]$.

Solution: If the DTFS coefficients $x[n]$ and $g[n]$ are $\tilde{c}_k$ and $\tilde{d}_k$ respectively, the DTFS coefficients of $h[n]$ are

$$\tilde{h}[n] \overset{\text{DTFS}}{\longleftrightarrow} 25\tilde{c}_k\tilde{d}_k\tilde{c}_k$$

k	$\tilde{d}_k$	$\tilde{c}_k$	Coefficients of $h[n]$
0	0.6	2	60
1	0.326	0.5854 + 0.4253i	1.3090 + 4.0287i
2	-0.1236	-0.0854 - 0.2629i	0.1910 - 0.1388i
3	-0.1236	-0.0854 + 0.2629i	0.1910 + 0.1388i
4	0.3236	0.5854 - 0.4253i	1.3090 - 4.0287i

Problem 6 (10 pts)

Let $x[n] = 1 + e^{j\omega_0 n}$ and $y[n] = 1 + \frac{1}{2}e^{j4\omega_0 n} + \frac{1}{2}e^{j3\omega_0 n}$ be two signals with a fundamental period N, such that $\omega_0 = 2\pi/N$.

Find the DTFS coefficients of their product $z[n] = x[n]y[n]$, assuming $N = 3$.

Solution:

$$z[n] = 1 + \frac{1}{2}e^{j4\omega_0 n} + \frac{1}{2}e^{j3\omega_0 n} + e^{j\omega_0 n} + \frac{1}{2}e^{j5\omega_0 n} + \frac{1}{2}e^{j4\omega_0 n}$$

$$z[n] = 1 + e^{j\omega_0 n} + \frac{1}{2}e^{j3\omega_0 n} + e^{j4\omega_0 n} + \frac{1}{2}e^{j5\omega_0 n}$$

The period of $z[n]$ is $N = 3$. We can then rewrite $z[n]$ as

$$z[n] = 1 + e^{j2\pi n/3} + \frac{1}{2}e^{j2\pi 3n/3} + e^{j2\pi 4n/3} + \frac{1}{2}e^{j2\pi 5n/3}$$

$$z[n] = 1 + e^{j2\pi n/3} + \frac{1}{2}e^{j2\pi 0n/3} + e^{j2\pi n/3} + \frac{1}{2}e^{j2\pi 2n/3}$$

$$z[n] = \frac{3}{2} + 2e^{j2\pi n/3} + \frac{1}{2}e^{j2\pi 2n/3}$$

The DTFS basis functions for the signal with period $N = 3$ are $e^{j\frac{2\pi}{3}kn}$, for $k = 0, ..., 2$. We can then conclude that :

$$\tilde{c}_0 = 3/2$$

$$\tilde{c}_1 = 2$$

$$\tilde{c}_2 = 1/2$$