UCLA DEPARTMENT OF ELECTRICAL ENGINEERING

EE113: DIGITAL SIGNAL PROCESSING

Midterm 1 Exam Solution
Date: October 28, 2019, Duration: 1 hour

INSTRUCTIONS:

- The exam has 3 problems and 6 printed pages.
- The exam is closed-book.
- One double-sided cheat sheets of A4 size is allowed.
- Calculator is allowed.

Your name:________________________________

Student ID:________________________________

Table 1: Score Table

Problem	a	b	c	d	e	Total	Score
1	3	3	3	3	3	15	
2	5	5	5			15	
3	10					10	
Sum						40	

Problem 1 (15 pts)

Consider the following system: $y[n] = 0.5x[3n] + 0.5x[2n-1]$

(a) (3pts) Is the system linear? Prove your answer.

(b) (3pts) Is the system time-invariant? Prove your answer.

(c) (3pts) Is the system causal? Prove your answer.

(d) (3pts) Is the system BIBO stable? Prove your answer.

(e) (3pts) Suppose that $x[n]$ is a periodic signal with a fundamental period of $N_x = 110$, what will be the fundamental period of $y[n]$, N_y?

Solution:

(a) The system is linear. (1 pt)

$$Sys\left\{\alpha_1 x_1[n] + \alpha_2 x_2[n]\right\} = 0.5\alpha_1 x_1[3n] + 0.5\alpha_2 x_2[3n] + 0.5\alpha_1 x_1[2n-1] + 0.5\alpha_2 x_2[2n-1]$$

$$=\alpha_1 \text{Sys}\{x_1[n]\} + \alpha_2 \text{Sys}\{x_2[n]\} \quad \text{(2 pts)}$$

(b) The system is time-varying. (1 pt)

$$Sys\left\{x[n-k]\right\} = 0.5x[3n-k] + 0.5x[2n-k-1] \text{ (1 pt)}$$

$$y[n-k] = 0.5x[3n-3k] + 0.5x[2n-2k-1] \text{ (1 pt)}$$

(c) The system is not causal. (1 pt)

For example, to obtain $y[n]$ at $n = 1$, we need the values of $x[n]$ at $n > 1$. Therefore, the system is not causal.

$$y[1] = 0.5x[3] + 0.5x[1] \text{ (2 pts)}$$

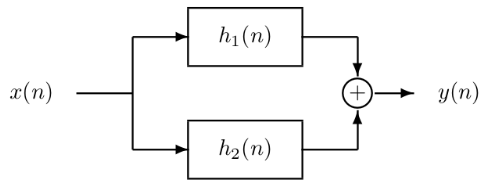
(d) The system is BIBO stable. (1 pt)

$$|x[n]| \leq B_x \ \forall n \Rightarrow |y[n]| \leq B_x \ \forall n \text{ (2 pts)}$$

(e) Let $y_1[n] = 0.5x[3n]$ and $y_2[n] = 0.5x[2n-1]$.
The period of $y_2[n]$ is $N_{y_2} = 110$. (1 pt)
The period of $y_2[n]$ is $N_{y_2} = 55$. (1 pt)
N_y is the least common multiple of N_{y_1} and N_{y_2}, therefore $N_y = 110$. (1 pt)
Full marks given for only stating the correct answer also.

Problem 2 (15 pts)

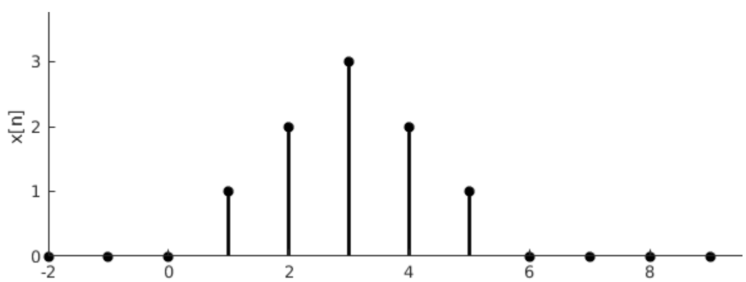
Consider the system composed of parallel connection of two LTI systems.



(a) (5 points) If unit-step response of the equivalent system (the response when the input is a unit-step function) is $y[n] = r[n+1] - r[n-1]$ and $h_1[n] = u[n] - 2u[n-1] + u[n-2]$, find and sketch $h_2[n]$.

(b) (5 points) Find the equivalent impulse response of the system $h_{eq}[n]$. The equivalent response is defined by the following relation: $y[n] = x[n] * h_{eq}[n]$.

(c) (5 points) Find the response of the system $y[n]$ for $x[n]$ shown in the figure below.



Solution:

(a) The system response to $u[n]$ is

$$y[n] = u[n] * (h_1[n] + h_2[n]) = u[n] * h_1[n] + u[n] * h_2[n] \text{ (1 pt)}$$

Furthermore,

$$\begin{aligned} u[n] * h_1[n] &= u[n] * (u[n] - 2u[n-1] + u[n-2]) \\ &= u[n] * u[n] - 2u[n-1] * u[n] + u[n-2] * u[n] \\ &= r[n+1] - 2r[n] + r[n-1] \text{ (2 pts)} \end{aligned}$$

Where we have used the fact that $u[n] * u[n] = r[n+1]$. Now we can find $h_2[n]$,

$$u[n] * h_2[n] = y[n] - u[n] * h_1[n]$$

$$u[n] * h_2[n] = r[n+1] - r[n-1] - r[n+1] + 2r[n] - r[n-1]$$

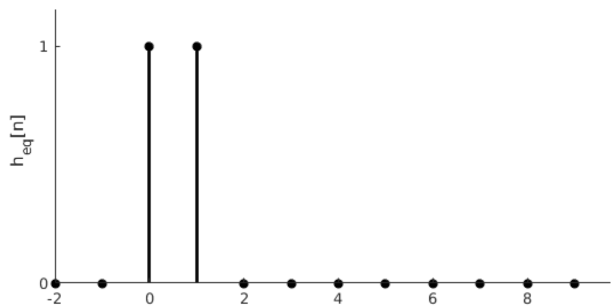
$$u[n] * h_2[n] = 2r[n] - 2r[n-1]$$

$$u[n] * h_2[n] = u[n] * 2u[n-1] - u[n] * 2u[n-2]$$

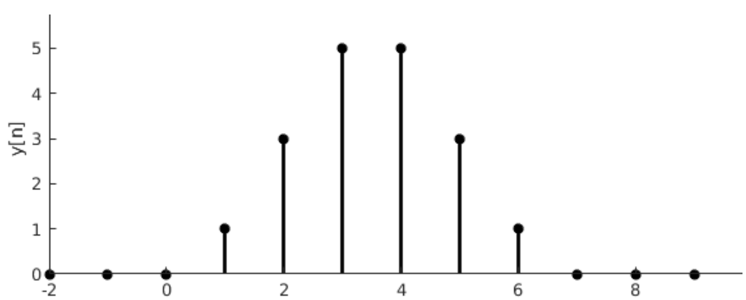
$$u[n] * h_2[n] = u[n] * (2u[n-1] - 2u[n-2])$$

Hence, $h_2[n] = 2u[n-1] - 2u[n-2] = 2\delta[n-1]$. (1 pt)
Sketch (1 pt)

(b) Since this is an addition of the outputs of two parallel systems, $h_{eq} = h_1[n] + h_2[n] = u[n] - 2u[n-1] + u[n-2] + 2u[n-1] - 2u[n-2] = u[n] - u[n-2]$. (Points awarded even if incorrect $h_2[n]$ is used.) 2 points awarded for the equation. 3 points for the correct answer.



(c) The response of the system is shown. Points awarded even if incorrect $h_{eq}[n]$ is used.



Problem 3 (10 pts)

Let $\tilde{x}[n]$ be a periodic signal with period N. Its DTFS representation is given by

$$\tilde{x}[n] = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn},$$

where $\tilde{c}_k$ are the DTFS coefficients.

Show that if $\tilde{x}[n]$ is a complex signal and conjugate symmetric ($\tilde{x}^*[n] = \tilde{x}[-n]$), then $\text{Im}\{\tilde{c}_k\} = 0$.

Solution 1:

$$\tilde{c}_k = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

We take the conjugate of both sides of the expression above

$$\tilde{c}_k^* = \sum_{n=0}^{N-1} x^*[n]e^{j\frac{2\pi}{N}kn} \text{ (2 pts)}$$

Using the fact that $\tilde{x}^*[n] = \tilde{x}[-n]$

$$\tilde{c}_k^* = \sum_{n=0}^{N-1} x[-n]e^{j\frac{2\pi}{N}kn} \text{ (2 pts)}$$

We make the substitution $m = -n$

$$\tilde{c}_k^* = \sum_{m=1-N}^{0} x[m]e^{-j\frac{2\pi}{N}km} \text{ (2 pts)}$$

We can freely shift the starting index of the summation above without changing its value and we shift it by $N-1$ to the right

$$\tilde{c}_k^* = \sum_{m=0}^{N-1} x[m]e^{-j\frac{2\pi}{N}km} \text{ (2 pts)}$$

The expressions for $\tilde{c}_k^*$ and $\tilde{c}_k$ are identical, therefore $c_k^* = c_k$. This implies that $\text{Im}\{\tilde{c}_k\} = 0$. (2 pts)

Solution 2:

$$\tilde{x}[n] = \sum_{k=0}^{N-1} \tilde{c}_k e^{j\frac{2\pi}{N}kn}$$

$$\tilde{x}^*[n] = \sum_{k=0}^{N-1} \tilde{c}_k^* e^{-j\frac{2\pi}{N}kn}$$

$$\tilde{x}[-n] = \sum_{k=0}^{N-1} \tilde{c}_k e^{-j\frac{2\pi}{N}kn}$$

$$\tilde{x}[-n] - \tilde{x}^*[n] = \sum_{k=0}^{N-1} (\tilde{c}_k - \tilde{c}_k^*) e^{-j\frac{2\pi}{N}kn} = 0 \text{ (4 pts)}$$

$$2\sum_{k=0}^{N-1} \text{Im}\{\tilde{c}_k\} e^{-j\frac{2\pi}{N}kn} = 0 \text{ (3 pts)}$$

This implies that either $\text{Im}\{\tilde{c}_k\}$ is 0 or orthogonal to $e^{-j\frac{2\pi}{N}kn}$. However, since $e^{-j\frac{2\pi}{N}kn}$ is an orthogonal basis, its null space is $\{\emptyset\}$. therefore $\text{Im}\{\tilde{c}_k\} = 0$. (3 pts)