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UCLA
Dept. of Electrical Engineering
EE113
Quiz 1, Fall 2012

This quiz consists of three problems. Please justify your answers clearly; a correct answer with no justification will not receive credit. Please write your name clearly on each page. Good Luck!

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1. **The various parts of this question are independent of one another.**

   (A) **Sampling** Consider the discrete-time sequence:
   $x(n) = cos(n\pi/8)$
   Find two different continuous-time signals that would produce this sequence if the sampling frequency (F_s) is 20 kHz.

   (B) **Eignefunctions and LTI systems** Can the function $5^n e^{jwn}$ be the eigenfunction of an LTI system? Why or why not?

$2500 \times 17 = 17500 + 25000 = 42500$

A) $\Omega = \frac{W}{T} = wF$ $\quad W = \frac{\pi}{8}, \; F = 20\,kHz$

$= (20\times10^3)\frac{\pi}{8}$

$= 2500\pi$

$\frac{\pi}{8} + 2\pi = \pi\left(\frac{1+16}{8}\right) = \pi\left(\frac{17}{8}\right)$

or

$= (20\times10^3)\frac{17\pi}{8}$

$= 42500\pi$

$\boxed{x(t) = \cos(2500\pi t) \quad x(t) = \cos(42500\pi t)}$

B) Yes, it can

$y(n) = \sum_{k=\infty}^{n} 5^{n-k} e^{jw(n-k)} h(n)$

$= \sum_{k=-\infty}^{n} 5^n 5^k e^{jwn} e^{jwk} h(n)$

$= 5^n e^{jwn} X(z^1)$

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2. For the following system, determine whether or not the system is causal, linear, time invariant, and memoryless:

$y(n) = \sum_{k=n-n_0}^{n+n_0} x(k)$

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$$S[ax_1(n) + bx_2(n)] = \sum_{k=n-n_0}^{n+n_0} (ax_1(n) + bx_2(n))$$

$$= \sum_{k=n-n_0}^{n+n_0} ax_1(n) + \sum_{k=n-n_0}^{n+n_0} bx_2(n)$$

Linear

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Not causal because output depends on future inputs

$$\text{ex: } y(1) = \sum_{k=1-n_0}^{1+n_0} x(k)$$

$$= x(1-n_0) + x(1-n_0+1) \dots + x(1+n_0-1) + x(1+n_0)$$

future inputs

10

$$S[x(n-l)] = \sum_{k=n-n_0}^{n+n_0} x(k-l) \qquad y(n-l) = \sum_{k=n-l-n_0}^{n-l+n_0} x(k)$$

$$= \sum_{m+l=n-n_0}^{n+n_0} x(m) \qquad m = k-l, \quad k = m+l$$

$$= \sum_{m=n-l+n_0}^{n-l+n_0} x(m)$$

T.I.

10

Not memoryless, because output depends on past inputs

3. Consider the LCCDE representing a causal LTI system:

$y(n) + (1/a)y(n-1) = x(n-1)$ $\qquad y(n) = \frac{-1}{a}y(n-1) + x(n-1)$

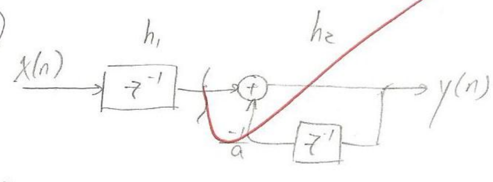
24.

(A) Draw a block diagram of the equation.

(B) Find the impulse response of the system, $h(n)$.

(C) For what range of a is the system BIBO?

A)



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B) $h(n) = h_1(n) * h_2(n)$

$h_1(n) = \delta(n-1)$

$h_2(n) = \left(\frac{-1}{a}\right)^n u(n)$

$h(n) = h_1(n) * h_2(n) = \sum_{k=0}^{n} \delta(n-1)\left(\frac{-1}{a}\right)^n$

$= \left(\frac{-1}{a}\right)^{n-1} u(n-1)$

$\boxed{h(n) = \left(\frac{-1}{a}\right)^{n-1} u(n-1)}$

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$h(-1) = 0$ since LTI

$h(n) + \frac{1}{a}h(n-1) = \delta(n-1)$

$h(0) + \frac{1}{a}h(-1) = \delta(-1)$

$h(0) = 0$

$h(1) + \frac{1}{a}h(0) = \delta(0)$

$h(1) = 1$

$h(2) + \frac{1}{a}h(1) = \delta(1)$

$h(2) = \frac{-1}{a}$

$h(3) + \frac{1}{a}h(2) = \delta(2)$

$h(3) = \frac{1}{a^2}$

$h(4) + \frac{1}{a}h(3) = \delta(3)$

$h(4) = \frac{-1}{a^3}$

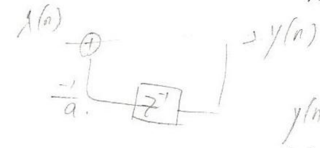
$h(5) + \frac{1}{a}h(4) = \delta(4)$

$h(5) = \frac{1}{a^4}$

$h(n) = \left(\frac{-1}{a}\right)^{n-1} u(n-1)$

c) $\boxed{|a| \geq 1}$

4.



$y(n) = \frac{-1}{a}y(n-1) + x(n)$

$h(n) = \frac{-1}{a}h(n-1) + \delta(n)$

$h(0) = \frac{-1}{a}h(-1) + \delta(0) = 1$

$h(1) = \frac{-1}{a}h(0) + \delta(1) = \frac{-1}{a}$

$h(2) = \frac{-1}{a}h(1) = \frac{1}{a^2}$

$h(3) = \frac{-1}{a}h(2) = \frac{-1}{a^3}$